4724 Core Mathematics 4

1	Attempt to factorise numerator and denominator		M1	$\frac{A}{\mathrm{f}(x)}+\frac{B}{\mathrm{g}(x)}$;fg= $6x^2-24x$
	Any (part) factorisation of both num and denom		A1	Corres identity/cover-up
	Final answer = $-\frac{5}{6x}, \frac{-5}{6x}, \frac{5}{-6x}, -\frac{5}{6}x^{-1}$ Not $-\frac{\frac{5}{6}}{x}$		A1	
			$\boxed{3}$	
2	Use parts with $u=x, \mathrm{d}v=\sec^2 x$		M1	result $\mathrm{f}(x)+/-\int \mathrm{g}(x)\,\mathrm{d}x$
	Obtain correct result $x\tan x-\int \tan x\,\mathrm{d}x$		A1	
	$\int \tan x\,\mathrm{d}x = k\ln\sec x$ or $k\ln\cos x$, where $k=1$ or -1		B1	or $k\ln\lvert\sec x\rvert$ or $k\ln\lvert\cos x\rvert$
	Final answer = $x\tan x-\ln\lvert\sec x\rvert+c$ or $x\tan x+\ln\lvert\cos x\rvert+c$		A1	
			$\boxed{4}$	
3 (i)	$1+\frac{1}{2}.2x+\frac{\frac{1}{2}.-\frac{1}{2}}{2}\left(4x^2 \text{ or } 2x^2\right)+\frac{\frac{1}{2}.-\frac{1}{2}.-\frac{3}{2}}{6}\left(8x^3 \text{ or } 2x^3\right)$		M1	
	$=1+x$		B1	
	$\ldots -\frac{1}{2}x^2+\frac{1}{2}x^3$ (AE fract coeffs)		A1 (**3**)	For both terms
(ii)	$(1+x)^{-3}=1-3x+6x^2-10x^3$		B1	or $(1+x)^3=1+3x+3x^2+x^3$
	Either attempt at their **(i)** multiplied by $(1+x)^{-3}$		M1	or **(i)** long div by $(1+x)^3$
	$1-2x$ ….	$\surd 1+(a-3)x$	A1	f.t. **(i)** = $1+ax+bx^2+cx^3$
	… + $\frac{5}{2}x^2$....	$\surd\ (-3a+b+6)x^2$	A1	
	… $-2x^3$	$\surd\,(6a-3b+c-10)x^3$	A1 (**5**)	(AE fract.coeffs)
(iii)	$-\frac{1}{2}<x<\frac{1}{2}$, or $\lvert x\rvert<\frac{1}{2}$		B1 (**1**)	
			$\boxed{9}$	

4		Attempt to expand $(1+\sin x)^2$ and integrate it	*M1	Minimum of $1+\sin^2 x$
		Attempt to change $\sin^2 x$ into $\mathrm{f}(\cos 2x)$	M1	
		Use $\sin^2 x = \frac{1}{2}(1-\cos 2x)$	A1	dep M1 + M1
		Use $\int \cos 2x \, dx = \frac{1}{2}\sin 2x$	A1	dep M1 + M1
		Use limits correctly on an attempt at integration	dep* M1	Tolerate g $(\frac{1}{4}\pi) - 0$
		$\frac{3}{8}\pi - \sqrt{2} + \frac{7}{4}$ AE(3-term)F	A1	WW 1.51… $\rightarrow$ M1 A0
			$\boxed{\mathbf{6}}$	
5	**(i)**	Attempt to connect du and dx, find $\frac{du}{dx}$ or $\frac{dx}{du}$	M1	But not e.g. du = dx
		Any correct relationship, however used, such as dx = 2u du	A1	or $\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$
		Subst with clear reduction (≥ 1 inter step) to **AG**	A1 (**3**)	WWW
	(ii)	Attempt partial fractions	M1	
		$\frac{2}{u} - \frac{2}{1+u}$	A1	
		√ $A \ln u + B \ln (1+u)$	√A1	Based on $\frac{A}{u} + \frac{B}{1+u}$
		Attempt integ, change limits & use on f(u)	M1	or re-subst & use 1 & 9
		$\ln \frac{9}{4}$ AEexactF (e.g. 2 ln 3 –2 ln 4 + 2 ln 2)	A1 (**5**)	Not involving ln 1
			$\boxed{\mathbf{8}}$	

6	**(i)**	Solve 0 = $t-3$ & subst into $x=t^2-6t+4$	M1	
		Obtain $x=-5$	A1 (**2**)	(–5,0) need not be quoted
		N.B. If **(ii)** completed first, subst $y=0$ into their cartesian eqn (M1) & find x (no f.t.) (A1)		

	(ii)	Attempt to eliminate t	M1	
		Simplify to $x=y^2-5$ ISW	A1 (**2**)	

	(iii)	Attempt to find $\frac{dy}{dx}$ or $\frac{dx}{dy}$ from cartes or para form	M1	Award anywhere in Que
		Obtain $\frac{dy}{dx} = \frac{1}{2t-6}$ or $\frac{1}{2y}$ or (-) $\frac{1}{2}(x+5)^{-\frac{1}{2}}$	A1	
		If t = 2, $x=-4$ and $y=-1$	B1	Awarded anywhere in **(iii)**
		Using their num (x, y) & their num $\frac{dy}{dx}$, find tgt eqn	M1	
		$x+2y+6=0$ AEF(without fractions) ISW	A1 (**5**)	
			$\boxed{\mathbf{9}}$	

7	**(i)**	Attempt direction vector between the 2 given points	M1	
		State eqn of line using format (**r**) = (either end) + s(dir vec)	M1	's' can be 't'
		Produce 2/3 eqns containing t and s	M1	2 different parameters
		Solve giving $t=3$, $s=-2$ or 2 or -1 or 1	A1	
		Show consistency	B1	
		Point of intersection = $(5,9,-1)$	A1 (**6**)	

	(ii)	Correct method for scalar product of 'any' 2 vectors	M1	Vectors from this question
		Correct method for magnitude of 'any' vector	M1	Vector from this question
		Use $\cos\theta = \frac{\mathbf{a.b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ for the correct 2 vectors $\begin{pmatrix}1\\4\\-2\end{pmatrix}$ & $\begin{pmatrix}2\\-1\\3\end{pmatrix}$	M1	Vects may be mults of dvs
		62.2 (62.188157…) 1.09 (1.0853881)	A1 (**4**)	
			$\boxed{\mathbf{10}}$	

8	**(i)**	$\frac{d}{dx}\left(y^3\right) = 3y^2\frac{dy}{dx}$	B1	
		Consider $\frac{d}{dx}(xy)$ as a product	M1	
		$= \quad x\frac{dy}{dx} + y$	A1	Tolerate omission of ‘6’
		$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$ ISW AEF	A1 (**4**)	
	(ii)	$x^3 = 2^4$ or 16 and $y^3 = 2^5$ or 32	*B1	
		Satisfactory conclusion	dep* B1	**AG**
		Substitute $\left(2^{\frac{4}{3}}, 2^{\frac{5}{3}}\right)$ into their $\frac{dy}{dx}$	M1	or the numerator of $\frac{dy}{dx}$
		Show or use calc to demo that num = 0, ignore denom **AG**	A1 (**4**)	
	(iii)	Substitute (a, a) into eqn of curve	M1	& attempt to state ‘a = …’
		$a = 3$ only with clear ref to $a \neq 0$	A1	
		Substitute $(3,3)$ or (their a, their a) into their $\frac{dy}{dx}$	M1	
		-1 only WWW	A1 (**4**)	from (their a,their a)
			$\boxed{\mathbf{12}}$	
9	**(i)**	$\frac{d\theta}{dt} = \ldots$	B1	
		$k(160-\theta)$	B1 (**2**)	The 2 @ ‘B1’ are indep
	(ii)	Separate variables with $(160-\theta)$ in denom; or invert	*M1	$\int \frac{1}{160-\theta}\,d\theta = \int k, \frac{1}{k}, 1\,dt$
		Indication that LHS = $\ln f(\theta)$	A1	If wrong ln, final 3@A = 0
		RHS = kt or $\frac{1}{k}t$ or t (+ c)	A1	
		Subst. $t = 0, \theta = 20$ into equation containing ‘c’	dep* M1	
		Subst $t = 5, \theta = 65$ into equation containing ‘c’ & ‘k’	dep* M1	
		$c = -\ln 140$ (-4.94) ISW	A1	
		$k = \frac{1}{5}\ln\frac{140}{95}$ (≈ 0.077 or 0.078) ISW	A1	
		Using their ‘c’ & ‘k’, subst t = 10 & evaluate θ	dep*M1	
		$\theta = 96(95.535714)$ $\left(95\frac{15}{28}\right)$	A1 (**9**)	
			$\boxed{\mathbf{11}}$	