4736 Decision Mathematics 1

1	(i)	Α	В	С	D		M1	A, B and C correct for first pass	
		614	416	1	198	(A=198)	A1	D = 198 on first pass	
		198	891	2	693	(A=693)	M1	sca at second and third passes	
		693	396	3	297		A1	Second and third passes correct	[4]
	(ii)	0					B1	0	[1]
	(iii)	To make the	he algor	ithm ter	minate		B1	So that it does not get stuck in a loop	[1]
								Total =	6

2	(i)	eg		Graph need not be simple or planar			
		•	M1	A graph with five vertices and at least three correct vertex orders			
			A1	A graph with five vertices of orders 1, 2, 2, 3, 4			
		*			[2]		
	(ii)	Semi-Eulerian	M1	Unless their graph was not connected, in which case the answer is 'neither'			
		It has <u>exactly</u> two odd nodes	A1	(Unless their graph was not connected, in which case follow this through)	[2]		
	(iii)	A tree with five vertices would only have four arcs, but this graph has six <u>Or</u> A tree must have at least two vertices of order 1	B2	Give B1 for an incomplete reason, eg 'too many arcs' or 'it has a cycle'	[2]		
	Total = 6						

ANSWERED ON INSERT

3	(i)	$ \begin{array}{rcrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$		M1 A1	Not selecting <i>CF</i> (working seen on list) Selecting correct arcs (working seen on list)	
		CD = 18 $FG = 20$ $CF = 22$ $EG = 23$ $EF = 26$	$A \longrightarrow_{C} F$	M1 A1	A spanning tree drawn Correct (minimum) spanning tree drawn	
		$\begin{array}{rcl} & - & 20 \\ \hline AC & = & 27 \\ \hline DE & = & 28 \\ \hline AD & = & 29 \\ \hline DG & = & 31 \\ \hline BE & = & 37 \end{array}$	Total weight = 100	B1	100 cao	[5]

(iii) $A - B - D - F - G - E$ stallM1 $A - B - D - F - G - E$ [2]Misses out vertex CA1Cannot continue because B, D and F have already been visited[2](iv) $B - A - C - D - F - G - E - B$ M1Tour starts $B - A - C - D - F - G - E - B$ [3]Upper bound = 148B1148 cao[3](v) B 2 9 E 6 46 A^{1} 0 D 3 25 G 7 A^{1} 0 D 3 25 G 7 29 25 G 7 56 B1Permanent labels correct C 4 27 F 5 39 B1Weight = 56B156 caoB156 cao	(ii)	Delete EG from spanning tree		Follow through from part (i) if possible	
77 + 23 + 26 = 126 Lower bound = 126M1 A1Adding two shortest arcs to MST 126 cao[3](iii) $A - B - D - F - G - E - stall$ M1 M1 $A - B - D - F - G - E$ [2](iii) $B - A - C - D - F - G - E - B$ Upper bound = 148M1 B1 $A - B - D - F - G - E$ [2](iv) $B - A - C - D - F - G - E - B$ Upper bound = 148M1 B1Tour starts $B - A - C - D - F - G - E - C - C - C - C - C - C - C - C - C$			B1	Weight of MST on reduced network	
Lower bound = 126A1126 cao[3](iii) $A - B - D - F - G - E$ - stallM1 $A - B - D - F - G - E$ [2]Misses out vertex CA1 $Cannot continue because B, D and F have already been visited[2](iv)B - A - C - D - F - G - E - BM1Tour starts B - A - C - D - F - CUpper bound = 148B1148 cao[3](v)B \boxed{2} 9E \boxed{6} 46(Accept correct working starting from G, if seen)A110D \boxed{3} 25G \boxed{7} 56A1Cacept correct, with no extrasA \boxed{1}03 25G \boxed{29 25}G \boxed{7} 56B1Permanent labels correct, with no extrasA \boxed{1}0C \boxed{4} 27F \boxed{5} 3939B1Order of labelling (correct or follow through their permanent labels)Weight = 56B1Route = A - B - D - GB1AB = 9AC = 27AG = 56B1C = 42BG = 47B = 9AC = 27AG = 54A1S17490A1A1these tone correct pairing seen or total seen (not just six weights)A1A1A2B1A1A1A2B1A2A2A3A1A2A2A2A2A3A2A4A2A4A5A5A2$					
(iii) $A - B - D - F - G - E$ stallM1 $A - B - D - F - G - E$ Misses out vertex CA1Cannot continue because B, D and F have already been visited(iv) $B - A - C - D - F - G - E - B$ M1Tour starts $B - A - C - D - F - G$ Upper bound = 148B1148 caoI3(v) B 2 9 E 6 9 B 2 9 E 6 46 M1Correct tour, starting and ending at B148 cao A 1 0 3 25 29 25 G 7 56 56 B1 A C 42 27 F 5 39 39 B1Weight = 56B1 $A - B - D - G$ caoRoute = $A - B - D - G$ B1 $A = 9$ $AC = 27$ $AG = 56$ $CG = 42$ $BG = 47$ $BC = 34$ 51 74 90 $A1$ A1Repeat AB and CG ($C - F - G$) = 51					
Misses out vertex CA1Cannot continue because B, D and F have already been visited[2](iv) $B-A-C-D-F-G-E-B$ M1Tour starts $B-A-C-D-F-$ Correct tour, starting and ending at B 148 cao[3](v) B 2 9 E 6 46 (v) B 2 9 E 6 46 M1At least three sets of temporary labels correct, with no extras(Accept correct or correct, with no extras A 1 0 3 25 29 25 G 7 56 B1 $B1$ D 3 25 29 25 G 7 56 B1 C 42 7 7 6 81 81 Order of labelling (correct or follow through their permanent labels)[4]Weight = 56 Route = $A - B - D - G$ B1 56 cao $A - B - D - G$ cao[2](vi) A, B, C and G are oddB1Identifying or using A, B, C, G (seen)A1 $AB = 9$ $AC = 27$ $AG = 56$ $CG = 42$ $BG = 47$ 74 $BC = 34$ 51 A1 $A1$ three totals correct, or explanation of how it is known that other pairings are too long					[3]
(iv) $B-A-C-D-F-G-E-B$ M1 A1Tour starts $B-A-C-D-F-$ Correct tour, starting and ending at B 148 cao(v) $B \boxed{2} 9 9$ $E \boxed{6} 46$ A1 46(Accept correct working starting from G, if seen)(Accept correct, with no extras remporary labels correct, with no extras $A \boxed{1} 0$ $D \boxed{3} 25$ $G \boxed{7} 56$ A1 56(Accept correct working starting from C, if seen) $A \boxed{1} 0$ $D \boxed{3} 25$ $G \boxed{7} 56$ A1 56(Accept correct, with no extras remporary labels all correct, with no extras $A \boxed{1} 27$ $F \boxed{5} 39$ B1Order of labelling (correct or follow through their permanent labels)Weight = 56 Route = $A \cdot B \cdot D \cdot G$ B1 56 cao B1S6 cao A $\cdot B - D \cdot G$ cao(vi) A, B, C and G are oddB1Identifying or using A, B, C, G (seen) $AB = 9$ $CG = 42$ 51 $BC = 34$ 74 90 M1 At least one correct pairing seen or total seen (not just six weights)A11 A11 A11 B1 B11 	(iii)	A-B-D-F-G-E - stall	M1	A-B-D-F-G-E	
A1 Upper bound = 148Correct tour, starting and ending at B 148 cao[3](v) $B = 2$ 9 9 9 $E = 6$ 46 46 (Accept correct working starting from G, if seen) At least three sets of temporary labels correct, with no extras Temporary labels all correct, with no extras Permanent labels correct(Accept correct working starting from G, if seen) At least three sets of temporary labels correct, with no extras Temporary labels all correct, with no extras Permanent labels correct $C = 42$ 27 27 27 $F = 5$ 39 39 $B1$ Weight = 56 Route = $A - B - D - G$ $B1$ $B1$ Order of labelling (correct or follow through their permanent labels)[4] $AB = 9$ $CG = 42$ 51 $AC = 27$ 74 90 $AG = 56$ $A1$ $B1$ $A1$ latentifying or using A, B, C, G (seen) $AB = 9$ $AC = 27$ $AG = 56CG = 4251BC = 3490A1A1latentifying or using A, B, C, G (seen)A1$		Misses out vertex C	A1		[2]
Upper bound = 148B1148 cao[3](v) B 2 9 E 6 46 $(Accept correct working starting from G, if seen)$ A 1 0 3 25 G 7 56 $A1$ a D 3 25 G 7 A^{-1} 0 0 3 25 G 7 29 25 G 7 56 $A1$ $B1$ D 3 25 G 7 29 25 G 7 56 $B1$ $B1$ $Crect or rect, with no extrasTemporary labels all correct, with no extrasPermanent labels correctC427F5127F539B1Crect of labelling (correct or follow through their permanent labels)Weight = 56B1A - B - D - G caoRoute = A - B - D - GB1A - B - D - G caoAB = 9AC = 27AG = 56CG = 42BG = 47BC = 34517490Repeat AB and CG (C - F - G) = 51$	(iv)	B-A-C-D-F-G-E-B	M1	Tour starts $B - A - C - D - F - C$	
(v) B 2 9 E 6 46 (Accept correct working starting from G , if seen) A 1 0 3 25 G 7 56 A^{-1} 0 3 25 G 7 56 B^{-1} C 42 7 F 5 39 B^{-1} C 4 27 F 5 39 B^{-1} C^{-1} C^{-1} A^{-1} A^{-1} B^{-1} B^{-1} C^{-1} 27 A^{-1} A^{-1} A^{-1} A^{-1} B^{-1} A^{-1} </td <td></td> <td></td> <td></td> <td>5 5</td> <td></td>				5 5	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		Upper bound = 148	B1		[3]
C 2925 56 $B1$ extras Permanent labels correct C 427 F 539 $B1$ Order of labelling (correct or follow through their permanent labels) $[4]$ Weight = 56 Route = $A - B - D - G$ $B1$ 56 cao $B1$ $A - B - D - G$ cao $[2]$ (vi) A, B, C and G are odd $B1$ Identifying or using A, B, C, G (seen) $[4]$ $AB = 9$ $CG = \frac{42}{51}$ $BG = \frac{47}{74}$ $BC = \frac{34}{90}$ $B1$ $A1$ At least one correct pairing seen or total seen (not just six weights) $A1$	(v)			if seen) At least three sets of temporary labels correct, with no extras	
4 27 5 39 through their permanent labels)[4]Weight = 56 Route = $A \cdot B - D - G$ B1 56 cao B1 $A - B - D - G$ cao[2](vi) A, B, C and G are oddB1Identifying or using A, B, C, G (seen)[2](vi) A, B, C and G are oddB1Identifying or using A, B, C, G (seen)[2] $AB = 9$ $AC = 27$ $AG = 56$ $CG = 42$ M1At least one correct pairing seen or total seen (not just six weights) $A1$ All three totals correct, or explanation of how it is known that other pairings are too long			B1		
Route = $A - B - D - G$ B1 $A - B - D - G$ cao[2](vi) A, B, C and G are oddB1Identifying or using A, B, C, G (seen)[3] $AB = 9$ $AC = 27$ $AG = 56$ M1At least one correct pairing seen or total seen (not just six weights) $AB = 42$ $BG = 47$ $BC = 34$ M1At least one correct, or explanation of how it is known that other pairings are too long			B1		[4]
(vi)A, B, C and G are oddB1Identifying or using A, B, C, G (seen) $AB = 9$ $AC = 27$ $AG = 56$ M1At least one correct pairing seen or total $CG = \underline{42}$ $BG = \underline{47}$ $BC = \underline{34}$ M1At least one correct pairing seen or total $CG = \underline{42}$ $BG = \underline{47}$ $BC = \underline{34}$ M1 $A1$ Here totals correct, or explanation of how it is known that other pairings are too long		Weight = 56	B1	56 cao	
$AB = 9 AC = 27 AG = 56$ $CG = \frac{42}{51} BG = \frac{47}{74} BC = \frac{34}{90}$ Repeat AB and CG (C - F - G) = 51 $AI At least one correct pairing seen or total seen (not just six weights)$ $AI At least one correct pairing seen or total seen (not just six weights)$ $AI At least one correct pairing seen or total seen (not just six weights)$ $AI be a constrained by a con$		Route = $A - B - D - G$	B1	<i>A</i> - <i>B</i> - <i>D</i> - <i>G</i> cao	[2]
$\begin{bmatrix} CG = \frac{42}{51} & BG = \frac{47}{74} & BC = \frac{34}{90} \\ \text{Repeat } AB \text{ and } CG (C - F - G) = 51 \end{bmatrix}$ A1 As seen (not just six weights) A1 three totals correct, or explanation of how it is known that other pairings are too long	(vi)				
51 74 90 A1All three totals correct, or explanation of how it is known that other pairings are too long		AB = 9 $AC = 27$ $AG = 56$	M1	· •	
Repeat AB and $CG(C - F - G) = 51$ how it is known that other pairings are too long		$CG = \underline{42}$ $BG = \underline{47}$ $BC = \underline{34}$			
long		51 74 90	A1		
		Repeat AB and CG $(C - F - G) = 51$			
weight - 500 + 51 = 551 D1 551 cao [4]		$W_{aight} = 300 + 51 = 351$	D1	0	[4]
Total = 23		w eigin - 300 + 31 = 331	DI	l.	

ANSWERED ON INSERT

4	(i)	8		B1	cao	[1]
	(ii)	1 comparison and 1 swap		B1	1 and 1	[1]
	(iii)	76 65 21 13 88 62 67 28 34		B1	Correct list (complete)	
		2 comparisons and 1 swap		B1	2 and 1	[2]
	(iv)		C S		Underlined values correct in 3 rd and 4 th	
		<u>76 65 21 13</u> 88 62 67 28 34	1 0	M1	passes, values not underlined may be left	
		<u>88 76 65 21 13</u> 62 67 28 34	4 4		blank	
				M1	Similarly for 5^{th} and 6^{th} passes, follow	
		<u>88 76 65 62 21 13</u> 67 28 34	3 2		through slips in previous passes	
		<u>88 76 67 65 62 21 13</u> 28 34	54	A1	Similarly for 7 th and 8 th passes, but cao	[3]
					(Dependent on both M marks)	
		<u>88 76 67 65 62 28 21 13</u> 34	3 2	M1	Reasonable attempt at Comp and Swap	
		88 76 67 65 62 34 28 21 13	4 3	A1	1 4 3 5 3 4 cao in figures	
				A1	0 4 2 4 2 3 cao in figures	[3]

(v)	Shuttle sort uses 23 comparisons and 17		Follow through their totals if possible	
	swaps			
	Shuttle sort is more efficient	M1	Choosing shuttle sort with a reason or	
	because		with totals seen (here)	
	although it uses the same number of swaps	A1	Correct reason stated (comparisons and	
	as bubble sort it uses fewer comparisons		swaps both compared, in words)	[2]
			Total =	12

5		Votio must spond at losst 9 minutes monoring	M1	Identifying why there is loss than 60	
5	(i)	Katie must spend at least 8 minutes preparing the first batch of cookies so she has at most	IVII	Identifying why there is less than 60 minutes of baking time (or seeing 52)	
		52 minutes of baking time.	A1	minutes of baking time (or seeing 52) Explaining why 4 is the greatest possible	
		$52 \div 12 = 4.3$, hence at most 4 batches	AI	number of batches	[2]
	(::)				[4]
	(ii)	The last batch takes 12 minutes to bake,	B1	Explaining why total time for preparation	
		so Katie has (at most) 48 minutes of	DI	cannot exceed 48 minutes	
		preparation time			
		$8x + 12y + 10z \le 48 \Longrightarrow 4x + 6y + 5z \le 24$	B1	$8x + 12y + 10z \le 48$ seen or explicitly	
		$3x + 12y + 10z \le 48 \implies 4x + 0y + 5z \le 24$ as given	DI	referred to	[2]
-	(iii)	Must be integer valued	B1		[2]
	× /			Integers	
	(iv)	P = 5x + 4y + 3z	B1	5x + 4y + 3z or any positive multiple of this	
		Assumes that she sells all the cookies	B1	Assumes she sells them all	
			DI	Assumes she sens them an	[2]
-	()	(batches) that she makes			[2]
	(v)	P x y z s t 1 -5 -4 -3 0 0 0	M1	Correct use of slack variable columns	
			A1		
		$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	A1 A1	Objective row correct (cao) Constraint rows correct (cao)	[3]
		0 4 0 5 0 1 24	AI	Constraint Tows correct (cao)	[J]
		$4 \div 1 = 4, 24 \div 4 = 6, 4 < 6$		Working need not be seen	
		Pivot on the 1 in the x column	B1	Correct pivot choice (row 2) (cao)	
		Troot on the T in the x column	DI		
		Pxyzst		Follow through their tableau and pivot	
		$\begin{array}{cccccccccccccccccccccccccccccccccccc$		choice, if possible	
		$\begin{array}{cccccccccccccccccccccccccccccccccccc$	M1	sca pivoting $(x, t \text{ cols}, P \text{ not decreased})$	
		P x y z s t 1 0 1 2 5 0 20 0 1 1 1 0 4 0 0 2 1 -4 1 8	A1	Correct tableau (final column contains no	
				negative values)	
		Row $1 = R1 + 5 \times R2$			
		Row $2 = R2 \div 1$	B1	Showing valid method,	
		Row $3 = R3 - 4 \times R2$		may imply row 2	
				Follow through their tableau, if reasonable	[4]
		x = 4, y = 0, z = 0, P = 20		(non-negative variables)	
			3.41		
		Katie should make 4 batches of plain	M1	Reading off values from tableau	
		cookies, and no chocolate chip or fruit	A 1	(<u>may be implied</u> from answer)	
		cookies, to give a profit of £20.	A1	Interpretation: 4 batches of plain cookies	
			A1	(may imply none of others) Interpretation: £20	
			AI		[3]
					[2]
			I		

