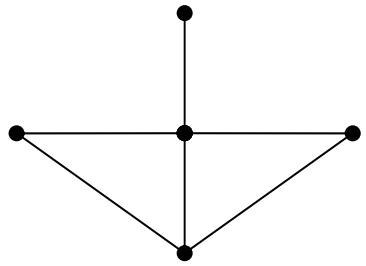
4736 Decision Mathematics 1

1	**(i)**	*A* *B* *C* *D* 614 416 1 198 (*A*=198) 198 891 2 693 (*A*=693) 693 396 3 297	M1 A1 M1 A1	*A*, *B* and *C* correct for first pass *D* = 198 on first pass sca at second <u>and</u> third passes Second and third passes correct	**[4]**
	(ii)	0	B1	0	**[1]**
	(iii)	To make the algorithm terminate	B1	So that it does not get stuck in a loop	**[1]**
				Total =	**6**

2	**(i)**	eg 	 M1 A1	Graph need not be simple or planar A graph with five vertices and at least three correct vertex orders A graph with five vertices of orders 1, 2, 2, 3, 4	**[2]**
	(ii)	Semi-Eulerian It has <u>exactly</u> two odd nodes	M1 A1	Unless their graph was not connected, in which case the answer is 'neither' (Unless their graph was not connected, in which case follow this through)	**[2]**
	(iii)	A tree with five vertices would only have four arcs, but this graph has six <u>Or</u> A tree must have at least two vertices of order 1	B2	Give B1 for an incomplete reason, eg 'too many arcs' or 'it has a cycle'	**[2]**
				Total =	**6**

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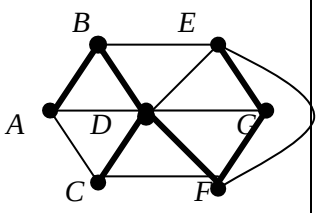
3	**(i)**	*AB* = 9 *DF* = 14 *BD* = 16 *CD* = 18 *FG* = 20 ~~*CF* = 22~~ *EG* = 23 ~~*EF* = 26~~ ~~*AC* = 27~~ ~~*DE* = 28~~ ~~*AD* = 29~~ ~~*DG* = 31~~ ~~*BE* = 37~~  Total weight = 100	M1 A1 M1 A1 B1	Not selecting *CF* (working seen on list) Selecting correct arcs (working seen on list) A spanning tree drawn Correct (minimum) spanning tree drawn 100 cao	**[5]**

	(ii)	Delete *EG* from spanning tree 100 – 23 = 77 Two shortest arcs from *E* are *EG* and *EF* 77 + 23 + 26 = 126 Lower bound = 126	 B1 M1 A1	Follow through from part (i) if possible Weight of MST on reduced network Adding two shortest arcs to MST 126 cao	**[3]**
	(iii)	*A* – *B* – *D* – *F* – *G* – *E* – stall Misses out vertex *C*	M1 A1	*A* – *B* – *D* – *F* – *G* – *E* <u>Cannot continue</u> because *B*, *D* and *F* have already been visited	**[2]**
	(iv)	*B* – *A* – *C* – *D* – *F* – *G* – *E* – *B* Upper bound = 148	M1 A1 B1	Tour starts *B* – *A* – *C* – *D* – *F* – Correct tour, starting and ending at *B* 148 cao	**[3]**
	(v)	*B* 2 9 / 9 *E* 6 46 / 46 *A* 1 0 / *D* 3 25 / 29 25 *G* 7 56 / 56 *C* 4 27 / 27 *F* 5 39 / 39 Weight = 56 Route = *A* -*B* - *D* - *G*	 M1 A1 B1 B1 B1	(Accept correct working starting from *G*, if seen) At least three sets of temporary labels correct, with no extras Temporary labels all correct, with no extras Permanent labels correct Order of labelling (correct or follow through their permanent labels) 56 cao *A* - *B* - *D* - *G* cao	**[4]** **[2]**
	(vi)	*A*, *B*, *C* and *G* are odd *AB* = 9 *AC* = 27 *AG* = 56 *CG* = <u>42</u> *BG* = <u>47</u> *BC* = <u>34</u> 51 74 90 Repeat *AB* and *CG* (*C* - *F* - *G*) = 51 Weight = 300 + 51 = 351	B1 M1 A1 B1	Identifying or using *A*, *B*, *C*, *G* (seen) At least one correct pairing seen or total seen (not just six weights) All three totals correct, or explanation of how it is known that other pairings are too long 351 cao	**[4]**
				Total =	**23**

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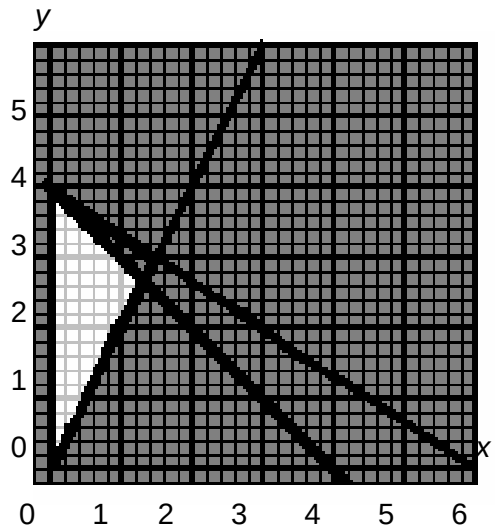
4	**(i)**	8	B1	cao	**[1]**
	(ii)	1 comparison and 1 swap	B1	1 and 1	**[1]**
	(iii)	76 65 21 13 88 62 67 28 34 2 comparisons and 1 swap	B1 B1	Correct list (complete) 2 and 1	**[2]**
	(iv)	C S <u>76 65 21 13</u> 88 62 67 28 34 1 0 <u>88 76 65 21 13</u> 62 67 28 34 4 4 <u>88 76 65 62 21 13</u> 67 28 34 3 2 <u>88 76 67 65 62 21 13</u> 28 34 5 4 <u>88 76 67 65 62 28 21 13</u> 34 3 2 <u>88 76 67 65 62 34 28 21 13</u> 4 3	 M1 M1 A1 M1 A1 A1	Underlined values correct in 3rd and 4th passes, values not underlined may be left blank Similarly for 5th and 6th passes, follow through slips in previous passes Similarly for 7th and 8th passes, but cao (Dependent on both M marks) Reasonable attempt at Comp <u>and</u> Swap 1 4 3 5 3 4 cao in figures 0 4 2 4 2 3 cao in figures	**[3]** **[3]**

(v)	Shuttle sort uses 23 comparisons and 17 swaps		Follow through their totals if possible	
	Shuttle sort is more efficient because	M1	Choosing shuttle sort with a reason or with totals seen (here)	
	although it uses the same number of swaps as bubble sort it uses fewer comparisons	A1	Correct reason stated (comparisons and swaps both compared, in words)	**[2]**
			Total =	**12**

5	**(i)**	Katie must spend at least 8 minutes preparing the first batch of cookies so she has at most 52 minutes of baking time. $52 \div 12 = 4.3$, hence at most 4 batches	M1 A1	Identifying why there is less than 60 minutes of baking time (or seeing 52) Explaining why 4 is the greatest possible number of batches	**[2]**
	(ii)	The last batch takes 12 minutes to bake, so Katie has (at most) 48 minutes of preparation time $8x + 12y + 10z \leq 48 \Rightarrow 4x + 6y + 5z \leq 24$ as given	B1 B1	Explaining why total time for preparation cannot exceed 48 minutes $8x + 12y + 10z \leq 48$ seen or explicitly referred to	**[2]**
	(iii)	Must be integer valued	B1	Integers	**[1]**
	(iv)	$P = 5x + 4y + 3z$ Assumes that she sells all the cookies (batches) that she makes	B1 B1	$5x + 4y + 3z$ or any positive multiple of this Assumes she sells them all	**[2]**
	(v)	P x y z s t *1 -5 -4 -3 0 0 0* *0* **1** *1 1 1 0 4* *0 4 6 5 0 1 24* $4 \div 1 = 4$, $24 \div 4 = 6$, $4 < 6$ Pivot on the 1 in the x column P x y z s t *1 0 1 2 5 0 20* *0 1 1 1 1 0 4* *0 0 2 1 -4 1 8* Row 1 = R1 + 5×R2 Row 2 = R2 ÷ 1 Row 3 = R3 - 4×R2 $x = 4$, $y = 0$, $z = 0$, $P = 20$ Katie should make 4 batches of plain cookies, and no chocolate chip or fruit cookies, to give a profit of £20.	M1 A1 A1 B1 B1 M1 A1 M1 A1 A1	Correct use of slack variable columns Objective row correct (cao) Constraint rows correct (cao) Working need not be seen Correct pivot choice (row 2) (cao) Follow through their tableau and pivot choice, if possible sca pivoting (x, t cols, P not decreased) Correct tableau (final column contains no negative values) Showing valid method, may imply row 2 Follow through their tableau, if reasonable (non-negative variables) Reading off values from tableau (may be implied from answer) Interpretation: 4 batches of plain cookies (may imply none of others) Interpretation: £20	**[3]** **[4]** **[3]**

	(vi)		M1	At least two of the lines $y = 2x$, $x+y = 4$ and $4x + 6y = 24$ drawn correctly	
			A1	All three lines drawn correctly and graph has both scales and labels	
			A1	Feasible region identified and correct	
				Follow through their feasible region if possible	**[3]**
		Vertices of feasible region are (0, 0), (0, 4) and $(1\frac{1}{3}, 2\frac{2}{3})$	M1 A1	At least two correct All (three) correct (1 dp or better)	**[2]**
		$x = 0, y = 4 \Rightarrow P = 16$ $x = 1, y = 3 \Rightarrow P = 17$ $(x = 1\frac{1}{3}, y = 2\frac{2}{3} \Rightarrow P = 17\frac{1}{3})$	M1 A1	Or a line of constant profit drawn (or gradient discussed) and used correctly on integer-valued coordinates For (1, 3) or 17 chosen (cao)	
		Make 1 batch of plain cookies and 3 batches of chocolate chip cookies	B1	Interpretation: 1 batch of plain, 3 batches of chocolate chip (cao)	**[3]**
				Total =	**25**