

Mark Scheme (Results)

January 2009

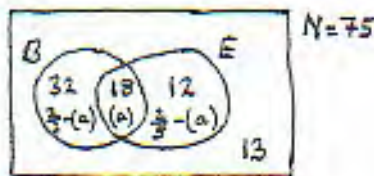
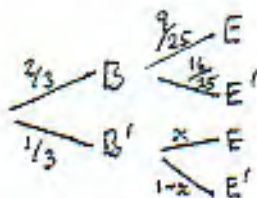
GCE

GCE Mathematics (6683/01)

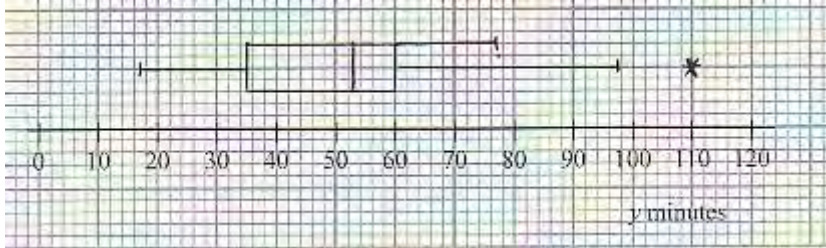
January 2009
6683 Statistics S1
Mark Scheme

Question Number	Scheme	Marks
1	<p>(a) $S_{xx} = 57.22 - \frac{(21.4)^2}{10} = 11.424$</p> <p>$S_{xy} = 313.7 - \frac{21.4 \times 96}{10} = 108.26$</p> <p>(b) $b = \frac{S_{xy}}{S_{xx}} = 9.4765\dots$</p> <p>$a = \bar{y} - b\bar{x} = 9.6 - 2.14b = (-10.679\dots)$</p> <p>$y = -10.7 + 9.48x$</p> <p>(c) Every (extra) <u>hour</u> spent using the programme produces about <u>9.5 marks improvement</u></p> <p>(d) $y = -10.7 + 9.48 \times 3.3, = 20.6$ awrt 21</p> <p>(e) Model may not be valid since [8h is] outside the range [0.5 - 4].</p>	<p>M1 A1</p> <p>A1 (3)</p> <p>M1 A1 M1 A1 (4)</p> <p>B1ft (1)</p> <p>M1,A1 (2)</p> <p>B1 (1)</p> <p style="text-align: right;">[11]</p>
	<p>(a) M1 for a correct expression 1st A1 for AWRT 11.4 for S_{xx} 2nd A1 for AWRT 108 for S_{xy} Correct answers only: One value correct scores M1 and appropriate A1, both correct M1A1A1</p> <p>(b) 1st M1 for using their values in correct formula 1st A1 for AWRT 9.5 2nd M1 for correct method for a (minus sign required) 2nd A1 for equation with a and b AWRT 3 sf (e.g. $y = -10.68 + 9.48x$ is fine) Must have a full equation with a and b correct to awrt 3 sf</p> <p>(c) B1ft for comment conveying the idea of <u>b marks per hour</u>. Must mention value of b but can fit their value of b. No need to mention “extra” but must mention “marks” and “hour(s)” e.g. “...9.5 times per hour...” scores B0</p> <p>(d) M1 for sub $x = 3.3$ into their regression equation from the end of part (b) A1 for awrt 21</p> <p>(e) B1 for a statement that says or implies that it may <u>not</u> be valid because <u>outside the range</u>. They do not have to mention the values concerned here namely 8 h or 0.5 - 4</p>	

Question Number	Scheme	Marks
2	<p>$E = \text{take regular exercise}$ $B = \text{always eat breakfast}$</p> <p>(a) $P(E \cap B) = P(E B) \times P(B)$ $= \frac{9}{25} \times \frac{2}{3} = 0.24$ or $\frac{6}{25}$ or $\frac{18}{75}$</p> <p>(b) $P(E \cup B) = \frac{2}{3} + \frac{2}{5} - \frac{6}{25}$ or $P(E' B')$ or $P(B' \cap E)$ or $P(B \cap E')$ $= \frac{62}{75}$ $= \frac{13}{25}$ $= \frac{12}{75}$ $= \frac{32}{75}$</p> <p>$P(E' \cap B') = 1 - P(E \cup B) = \frac{13}{75}$ or 0.173</p> <p>(c) $P(E B) = 0.36 \neq 0.40 = P(E)$ or $P(E \cap B) = \frac{6}{25} \neq \frac{2}{5} \times \frac{2}{3} = P(E) \times P(B)$ So E and B are <u>not</u> statistically independent</p>	<p>M1 A1 (2)</p> <p>M1 A1 M1 A1 (4)</p> <p>M1 A1 (2)</p> <p>[8]</p>
	<p>(a) M1 for $\frac{9}{25} \times \frac{2}{3}$ or $P(E B) \times P(B)$ <u>and</u> at least one correct value seen. A1 for 0.24 or exact equiv. NB $\frac{2}{5} \times \frac{2}{3}$ alone or $\frac{2}{5} \times \frac{9}{25}$ alone scores M0A0. Correct answer scores full marks.</p> <p>(b) 1st M1 for use of the addition rule. Must have 3 terms and some values, can ft their (a) <u>Or</u> a full method for $P(E' B')$ requires $1 - P(E B')$ and equation for $P(E B')$: $(a) + \frac{x}{3} = \frac{2}{5}$ <u>Or</u> a full method for $P(B' \cap E)$ <u>or</u> $P(B \cap E')$ [or other valid method] 2nd M1 for a method leading to answer e.g. $1 - P(E \cup B)$ <u>or</u> $P(B') \times P(E' B')$ <u>or</u> $P(B') - P(B' \cap E)$ <u>or</u> $P(E') - P(B \cap E')$ <u>Venn Diagram</u> 1st M1 for diagram with attempt at $\frac{2}{5} - P(B \cap E)$ or $\frac{2}{3} - P(B \cap E)$. Can ft their (a) 1st A1 for a correct first probability as listed or 32, 18 and 12 on Venn Diagram 2nd M1 for attempting 75 - their (18 + 32 + 12)</p> <p>(c) M1 for identifying suitable values to test for independence e.g. $P(E) = 0.40$ and $P(E B) = 0.36$ <u>Or</u> $P(E) \times P(B) = \dots$ and $P(E \cap B) = \text{their (a)}$ [but their (a) $\neq \frac{2}{5} \times \frac{2}{3}$]. Values seen somewhere A1 for correct values and a correct comment</p> <p>Diagrams You may see these or find these useful for identifying probabilities.</p>	<p>Common Errors</p> <p>(a) $\frac{9}{25}$ is M0A0</p> <p>(b) $P(E \cup B) = \frac{53}{75}$ scores M1A0 $1 - P(E \cup B) = \frac{22}{75}$ scores M1A0</p> <p>(b) $P(B') \times P(E') = \frac{1}{3} \times \frac{3}{5}$ scores 0/4</p>



Question Number	Scheme	Marks																		
3	<p>(a) $E(X) = 0 \times 0.4 + 1 \times 0.3 + \dots + 3 \times 0.1, = 1$</p> <p>(b) $F(1.5) = [P(X \leq 1.5) =] P(X \leq 1), = 0.4 + 0.3 = 0.7$</p> <p>(c) $E(X^2) = 0^2 \times 0.4 + 1^2 \times 0.3 + \dots + 3^2 \times 0.1, = 2$ $\text{Var}(X) = 2 - 1^2, = 1$ (*)</p> <p>(d) $\text{Var}(5 - 3X) = (-3)^2 \text{Var}(X), = 9$</p> <p>(e)</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Total</th> <th>Cases</th> <th>Probability</th> </tr> </thead> <tbody> <tr> <td rowspan="3">4</td> <td>$(X = 3) \cap (X = 1)$</td> <td>$0.1 \times 0.3 = 0.03$</td> </tr> <tr> <td>$(X = 1) \cap (X = 3)$</td> <td>$0.3 \times 0.1 = 0.03$</td> </tr> <tr> <td>$(X = 2) \cap (X = 2)$</td> <td>$0.2 \times 0.2 = 0.04$</td> </tr> <tr> <td rowspan="2">5</td> <td>$(X = 3) \cap (X = 2)$</td> <td>$0.1 \times 0.2 = 0.02$</td> </tr> <tr> <td>$(X = 2) \cap (X = 3)$</td> <td>$0.2 \times 0.1 = 0.02$</td> </tr> <tr> <td>6</td> <td>$(X = 3) \cap (X = 3)$</td> <td>$0.1 \times 0.1 = 0.01$</td> </tr> </tbody> </table> <p>Total probability = $0.03 + 0.03 + 0.04 + 0.02 + 0.02 + 0.01 = 0.15$</p>	Total	Cases	Probability	4	$(X = 3) \cap (X = 1)$	$0.1 \times 0.3 = 0.03$	$(X = 1) \cap (X = 3)$	$0.3 \times 0.1 = 0.03$	$(X = 2) \cap (X = 2)$	$0.2 \times 0.2 = 0.04$	5	$(X = 3) \cap (X = 2)$	$0.1 \times 0.2 = 0.02$	$(X = 2) \cap (X = 3)$	$0.2 \times 0.1 = 0.02$	6	$(X = 3) \cap (X = 3)$	$0.1 \times 0.1 = 0.01$	<p>M1, A1 (2)</p> <p>M1, A1 (2)</p> <p>M1, A1 M1, A1cso (4)</p> <p>M1, A1 (2)</p> <p>B1B1B1</p> <p>M1 A1</p> <p>A1 (6) [16]</p>
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6	$(X = 3) \cap (X = 3)$	$0.1 \times 0.1 = 0.01$																		
ALT	<p>(a) M1 for at least 3 terms seen. Correct answer only scores M1A1. Dividing by $k (\neq 1)$ is M0.</p> <p>(b) M1 for $F(1.5) = P(X \leq 1)$. [Beware: $2 \times 0.2 + 3 \times 0.1 = 0.7$ but scores M0A0]</p> <p>(c) 1st M1 for at least 2 non-zero terms seen. $E(X^2) = 2$ alone is M0. Condone calling $E(X^2) = \text{Var}(X)$. 1st A1 is for an answer of 2 or a fully correct expression. 2nd M1 for $-\mu^2$, condone $2 - 1$, unless clearly $2 -$. Allow $2 - \mu^2$, with $= 1$ even if $E(X) \neq 1$ 2nd A1 for a fully correct solution with no incorrect working seen, both Ms required. $\frac{\sum (x - \mu)^2 \times P(X = x)}{}$</p> <p>1st M1 for an attempt at a full list of $(x - \mu)^2$ values and probabilities. 1st A1 if all correct 2nd M1 for at least 2 non-zero terms of $(x - \mu)^2 \times P(X = x)$ seen. 2nd A1 for $0.4 + 0.2 + 0.4 = 1$</p> <p>(d) M1 for use of the correct formula. $-3^2 \text{Var}(X)$ is M0 unless the final answer is > 0.</p> <p>(e) Can follow through their $\text{Var}(X)$ for M1</p>																			
ALT	<p>1st B1 for all cases listed for a total of 4 or 5 or 6 . e.g. (2,2) counted twice for a total of 4 is B0 2nd B1 for all cases listed for 2 totals 3rd B1 for a complete list of all 6 cases } These may be highlighted in a table</p> <p><u>Using Cumulative probabilities</u> 1st B1 for one or more cumulative probabilities used e.g. 2 then 2 or more or 3 then 1 or more 2nd B1 for both cumulative probabilities used. 3rd B1 for a complete list 1, 3; 2, ≥ 2; 3, ≥ 1 M1 for one correct pair of correct probabilities multiplied 1st A1 for all 6 correct probabilities listed (0.03, 0.03, 0.04, 0.02, 0.02, 0.01) needn't be added. 2nd A1 for 0.15 or exact equivalent only as the final answer.</p>																			

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4	<p>(a) $Q_2 = 53, Q_1 = 35, Q_3 = 60$</p> <p>(b) $Q_3 - Q_1 = 25 \Rightarrow Q_1 - 1.5 \times 25 = -2.5$ (no outlier) $Q_3 + 1.5 \times 25 = 97.5$ (so 110 is an outlier)</p> <p>(c) </p> <p>(d) $\sum y = 461, \sum y^2 = 24\ 219 \therefore S_{yy} = 24219 - \frac{461^2}{10}, = 2966.9$ (*)</p> <p>(e) $r = \frac{-18.3}{\sqrt{3463.6 \times 2966.9}}$ or $\frac{-18.3}{3205.64\dots} = -0.0057$ AWRT - 0.006 or -6×10^{-3}</p> <p>(f) r suggests correlation is close to zero so parent's claim is not justified</p>	<p>B1, B1, B1 (3)</p> <p>M1</p> <p>A1 (2)</p> <p>M1</p> <p>A1ft</p> <p>A1ft (3)</p> <p>B1, B1, B1cso (3)</p> <p>M1 A1 (2)</p> <p>B1 (1)</p> <p>[14]</p>
	<p>(a) 1st B1 for median 2nd B1 for lower quartile 3rd B1 for upper quartile</p> <p>(b) M1 for attempt to find one limit A1 for both limits found and correct. No explicit comment about outliers needed.</p> <p>(c) M1 for a box and two whiskers 1st A1ft for correct position of box, median and quartiles. Follow through their values. 2nd A1ft for 17 and 77 or "their" 97.5 and *. If 110 is not an outlier then score A0 here. Penalise no gap between end of whisker and outlier. Must label outlier, needn't be with *. Accuracy should be within the correct square so 97 or 98 will do for 97.5</p> <p>(d) 1st B1 for $\sum y$ N.B. $(\sum y)^2 = 212521$ and can imply this mark 2nd B1 for $\sum y^2$ or at least three correct terms of $\sum (y - \bar{y})^2$ seen. 3rd B1 for complete correct expression seen leading to 2966.9. So all 10 terms of $\sum (y - \bar{y})^2$</p> <p>(e) M1 for attempt at correct expression for r. Can ft their S_{yy} for M1.</p> <p>(f) B1 for comment <u>rejecting</u> parent's claim on basis of <u>weak or zero</u> correlation Typical error is "negative correlation so comment is true" which scores B0 Weak negative or weak positive correlation is OK as the basis for their rejection.</p>	

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5	<p>(a) 8-10 hours: width = 10.5 - 7.5 = 3 represented by 1.5cm 16-25 hours: width = 25.5 - 15.5 = 10 so represented by <u>5 cm</u> 8- 10 hours: height = fd = 18/3 = 6 represented by 3 cm 16-25 hours: height = fd = 15/10 = 1.5 represented by <u>0.75 cm</u></p> <p>(b) $Q_2 = 7.5 + \frac{(52-36)}{18} \times 3 = 10.2$ $Q_1 = 5.5 + \frac{(26-20)}{16} \times 2 [= 6.25 \text{ or } 6.3]$ or $5.5 + \frac{(26.25-20)}{16} \times 2 [= 6.3]$ $Q_3 = 10.5 + \frac{(78-54)}{25} \times 5 [= 15.3]$ or $10.5 + \frac{(78.75-54)}{25} \times 5 [= 15.45 \text{ \textbackslash} 15.5]$ IQR = (15.3 - 6.3) = 9</p> <p>(c) $\sum fx = 1333.5 \Rightarrow \bar{x} = \frac{1333.5}{104} =$ AWRT <u>12.8</u></p> <p>(d) $\sum fx^2 = 27254 \Rightarrow \sigma_x = \sqrt{\frac{27254}{104} - \bar{x}^2} = \sqrt{262.05 - \bar{x}^2}$ AWRT <u>9.88</u> $Q_3 - Q_2 [= 5.1] > Q_2 - Q_1 [= 3.9]$ or $Q_2 < \bar{x}$</p> <p>(e) So data is positively skew</p> <p>Use median and IQR, since data is skewed <u>or</u> not affected by extreme values or outliers</p>	<p>B1 M1 A1 (3)</p> <p>M1 A1</p> <p>A1</p> <p>A1 A1ft (5)</p> <p>M1 A1</p> <p>M1 A1 (4)</p> <p>B1ft dB1 (2)</p> <p>B1 B1 (2)</p> <p>[16]</p>
	<p>(a) M1 For attempting both frequency densities $\frac{18}{3} (= 6)$ and $\frac{15}{10}$, <u>and</u> $\frac{15}{10} \times \text{SF}$, where $\text{SF} \neq 1$</p> <p>(b) NB Wrong class widths(2 and 9) gives $\frac{h}{1.66\dots} = \frac{3}{9} \rightarrow h = \frac{5}{9}$ or 0.55... and scores M1A0</p> <p>M1 for identifying correct interval and a correct fraction e.g. $\frac{\frac{1}{2}(104)-36}{18}$. Condone 52.5 or 53</p> <p>1st A1 for 10.2 for median. Using $(n + 1)$ allow awrt 10.3</p> <p>2nd A1 for a correct expression for either Q_1 or Q_3 (allow 26.25 and 78.75) NB: <u>Must see</u></p> <p>3rd A1 for correct expressions for both Q_1 and Q_3 <u>some</u></p> <p>(c) 4th A1ft for IQR, ft their quartiles. Using $(n + 1)$ gives 6.28 and 15.45 <u>method</u></p> <p>1st M1 for attempting $\sum fx$ and \bar{x}</p> <p>(d) 2nd M1 for attempting $\sum fx^2$ and $\sigma_x, \sqrt{\quad}$ is needed for M1. Allow $s =$ awrt 9.93</p> <p>1st B1ft for suitable test, values need not be seen but statement must be compatible with values used. Follow through their values</p> <p>2nd dB1 Dependent upon their test showing positive and for stating positive skew If their test shows negative skew they can score 1st B1 but lose the second</p> <p>(e) 1st B1 for choosing median and IQR. Must mention <u>both</u>. } <u>Award independently</u> 2nd B1 for suitable reason } e.g. “use median because data is skewed” scores B0B1 since IQR is not mentioned</p>	

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6	<p>(a) $P(X < 39) = P\left(Z < \frac{39-30}{5}\right)$ $= P(Z < 1.8) = \underline{0.9641}$ (allow awrt 0.964)</p> <p>(b) $P(X < d) = P\left(Z < \frac{d-30}{5}\right) = 0.1151$ $1 - 0.1151 = 0.8849$ (allow ± 1.2) $\Rightarrow z = -1.2$ $\therefore \frac{d-30}{5} = -1.2$ $\underline{d = 24}$</p> <p>(c) $P(X > e) = 0.1151$ so $e = \mu + (\mu - \text{their } d)$ or $\frac{e-30}{5} = 1.2$ or $-$ their z $\underline{e = 36}$</p> <p>(d) $P(d < X < e) = 1 - 2 \times 0.1151$ $= 0.7698$ AWRT $\underline{0.770}$</p>	<p>M1 A1 (2)</p> <p>M1 B1 M1A1 (4)</p> <p>M1 A1 (2)</p> <p>M1 A1 (2)</p> <p>[10]</p>
	<p>Answer only scores all marks in each section BUT check (b) and (c) are in correct order</p> <p>(a) M1 for standardising with σ, $z = \pm \frac{39-30}{5}$ is OK A1 for 0.9641 or awrt 0.964 but if they go on to calculate $1 - 0.9641$ they get M1A0</p> <p>(b) 1st M1 for attempting $1 - 0.1151$. Must be seen in (b) in connection with finding d B1 for $z = \pm 1.2$. They must state $z = \pm 1.2$ or imply it is a z value by its use. This mark is only available in part (b). 2nd M1 for $\left(\frac{d-30}{5}\right) =$ their negative z value (or equivalent)</p> <p>(c) M1 for a full method to find e. If they used $z = 1.2$ in (b) they can get M1 for $z = \pm 1.2$ here If they use symmetry about the mean $\mu + (\mu - \text{their } d)$ then ft their d for M1 Must explicitly <u>see</u> the method used unless the answer is correct.</p> <p>(d) M1 for a complete method or use of a correct expression e.g. “their 0.8849” - 0.1151 <u>or If their $d <$ their e using their values with $P(X < e) - P(X < d)$</u> If their $d \geq$ their e then they can only score from an argument like $1 - 2 \times 0.1151$ A negative probability or probability > 1 for part (d) scores M0A0</p>	