ADVANCED GCE

MATHEMATICS

4723

Core Mathematics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Thursday 15 January 2009
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Turn over

1 Find

(i) $\int 8e^{-2x}\,dx$,

(ii) $\int (4x+5)^6\,dx$. **[5]**

2 **(i)** Use Simpson's rule with four strips to find an approximation to

$$\int_4^{12} \ln x\,dx,$$

giving your answer correct to 2 decimal places. **[4]**

(ii) Deduce an approximation to $\int_4^{12} \ln(x^{10})\,dx$. **[1]**

3 **(i)** Express $2\tan^2\theta - \frac{1}{\cos\theta}$ in terms of $\sec\theta$. **[3]**

(ii) Hence solve, for $0° < \theta < 360°$, the equation

$$2\tan^2\theta - \frac{1}{\cos\theta} = 4.$$ **[4]**

4 For each of the following curves, find $\frac{dy}{dx}$ and determine the exact x-coordinate of the stationary point:

(i) $y = (4x^2+1)^5$, **[3]**

(ii) $y = \frac{x^2}{\ln x}$. **[4]**

5 The mass, M grams, of a certain substance is increasing exponentially so that, at time t hours, the mass is given by

$$M = 40e^{kt},$$

where k is a constant. The following table shows certain values of t and M.

t	0	21	63
M		80	

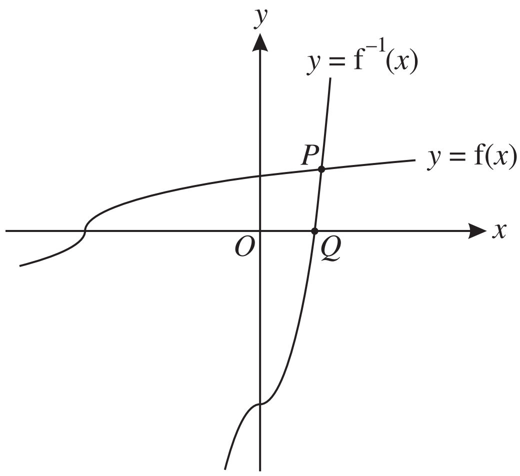
(i) In either order,

(a) find the values missing from the table, **[3]**

(b) determine the value of k. **[2]**

(ii) Find the rate at which the mass is increasing when $t = 21$. **[3]**

6



The function f is defined for all real values of x by

$$\mathrm{f}(x) = \sqrt[3]{\tfrac{1}{2}x + 2}.$$

The graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ meet at the point P, and the graph of $y = \mathrm{f}^{-1}(x)$ meets the x-axis at Q (see diagram).

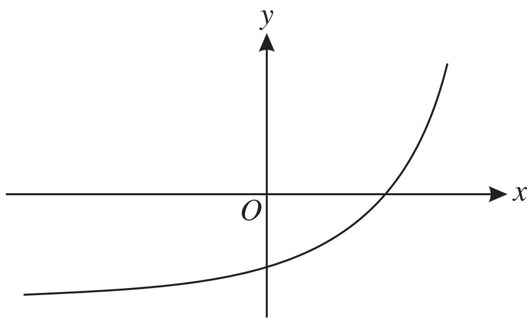
(i) Find an expression for $\mathrm{f}^{-1}(x)$ and determine the x-coordinate of the point Q. **[3]**

(ii) State how the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ are related geometrically, and hence show that the x-coordinate of the point P is the root of the equation

$$x = \sqrt[3]{\tfrac{1}{2}x + 2}.$$ **[2]**

(iii) Use an iterative process, based on the equation $x = \sqrt[3]{\tfrac{1}{2}x + 2}$, to find the x-coordinate of P, giving your answer correct to 2 decimal places. **[4]**

7



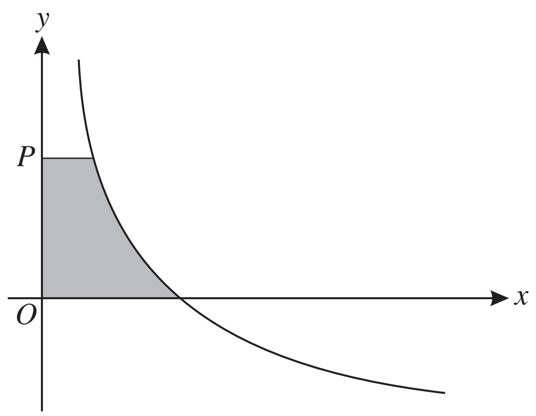
The diagram shows the curve $y = \mathrm{e}^{kx} - a$, where k and a are constants.

(i) Give details of the pair of transformations which transforms the curve $y = \mathrm{e}^{x}$ to the curve $y = \mathrm{e}^{kx} - a$. **[3]**

(ii) Sketch the curve $y = |\mathrm{e}^{kx} - a|$. **[2]**

(iii) Given that the curve $y = |\mathrm{e}^{kx} - a|$ passes through the points $(0,\ 13)$ and $(\ln 3,\ 13)$, find the values of k and a. **[4]**

8



The diagram shows the curve with equation

$$y = \frac{6}{\sqrt{x}} - 3.$$

The point P has coordinates $(0, p)$. The shaded region is bounded by the curve and the lines $x = 0$, $y = 0$ and $y = p$. The shaded region is rotated completely about the y-axis to form a solid of volume V.

(i) Show that $V = 16\pi\left(1 - \frac{27}{(p+3)^3}\right)$. **[6]**

(ii) It is given that P is moving along the y-axis in such a way that, at time t, the variables p and t are related by

$$\frac{\mathrm{d}p}{\mathrm{d}t} = \tfrac{1}{3}p + 1.$$

Find the value of $\frac{\mathrm{d}V}{\mathrm{d}t}$ at the instant when $p = 9$. **[4]**

9 **(i)** By first expanding $\cos(2\theta + \theta)$, prove that

$$\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta.$$ **[4]**

(ii) Hence prove that

$$\cos 6\theta \equiv 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1.$$ **[3]**

(iii) Show that the only solutions of the equation

$$1 + \cos 6\theta = 18\cos^2\theta$$

are odd multiples of 90°. **[5]**