

4723 Core Mathematics 3

1	Obtain integral of form $k(2x-7)^{-1}$ Obtain correct $-5(2x-7)^{-1}$ Include ... + c	M1 any constant k A1 or equiv B1 3 at least once; following any integral 3
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2 (i)	Use $\sin 2\theta = 2\sin\theta\cos\theta$ Attempt value of $\sin\theta$ from $k\sin\theta\cos\theta = 5\cos\theta$ Obtain $\frac{5}{12}$	B1 M1 any constant k ; or equiv A1 3 or exact equiv; ignore subsequent work
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(ii)	Use $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ or $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$ Attempt to produce equation involving $\cos\theta$ only Obtain $3\cos^2\theta + 8\cos\theta - 3 = 0$ Attempt solution of 3-term quadratic equation Obtain $\frac{1}{3}$ as only final value of $\cos\theta$	B1 or equiv M1 using $\sin^2\theta = \pm 1 \pm \cos^2\theta$ or equiv A1 or equiv M1 using formula or factorisation or equiv A1 5 or exact equiv; ignore subsequent work 8
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3 (i)	Obtain or clearly imply $60\ln x$ Obtain $(60\ln 20 - 60\ln 10)$ and hence $60\ln 2$	B1 B1 2 with no error seen
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(ii)	Attempt calculation of form $k(y_0 + 4y_1 + y_2)$ Identify k as $\frac{5}{3}$ Obtain $\frac{5}{3}(6 + 4 \times 4 + 3)$ and hence $\frac{125}{3}$ or 41.7	M1 any constant k ; using y -value attempts A1 A1 3 or equiv
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(iii)	Equate answers to parts (i) and (ii) Obtain $60\ln 2 = \frac{125}{3}$ and hence $\frac{25}{36}$	M1 provided $\ln 2$ involved A1 2 AG; necessary detail required including clear use of an exact value from (ii) 7
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4 (i)	Attempt correct process for composition Obtain $(7 \text{ and hence } 0)$	M1 numerical or algebraic A1 2
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(ii)	Attempt to find x -intercept Obtain $x \leq 7$	M1 A1 2 or equiv; condone use of <
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(iii)	Attempt correct process for finding inverse Obtain $\pm(2-y)^3 - 1$ or $\pm(2-x)^3 - 1$ Obtain correct $(2-x)^3 - 1$	M1 A1 A1 3 or equiv in terms of x
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(iv)	Refer to reflection in $y = x$	B1 1 or clear equiv 8

5 (i)	Obtain derivative of form $kx(x^2 + 1)^7$ Obtain $16x(x^2 + 1)^7$ Equate first derivative to 0 and confirm $x = 0$ or substitute $x = 0$ and verify first derivative zero Refer, in some way, to $x^2 + 1 = 0$ having no root	M1 any constant k A1 or equiv M1 AG; allow for deriv of form $kx(x^2 + 1)^7$ A1 4 or equiv

(ii)	Attempt use of product rule Obtain $16(x^2 + 1)^7 + \dots$ Obtain $\dots + 224x^2(x^2 + 1)^6$ Substitute 0 in attempt at second derivative Obtain 16	*M1 obtaining $\dots + \dots$ form A1 $\sqrt{}$ follow their $kx(x^2 + 1)^7$ A1 $\sqrt{}$ follow their $kx(x^2 + 1)^7$; or unsimplified equiv M1 dep *M A1 5 from second derivative which is correct at some point
9		

6	Integrate e^{3x} to obtain $\frac{1}{3}e^{3x}$ or $e^{-\frac{1}{2}x}$ to obtain $-2e^{-\frac{1}{2}x}$ Obtain indefinite integral of form $m_1e^{3x} + m_2e^{-\frac{1}{2}x}$ Obtain correct $\frac{1}{3}ke^{3x} - 2(k-2)e^{-\frac{1}{2}x}$ Obtain $e^{3\ln 4} = 64$ or $e^{-\frac{1}{2}\ln 4} = \frac{1}{2}$ Apply limits and equate to 185 Obtain $\frac{64}{3}k - (k-2) - \frac{1}{3}k + 2(k-2) = 185$ Obtain $\frac{17}{2}$	B1 or both M1 any constants m_1 and m_2 A1 or equiv B1 or both M1 including substitution of lower limit A1 or equiv A1 7 or equiv
7		

7 (a)	<u>Either:</u> State or imply either $\frac{dA}{dr} = 2\pi r$ or $\frac{dA}{dt} = 250$ Attempt manipulation of derivatives to find $\frac{dr}{dt}$ Obtain correct $\frac{250}{2\pi r}$ Obtain 1.6 <u>Or:</u> Attempt to express r in terms of t Obtain $r = \sqrt{\frac{250t}{\pi}}$ Differentiate $kt^{\frac{1}{2}}$ to produce $\frac{1}{2}kt^{-\frac{1}{2}}$ Substitute $t = 7.6$ to obtain 1.6	B1 or both M1 using multiplication / division A1 or equiv A1 4 or equiv; allow greater accuracy M1 using $A = 250t$ A1 or equiv M1 any constant k A1 (4) allow greater accuracy

- (b) State $\frac{dm}{dt} = -150ke^{-kt}$ B1
 Equate to $(\pm)3$ and attempt value for t M1 using valid process; condone sign confusion
 Obtain $-\frac{1}{k}\ln\left(\frac{1}{50k}\right)$ or $\frac{1}{k}\ln(50k)$ or $\frac{\ln 50 + \ln k}{k}$ A1 3 or equiv but with correct treatment of signs
7

- 8 (i) State scale factor is $\sqrt{2}$ B1 allow 1.4
 State translation is in negative x -direction ... B1 or clear equiv
 ... by $\frac{3}{2}$ units B1 3
- (ii) Draw (more or less) correct sketch of $y = \sqrt{2x+3}$ B1 'starting' at point on negative x -axis
 Draw (more or less) correct sketch of $y = \frac{N}{x^3}$ B1 showing both branches
 Indicate one point of intersection B1 3 with both sketches correct
 [SC: if neither sketch complete or correct but diagram correct for both in first quadrant B1]
- (iii) (a) Substitute 1.9037 into $x = N^{\frac{1}{3}}(2x+3)^{-\frac{1}{6}}$ M1 or into equation $\sqrt{2x+3} = \frac{N}{x^3}$; or equiv
 Obtain 18 or value rounding to 18 A1 2 with no error seen
- (b) State or imply $2.6282 = N^{\frac{1}{3}}(2 \times 2.6022 + 3)^{-\frac{1}{6}}$ B1
 Attempt solution for N M1 using correct process
 Obtain 52 A1 3 concluding with integer value
11

- 9 (i) Identify $\tan 55^\circ$ as $\tan(45^\circ + 10^\circ)$ B1 or equiv
 Use correct angle sum formula for $\tan(A+B)$ M1 or equiv
 Obtain $\frac{1+p}{1-p}$ A1 3 with $\tan 45^\circ$ replaced by 1
- (ii) Either: Attempt use of identity for $\tan 2A$ *M1 linking 10° and 5°
 Obtain $p = \frac{2t}{1-t^2}$ A1
 Attempt solution for t of quadratic equation M1 dep *M
 Obtain $\frac{-1 + \sqrt{1+p^2}}{p}$ A1 4 or equiv; and no second expression
- Or (1): Attempt expansion of $\tan(60^\circ - 55^\circ)$ *M1
 Obtain $\frac{\sqrt{3} - \frac{1+p}{1-p}}{1 + \sqrt{3} \frac{1+p}{1-p}}$ A1√ follow their answer from (i)
 Attempt simplification to remove denominators M1 dep *M
 Obtain $\frac{\sqrt{3}(1-p) - (1+p)}{1-p + \sqrt{3}(1+p)}$ A1 (4) or equiv

<u>Or (2):</u> State or imply $\tan 15^\circ = 2 - \sqrt{3}$	B1
Attempt expansion of $\tan(15^\circ - 10^\circ)$	M1 with exact attempt for $\tan 15^\circ$
Obtain $\frac{2 - \sqrt{3} - p}{1 + p(2 - \sqrt{3})}$	A2 (4)
<u>Or (3):</u> State or imply $\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1}$	B1 or exact equiv
Attempt expansion of $\tan(15^\circ - 10^\circ)$	M1 with exact attempt for $\tan 15^\circ$
Obtain $\frac{\sqrt{3}-1-p\sqrt{3}-p}{\sqrt{3}+1+p\sqrt{3}-p}$	A2 (4) or equiv
<u>Or (4):</u> Attempt expansion of $\tan(10^\circ - 5^\circ)$	*M1
Obtain $t = \frac{p-t}{1+pt}$	A1
Attempt solution for t of quadratic equation	M1 dep *M
Obtain $\frac{-2 + \sqrt{4+4p^2}}{2p}$	A1 (4) or equiv; and no second expression

(iii) Attempt expansion of both sides	M1
Obtain $3\sin\theta\cos 10^\circ + 3\cos\theta\sin 10^\circ =$ $7\cos\theta\cos 10^\circ + 7\sin\theta\sin 10^\circ$	A1 or equiv
Attempt division throughout by $\cos\theta\cos 10^\circ$	M1 or by $\cos\theta$ (or $\cos 10^\circ$) only
Obtain $3t + 3p = 7 + 7pt$	A1 or equiv
Obtain $\frac{3p-7}{7p-3}$	A1 5 or equiv

12
