

4724 Core Mathematics 4

1	<u>Long division method</u>		
	Correct leading term x^2 in quotient	B1	
	Evidence of correct div process	M1	Sufficient to convince
	(Quotient =) $x^2 + 6x - 4$	A1	
	(Remainder =) $11x + 9$	A1	
	<u>Identity method</u>		
	$x^4 + 11x^3 + 28x^2 + 3x + 1 = Q(x^2 + 5x + 2) + R$	M1	
	$Q = ax^2 + bx + c$ or $x^2 + bx + c$; $R = dx + e$ & ≥ 3 ops	M1	N.B. $a = 1 \Rightarrow 1$ of the 3 ops
	$a = 1, b = 6, c = -4, d = 11, e = 9$ (for all 5)	A2	S.R. <u>B1</u> for 3 of these
		4	
2 (i)	Find at least 2 of $(\overrightarrow{AB}$ or $\overrightarrow{BA}), (\overrightarrow{BC}$ or $\overrightarrow{CB}), (\overrightarrow{AC}$ or $\overrightarrow{CA})$	M1	irrespect of label; any notation
	Use correct method to find scal prod of any 2 vectors	M1	<u>or</u> use corr meth for modulus
	Use $\overrightarrow{AB} \cdot \overrightarrow{BC} = 0$ or $\frac{\overrightarrow{AB} \cdot \overrightarrow{BC}}{ \overrightarrow{AB} \overrightarrow{BC} } = 0$	M1	or use $ \overrightarrow{AB} ^2 + \overrightarrow{BC} ^2 = \overrightarrow{AC} ^2$
	Obtain $p = 1$	(dep 3 @ M1) A1	4

(ii)	Use equal ratios of appropriate vectors	M1	or scalar product method
	Obtain $p = -8$	A1	2
		6	
3	Use $\cos 2x = a \cos^2 x + b \pm \cos^2 x - \sin^2 x / 1 - 2\sin^2 x$	*M1	
	Obtain $\lambda + \mu \sec^2 x$	dep*M1	using 'reasonable' Pythag attempt
	$\int \lambda + \mu \sec^2 x \, dx = \lambda x + \mu \tan x$	A1	(λ or μ may be 0 here/prev line)
	Obtain correct result $2x - \tan x$	A1	no follow-through
	$\frac{1}{6} \pi - \sqrt{3} + 1$ ISW	A1	exact answer required
		5	

4	Attempt to connect du and dt or find $\frac{du}{dt}$ or $\frac{dt}{du}$	M1	not $du = dt$ but no accuracy
	$du = \frac{1}{t} dt$ or $\frac{du}{dt} = \frac{1}{t}$ or $dt = e^{u-2} du$ or $\frac{dt}{du} = e^{u-2}$	A1	
	Indef int $\rightarrow \int \frac{1}{u^2} (du)$	A1	no t or dt in evidence
	$= -\frac{1}{u}$	A1	
	Attempt to change limits if working with $f(u)$	M1	or re-subst & use 1 and e
	$\frac{1}{6}$ ISW	A1	ln e must be changed to 1, ln 1 to 0
		6	

5	(i) $(1+x)^{\frac{1}{3}} = 1 + \frac{1}{3}x + \dots$	B1	
	$\dots -\frac{1}{9}x^2$	B1 2	$-\frac{2}{18}x^2$ acceptable
	(ii) (a) $(8+16x)^{\frac{1}{3}} = 8^{\frac{1}{3}}(1+2x)^{\frac{1}{3}}$	B1	not $16^{\frac{1}{3}}(\frac{1}{2}+x)^{\frac{1}{3}}$
	$(1+2x)^{\frac{1}{3}} =$ their (i) expansion with $2x$ replacing x	M1	not dep on prev B1
	$= 1 + \frac{2}{3}x - \frac{4}{9}x^2 + \dots$	$\sqrt{A1}$	$-\frac{8}{18}x^2$ acceptable
	Required expansion = 2 (expansion just found)	$\sqrt{B1}$ 4	accept equiv fractions
N.B. If not based on part (i), award M1 for $8^{\frac{1}{3}} + \frac{1}{3} \cdot 8^{-\frac{2}{3}}(16x) + \frac{\frac{1}{3} \cdot -\frac{2}{3}}{1.2} 8^{-\frac{5}{3}}(16x)^2$, allowing $16x^2$ for $(16x)^2$, with 3 @ A1 for $2\dots + \frac{4}{3}x\dots - \frac{8}{9}x^2$, accepting equivalent fractions & ISW			
	(ii) (b) $-\frac{1}{2} < x < \frac{1}{2}$ or $ x < \frac{1}{2}$	B1 1	no equality
		7	
6	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	M1	quoted/implied
	$\frac{dx}{dt} = 9 - \frac{9}{9t}$ ISW	B1	
	$\frac{dy}{dt} = 3t^2 - \frac{3t^2}{t^3}$ ISW	B1	
	Stating/implying $\frac{3t^2 - \frac{3}{t}}{9 - \frac{1}{t}} = 3 \Rightarrow t^2 = 9$ or $t^3 - 9t = 0$	A1	WWW, totally correct at this stage
	$t = 3$ as final ans with clear log indication of invalidity of -3 ; ignore (non) mention of $t = 0$	A2	S.R. A1 if $t = \pm 3$ or $t = -3$ or $(t = 3 \text{ \& wrong/no indication})$
		6	
7	Treat $\frac{d}{dx}(x^2y)$ as a product	M1	
	$\frac{d}{dx}(y^3) = 3y^2 \frac{dy}{dx}$	B1	
	$3x^2 + 2x^2 \frac{dy}{dx} + 4xy = 3y^2 \frac{dy}{dx}$	A1	Ignore $\frac{dy}{dx} =$ if not used
	Subst (2, 1) and solve for $\frac{dy}{dx}$ or vice-versa	M1	
	$\frac{dy}{dx} = -4$ WWW	A1	
	grad normal = $-\frac{1}{\text{their } \frac{dy}{dx}}$	$\sqrt{A1}$	stated or used
	Find eqn of line, through (2, 1), with either gradient	M1	using their $\frac{dy}{dx}$ or $-\frac{1}{\text{their } \frac{dy}{dx}}$
	$x - 4y + 2 = 0$	A1	AEF with integral coefficients
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8 (i)	$-\sin x e^{\cos x}$	B1	1
(ii)	$\int \sin x e^{\cos x} dx = -e^{\cos x}$	B1	anywhere in part (ii)
	Parts with split $u = \cos x, dv = \sin x e^{\cos x}$	M1	result $f(x) + / - \int g(x) dx$
	Indef Integ, 1st stage $-\cos x e^{\cos x} - \int \sin x e^{\cos x} dx$	A1	accept $\dots - \int -e^{\cos x} \cdot -\sin x dx$
	Second stage $= -\cos x e^{\cos x} + e^{\cos x}$	*A1	
	Final answer = 1	dep*A2	6

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9 (i)	P is $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$	B1	
	direction vector of ℓ is $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and of $\overrightarrow{OP}$ is their P	$\sqrt{B1}$	
	Use $\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{ \mathbf{a} \mathbf{b} }$ for $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and their OP	M1	
	$\theta = 35.3$ or better (0.615... rad)	A1	4

(ii)	Use $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 3+t \\ 1-t \\ 1+2t \end{pmatrix} = 0$	M1	
	$1(3+t) - 1(1-t) + 2(1+2t) = 0$	A1	
	$t = -\frac{2}{3}$	A1	
	Subst. into $\begin{pmatrix} 3+t \\ 1-t \\ 1+2t \end{pmatrix}$ to produce $\begin{pmatrix} 7/3 \\ 5/3 \\ -1/3 \end{pmatrix}$ ISW	A1	4

(iii)	Use $\sqrt{x^2 + y^2 + z^2}$ where $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ is part (ii) answer	M1	
	Obtain $\sqrt{\frac{75}{9}}$ AEF, 2.89 or better (2.8867513...)	A1	2

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10 (i) $\frac{\frac{1}{3}}{3-x} \dots\dots\dots - \frac{\frac{1}{3}}{6-x}$ B1+1 2

(ii) (a) Separate variables $\int \frac{1}{(3-x)(6-x)} dx = \int k dt$ M1 or invert both sides

Style: For the M1, dx & dt must appear on correct sides or there must be $\int$ sign on both sides

Change $\frac{1}{(3-x)(6-x)}$ into partial fractions from (i) $\sqrt{B1}$

$\int \frac{A}{3-x} dx = \left(-A \text{ or } -\frac{1}{A}\right) \ln(3-x)$ B1 or $\int \frac{B}{6-x} dx = \left(-B \text{ or } -\frac{1}{B}\right) \ln(6-x)$

$-\frac{1}{3} \ln(3-x) + \frac{1}{3} \ln(6-x) = kt (+c)$ $\sqrt{A1}$ f.t. from wrong multiples in (i)

Subst ($x = 0, t = 0$) & ($x = 1, t = 1$) into eqn with 'c' M1 and solve for 'k'

Use $\ln a + \ln b = \ln ab$ or $\ln a - \ln b = \ln \frac{a}{b}$ M1

Obtain $k = \frac{1}{3} \ln \frac{5}{4}$ with sufficient working & WWW A1 7 AG

(b) Substitute $k = \frac{1}{3} \ln \frac{5}{4}$, $t = 2$ & their value of 'c' *M1

Reduce to an eqn of form $\frac{6-x}{3-x} = \lambda$ dep*M1 where λ is a const

Obtain $x = \frac{27}{17}$ or 1.6 or better (1.5882353...) A2 4 S.R. A1 $\sqrt{}$ for $x = \frac{3\lambda - 6}{\lambda - 1}$

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