OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

Core Mathematics 3

4723

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Wednesday 20 January 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Find $\int \frac{10}{(2x-7)^2}\,\mathrm{d}x$. **[3]**

2 The angle θ is such that $0^\circ < \theta < 90^\circ$.

(i) Given that θ satisfies the equation $6\sin 2\theta = 5\cos\theta$, find the exact value of $\sin\theta$. **[3]**

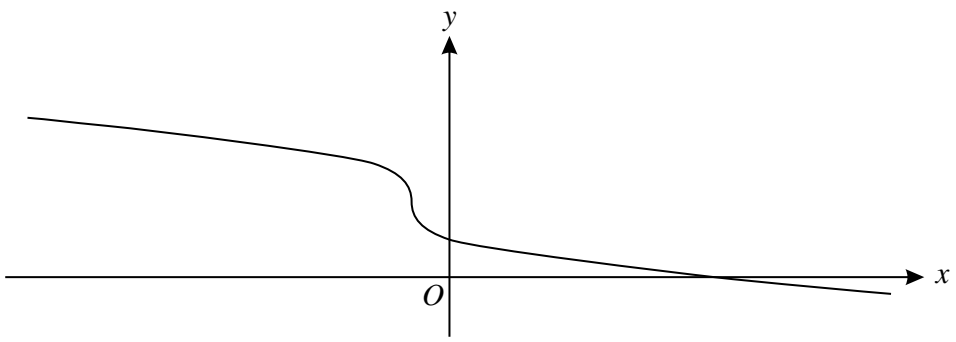
(ii) Given instead that θ satisfies the equation $8\cos\theta\,\mathrm{cosec}^2\theta = 3$, find the exact value of $\cos\theta$. **[5]**

3 **(i)** Find, in simplified form, the exact value of $\int_{10}^{20} \frac{60}{x}\,\mathrm{d}x$. **[2]**

(ii) Use Simpson's rule with two strips to find an approximation to $\int_{10}^{20} \frac{60}{x}\,\mathrm{d}x$. **[3]**

(iii) Use your answers to parts **(i)** and **(ii)** to show that $\ln 2 \approx \frac{25}{36}$. **[2]**

4



The function f is defined for all real values of x by

$$\mathrm{f}(x) = 2 - \sqrt[3]{x+1}.$$

The diagram shows the graph of $y = \mathrm{f}(x)$.

(i) Evaluate $\mathrm{ff}(-126)$. **[2]**

(ii) Find the set of values of x for which $\mathrm{f}(x) = |\mathrm{f}(x)|$. **[2]**

(iii) Find an expression for $\mathrm{f}^{-1}(x)$. **[3]**

(iv) State how the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ are related geometrically. **[1]**

5 The equation of a curve is $y = (x^2 + 1)^8$.

(i) Find an expression for $\frac{dy}{dx}$ and hence show that the only stationary point on the curve is the point for which $x = 0$. **[4]**

(ii) Find an expression for $\frac{d^2y}{dx^2}$ and hence find the value of $\frac{d^2y}{dx^2}$ at the stationary point. **[5]**

6 Given that

$$\int_0^{\ln 4} \left(ke^{3x} + (k-2)e^{-\frac{1}{2}x}\right) dx = 185,$$

find the value of the constant k. **[7]**

7 **(a)** Leaking oil is forming a circular patch on the surface of the sea. The area of the patch is increasing at a rate of 250 square metres per hour. Find the rate at which the radius of the patch is increasing at the instant when the area of the patch is 1900 square metres. Give your answer correct to 2 significant figures. **[4]**

(b) The mass of a substance is decreasing exponentially. Its mass now is 150 grams and its mass, m grams, at a time t years from now is given by

$$m = 150e^{-kt},$$

where k is a positive constant. Find, in terms of k, the number of years from now at which the mass will be decreasing at a rate of 3 grams per year. **[3]**

8 **(i)** The curve $y = \sqrt{x}$ can be transformed to the curve $y = \sqrt{2x+3}$ by means of a stretch parallel to the y-axis followed by a translation. State the scale factor of the stretch and give details of the translation. **[3]**

(ii) It is given that N is a positive integer. By sketching on a single diagram the graphs of $y = \sqrt{2x+3}$ and $y = \frac{N}{x^3}$, show that the equation

$$\sqrt{2x+3} = \frac{N}{x^3}$$

has exactly one real root. **[3]**

(iii) A sequence $x_1, x_2, x_3, \ldots$ has the property that

$$x_{n+1} = N^{\frac{1}{3}}(2x_n + 3)^{-\frac{1}{6}}.$$

For certain values of x_1 and N, it is given that the sequence converges to the root of the equation $\sqrt{2x+3} = \frac{N}{x^3}$.

(a) Find the value of the integer N for which the sequence converges to the value 1.9037 (correct to 4 decimal places). **[2]**

(b) Find the value of the integer N for which, correct to 4 decimal places, $x_3 = 2.6022$ and $x_4 = 2.6282$. **[3]**

[Question 9 is printed overleaf.]

9 The value of $\tan 10^\circ$ is denoted by p. Find, in terms of p, the value of

(i) $\tan 55^\circ$, **[3]**

(ii) $\tan 5^\circ$, **[4]**

(iii) $\tan\theta$, where θ satisfies the equation $3\sin(\theta + 10^\circ) = 7\cos(\theta - 10^\circ)$. **[5]**