GCE

Mathematics

Advanced Subsidiary GCE

Unit **4721:** Core Mathematics 1

Mark Scheme for January 2011

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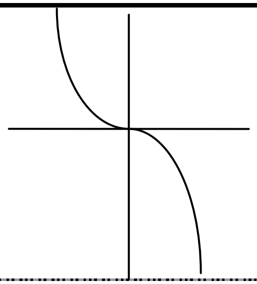
1 (i)	$\sqrt{(-2-6)^2+(7-1)^2}$ = 10	**M1** **A1**	**2**	Use of $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	3 out of 4 substitutions correct Look out for no square root, $(x_2+x_1)^2$ etc. **M0**
(ii)	$\frac{7-1}{-2-6}$ $=-\frac{3}{4}$	**M1** **A1**	**2**	uses $\frac{y_2-y_1}{x_2-x_1}$ o.e. **ISW**	3 out of 4 substitutions correct Allow –0.75 $\frac{3}{-4}$ etc.
(iii)	Gradient of given line = $\frac{4}{3}$ $-\frac{3}{4}\times\frac{4}{3}=-1$ So lines are perpendicular	**M1** **B1ft** **B1**	**3** **7**	Attempt to rearrange equation to make *y* the subject **OR** attempt to find the gradient using points on the line Correct conclusion for their gradients States $-\frac{3}{4}\times\frac{4}{3}=-1$ or "negative reciprocal" relating to the correct values **www**	Must at least isolate *y*
2	$2x^3+9x^2-2px^2-9px+10x-10p$ $=2x^3+qx^2-8x-4q$ *p* = 2 and *q* = 5	**M1*** **DM1** **A1**	**3** **3**	Attempt to expand both sides **OR** to substitute 2 values of *x* into both expressions **OR** to express at least one side as a product of three factors Valid method to obtain either p or q Both values correct	If expanding, minimum of 5 terms on LHS and 3terms on RHS If comparing coefficients, must be of corresponding terms **SR** Spotted solutions **B1** one correct **B2** other correct
3 (i)	$8^{\frac{1}{2}}$	**B1**	**1**		Allow $8^{0.5}$ Condone p= $\frac{1}{2}$, just "$\frac{1}{2}$" seen as answer **www**
(ii)	8^{-2}	**B1**	**1**		Condone p = -2, just "-2" seen as answer **www** $\frac{1}{8^2}$ only not enough
(iii)	$2^8=\left(8^{\frac{1}{3}}\right)^8$ $=8^{\frac{8}{3}}$	**M1** **M1** **A1**	**3** **5**	2^8 or $2^6=8^2$ soi $2=8^{\frac{1}{3}}$ soi o.e.	Condone p= $\frac{8}{3}$, just "$\frac{8}{3}$" seen as answer **www** 2^3= 8 not enough for second **M** mark

4	$u^2 - 5u + 4 = 0$ $(u-1)(u-4) = 0$ $u = 1$ or $u = 4$ $3x - 2 = \pm 1$ or $3x - 2 = \pm 2$ $x = 1$ or $\frac{1}{3}$ or $\frac{4}{3}$ or 0	**M1*** **DM1** **A1** **M1** **A1** **A1**	 **6** **6**	Use the given substitution to obtain a quadratic or factorise into 2 brackets each containing $(3x-2)^2$ Correct method to solve a quadratic Correct values for u Attempt to square root and rearrange to obtain x **OR** to expand, rearrange and solve quadratic (at least one) 2 correct values All 4 correct values ($\frac{0}{3}$ = **A0**)	**No marks** if evidence of "square rooting" e.g. "$(3x-2)^2$ - 5$(3x-2)$ + 2 (or 4) = 0" **No marks** if straight to quadratic formula to get x = "1" x = "4" and no further working **SR 1)** If **M0** Spotted solutions **www B1** each Justifies 4 solutions exactly **B2** **SR 2)** If first 3 marks awarded, spotted solutions 2 correct **B1** Other 2 correct **B1** Justifies 4 solutions exactly **B1** <u>Alternative scheme for candidates who multiply out:</u> Attempt to expand $(3x-2)^4$ and $(3x-2)^2$ **M1** $81x^4 - 216x^3 + 171x^2 - 36x = 0$ **A1** $x = 0$ a solution or x a factor of the quartic **A1** Attempt to use factor theorem to factorise their cubic **M1*** Correct method to solve quadratic **DM1** All 4 solutions correct **A1**
5 (i)		**M1** **A1**	 **2**	Negative cubic through (0, 0) (may have max and min) Must have reasonable rotational symmetry. Cannot be a finite "plot". Allow negative gradient at origin. Correct curvature at both ends.	Must be continuous. Allow slight curve towards or away from y-axis at one end, but not both.
(ii)	$y = -(x-3)^3$	**M1** **A1**	 **2**	$\pm(x-3)^3$ seen or $y = (3-x)^3$	 Must have "$y =$ " for A mark **SR** $y = -(x-3)^2$ **B1**
(iii)	Stretch scale factor 5 parallel to y-axis	**B1** **B1**	 **2** **6**	o.e. e.g. scale factor $\frac{1}{\sqrt[3]{5}}$ parallel to the x axis.	Allow "factor" for "scale factor" For "parallel to the y axis" allow "vertically", "in the y direction". **Do not accept** "in/on/across/up/along the y axis"

6 (i)	$y = 5x^{-2} - \frac{1}{4}x^{-1} + x$ $\frac{dy}{dx} = -10x^{-3} + \frac{1}{4}x^{-2} + 1$	**M1** **A1** **A1** **A1**	 **4**	x^{-2} used for $\frac{1}{x^2}$ **OR** x^{-1} used for $\frac{1}{x}$ soi, **OR** x correctly differentiated kx^{-3} or kx^{-2} from differentiating Two fully correct terms Completely correct	Look out for: $y = 5x^{-2} - 4x^{-1} + x$ followed by $\frac{dy}{dx} = -10x^{-3} + 4x^{-2} + 1$ and then the correct answer. This is **M1 A1 A1 A0** $4x^{-1}$ is **NOT** a misread
(ii)	$\frac{d^2y}{dx^2} = 30x^{-4} - \frac{1}{2}x^{-3}$	**M1** **A1**	 **2** **6**	Attempt to differentiate their $\frac{dy}{dx}$ (one term correctly differentiated) Completely correct	Allow a sign slip in coefficient for M mark **NB** Only penalise "+ c" first time seen in the question

7 (i)	$4(x^2+3x)-3$ $=4\left[\left(x+\frac{3}{2}\right)^2-\frac{9}{4}\right]-3$ $=4\left(x+\frac{3}{2}\right)^2-12$	**B1** **B1** **M1** **A1**	**4**	$p=4$ $q=\frac{3}{2}$ $r=-3-4q^2$ or $r=-\frac{3}{4}-q^2$ $r=-12$ (from $q=\pm1.5$)	If *p*, *q*, *r* found correctly, then **ISW** slips in format. $4(x+1.5)^2+12$ **B1 B1 M0 A0** $4(x+1.5)-12$ **B1 B1 M1 A1 (BOD)** $4(x+1.5x)^2-12$ **B1 B0 M1 A0** $4(x^2+1.5)^2-12$ **B1 B0 M1 A0** $4(x-1.5)^2-12$ **B1 B0 M1 A1** $4x(x+1.5)^2-12$ **B0 B1M1A1**
(ii)	$\frac{-12\pm\sqrt{12^2-4\times4\times-3}}{2\times4}$ $=\frac{-12\pm\sqrt{192}}{8}$ $=\frac{-12\pm8\sqrt{3}}{8}$ $=-\frac{3}{2}\pm\sqrt{3}$ OR: $4\left(x+\frac{3}{2}\right)^2-12=0$ $x+\frac{3}{2}=\pm\sqrt{3}$ $x=-\frac{3}{2}\pm\sqrt{3}$	**M1** **A1** **B1** **A1** **M1** **A1ft** **A1** **A1**	**4**	Correct method to solve quadratic $\frac{-12\pm\sqrt{192}}{8}$ or $\frac{-3\pm\sqrt{12}}{2}$ √192=8√3 or √12=2√3 from correct b^2-4ac $\frac{-3\pm2\sqrt{3}}{2}$ or $-\frac{12}{8}\pm\sqrt{3}, -\frac{6}{4}\pm\sqrt{3}$ Must have ± for method mark *x* +1.5 **ft** *x* + *q* from part(i) www in LHS in part (ii) ±√3 Do not **ISW**	Not for 2(*x* + *q*) = ... **SR** One correct root **www B1**
(iii)	$12^2-4\times4\times(-k)=0$ $144+16k=0$ $k=-9$ **OR (see next page)**	**M1** **A1** **A1**		Attempts $b^2-4ac=0$ or $\sqrt{b^2-4ac}=0$ involving k. If b^2-4ac not quoted then expression must be correct. Correct, unsimplified expression	Other alternative methods a) Attempt to factorise into two equal brackets, (may divide by 4 first – must be correct) **M1** Equate coefficient of *x* to 12 (or 3) **A1** $k=-9$ **A1** b) Uses differentiation to find x ordinate of turning point and uses this to form equation in *k* **M1** Correct equation in *k* **A1** $k=-9$ **A1**

7(iii) cont.	$4x^2+12x=k$				Must involve k in their working to gain the method marks in this scheme
	$4(x+\frac{3}{2})^2-9=k$	**M1**		Attempts completing the square in given equation or factorises to $(2x+3)^2-9=k$	
	Equal roots when $x=-\frac{3}{2}$	**M1**		Substitutes $x=-\frac{3}{2}$	
	$k=-9$	**A1**	**3** **[11]**		
8 (i)	$\frac{dy}{dx}=6-2x$	**M1** **A1**		Attempt to differentiate $\pm y$ Correct expression **cao**	One correct non-zero term
	When $x=5$, $6-2x=-4$	**M1**		Substitute $x=5$ into their $\frac{dy}{dx}$	
	When $x=5$, $y=12$	**B1**		Correct y coordinate	
	$y-12=-4(x-5)$	**M1**		Correct equation of straight line through (5, their y), their non-zero, numerical gradient	Allow $\frac{y-12}{x-5}$ = their gradient
	$4x+y-32=0$	**A1**	**6**	Shows rearrangement to correct form	If using $y=mx+c$ must attempt at evaluating c Allow any correct form e.g. $0=2y+8x-64$ etc.
(ii)	Q is point (8, 0)	**B1ft**		ft from line in (i)	.
	Midpoint of $PQ=\left(\frac{5+8}{2},\frac{12+0}{2}\right)$	**M1**		Uses $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ o.e. for their P,Q	
	$=\left(\frac{13}{2},6\right)$	**A1**	**3**		Do not accept $\left(\frac{13}{2},\frac{12}{2}\right)$
(iii)	$6-2x=0$	**M1**		Solution of their $\frac{dy}{dx}=0$	Alternatives for Method Mark a) attempts completion of square with $\pm(x-3)^2$ b) attempts to solve quadratic (usual scheme) and to find the mid-point of the two roots c) attempts to use $x=-\frac{b}{2a}$ (allow one sign slip on substitution)
	(Line of symmetry is) $x=3$	**A1**	**2**	Allow from $\pm[16-(x-3)^2]$, $\pm[6-2x=0]$	
(iv)	$x<3$	**M1**		$x<$ their3 or $x>$ their3 **OR** attempt to solve their $\frac{dy}{dx}>0$	May solve $\frac{dy}{dx}=0$ then use $\frac{d^2y}{dx^2}<0$ implies maximum point for the method mark, or sketch of curve
		A1	**2** **[13]**	Allow from $\pm[16-(x-3)^2]$**,** $\pm[6-2x=0]$ in (iii)	Allow $x\le 3$

9 (i)	Centre (4, 1) $(x-4)^2+(y-1)^2-16-1-3=0$ $(x-4)^2+(y-1)^2=20$ Radius $=\sqrt{20}$	B1 M1 A1	 3	Correct centre Correct method to find r^2 Correct radius	 r^2 =(± their 4)^2 + (±their 1)^2 + 3 soi $\pm\sqrt{20}$ is **A0** Ignore incorrect simplification of $\sqrt{20}$
(ii)	$k=1\pm\sqrt{20}$ $k=1\pm2\sqrt{5}$	M1 A1ft A1	 3	y ordinate of their centre ± their radius or Both correct, unsimplified values **cao**	Alternatives for method mark : a) Substitutes k for y and uses $b^2-4ac=0$ to obtain quadratic in k b) Recognises x = 4 is equation of normal, substitutes into circle equation and solves for k. **SR** k = 1+ √20 or k = 1 – √20 or better **www B1**
(iii)	$MT^2=r^2-2^2$ MT = 4 ST = 8	M1 A1ft A1	 3	Correct use of Pythagoras' theorem involving MT (or SM) Correct value of MT for their r **cao**	**SR** ST=8 from particular S and T co-ordinates [e.g. horizontal chord calculated as (0,3) and (8,3)] **B1** Justifies solution the same for all possible chords **B2**
(iv)	$x=2y+12$ $(2y+8)^2+(y-1)^2=20$ $4y^2+32y+64+y^2-2y+1=20$ $5y^2+30y+45=0$ $y^2+6y+9=0$ $(y+3)^2=0$ $y=-3$ $x=6$	M1* A1 A1 DM1 A1 A1		Attempt to solve equations simultaneously Correct unsimplified expression, may be $(12+2y)^2+y^2-8(12+2y)-2y-3=0$ Obtain correct 3 term quadratic Correct method to solve quadratic of form $ax^2+bx+c=0$ $(b\neq0)$ y value correct, no extra solutions x value correct **ISW**	Must be a clear attempt to reduce to one variable using equation of line and either form of equation of circle. Condone poor algebra for first mark. If y eliminated: $(x-4)^2+\left(\frac{1}{2}x-7\right)^2=20$ Or $x^2+\left(\frac{1}{2}x-6\right)^2-8x-2\left(\frac{1}{2}x-6\right)-3=0$ Leading to $x^2-12x+36=0$
	OR $y-1=-2(x-4)$ Solve simultaneously with $y=\frac{1}{2}x-6$ $x=6$ $y=-3$ States line is tangent as meets at one point or verifies (6, -3) lies on circle	 M1 A1 M1 A1 A1 B1	 6 15	 Attempt to find equation of radius/normal Correct equation Allow showing distance between (6,-3) and (4,1) = √20	 **SR** Correct coordinates spotted or from trial and improvement **www B2**

Allocation of method mark for solving a quadratic

e.g. $4x^2 + 12x - 3 = 0$

By factorisation
– when expanded, quadratic term and one other term must be correct (with correct sign):

$(2x+1)(2x-3) = 0$	M1	$4x^2$ and -3 obtained from expansion
$(4x+4)(x+2) = 0$	M1	$4x^2$ and $+12x$ obtained from expansion
$(4x-1)(x-3) = 0$	M0	only x^2 term correct

By formula
- if the formula is quoted correctly first, allow one sign slip in substituting values into it:
$a = 4$, $b = 12$, $c = -3$

$$\frac{12 \pm \sqrt{(12)^2 - 4\times 4\times -3}}{8}$$ gains M1 (minus sign incorrect at start of formula)

$$\frac{-12 \pm \sqrt{(12)^2 - 4\times 4\times 3}}{8}$$ gains M1 (3 for c instead of −3)

$$\frac{12 \pm \sqrt{(12)^2 - 4\times 4\times 3}}{8}$$ M0 (2 sign errors: initial sign and c incorrect)

- if the formula is not quoted, then no errors at all are allowed in substitution.

By completing the square

$$4x^2 + 12x - 3 = 0$$
$$4\left[\left(x+\frac{3}{2}\right)^2 - \frac{9}{4}\right] - 3 = 0$$
$$\left(x+\frac{3}{2}\right)^2 = 3$$
$$x + \frac{3}{2} = \pm\sqrt{3}$$

The method mark is awarded only at the last line of working
i.e. when ±√ combined constants is seen.

N.B. The value of the combined constants does not have to be correct for the M1 mark

Condone "invisible brackets" if justified by correct later working

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)
Telephone: 01223 553998
Facsimile: 01223 552627
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