

# **Mathematics (MEI)**

Advanced Subsidiary GCE

Unit **4752**: Concepts for Advanced Mathematics

## **Mark Scheme for January 2011**

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## SECTION A


1	11.4 o.e.	2	<b>M1</b> for $12/3 + 12/4 + 12/5 + 12/6$ o.e.	<b>M0</b> unless four terms summed
2	$\frac{1}{2}x^6 + 4x^{\frac{1}{2}} + c$	4	<b>B1</b> for $\frac{1}{2}x^6$ , <b>M1</b> for $kx^{\frac{1}{2}}$ , <b>A1</b> for $k = 4$ <del>4</del> or <b>1</b> , <b>B1</b> for $+c$ dependent on at least one power increased	<del>3</del> allow $\frac{3}{6}x^6$ isw,
3	$\frac{1}{2} \times 1.5 \times (0.6 + 0.7 + 2(2.3+3.1+2.8+1.8))$  = 15.975 rounded to 2 s.f. or more	<b>M2</b>  <b>A1</b>	<b>M1</b> if one error or <b>M2</b> for sum of 5 unsimplified individual trapezia: 2.175, 4.05, 4.425, 3.45, 1.875	basic shape of formula must be correct. Must be 5 strips. <b>M0</b> if pair of brackets omitted or $h = 7.5$ or 1. allow recovery of brackets omitted to obtain correct answer. <b>M0</b> for other than 5 trapezia isw only if 15.975 clearly identified as cross-sectional area
4	(i) (3, 15)	<b>B2</b>	<b>B1</b> for each coordinate	s.c. <b>B0</b> for (3, 5)
4	(ii) (1.5, 5)	<b>B2</b>	<b>B1</b> for each coordinate	s.c. <b>B0</b> for (3, 5)
5	$ar = 6$ and $ar^4 = -48$ $r = -2$ tenth term = 1536  $\frac{-3(1-(-2)^n)}{1-(-2)}$ o.e.  $(-2)^n - 1$	<b>M1</b> <b>M1</b> <b>A1</b>  <b>M1</b>  <b>A1</b>	<b>B2</b> for $r = -2$ www <b>B3</b> for 1536 www  allow <b>M1</b> for $a = 6$ ÷ their $r$ and substitution in GP formula with their $a$ and $r$  c.a.o.	ignore incorrect lettering such as $d = -2$  condone the omission of the brackets round “-2” in the numerator and / or the denominator

6	$a+2d = 24$ and $a + 9d = 3$  $d = -3; a = 30$  $S_{50} - S_{20}$  $-2205$ cao	<b>M1</b> <b>A1</b> <b>A1</b>  <b>M1</b>  <b>A1</b>	if <b>M0</b> , <b>B2</b> for either, <b>B3</b> for both  ft their $a$ and $d$ ; <b>M1</b> for $S_{30} = \frac{30}{2}(u_{21} + u_{50})$ o.e.  <b>B2</b> for $-2205$ www	do not award <b>B2</b> or <b>B3</b> if values clearly obtained fortuitously  $S_{50} = -2175; S_{20} = 30$ $u_{21} = 30 - 20 \times 3 = -30$ $u_{50} = 30 - 49 \times 3 = -117$
7	(i) $17 \log_{10} x$ or $\log_{10} x^{17}$	<b>B2</b>	<b>M1</b> for $5 \log_{10} x$ or $12 \log_{10} x$ or $\log_{10} x^{12}$ as part of the first step	condone omission of base
7	(ii) $-b$	<b>B2</b>	<b>M1</b> for $\log_a 1 = 0$ or $\log_a a = 1$ soi	allow $0 - b$
8	substitution of $\sin^2 \theta = 1 - \cos^2 \theta$ $-5 \cos^2 \theta = \cos \theta$ $\theta = 90$ and $270$ , 102 258  101 and 259	<b>M1</b> <b>A1</b> <b>A1</b> <b>A1</b> <b>A1</b>  <b>SC</b> <b>1</b>	soi or better  accept 101.5(...) and 258.(46...) rounded to 3 or more sf; if <b>M0</b> , allow <b>B1</b> for both of 90 and 270 and <b>B1</b> for 102 and <b>B1</b> for 258 (to 3 or more sf)	if the 4 correct values are presented, ignore any extra values which are outside the required range, but apply a penalty of minus 1 for extra values in the range  if given in radians deduct 1 mark from total awarded (1.57, 1.77, 4.51, 4.71)

9	<p>area sector = <math>\frac{1}{2} \times r^2 \times \frac{\pi}{6} \left[ = \frac{\pi r^2}{12} \right]</math></p> <p>area triangle = <math>\frac{1}{2} \times a^2 \times \sin \frac{\pi}{6} \left[ = \frac{a^2}{4} \right]</math></p> <p><math>\frac{1}{2} a^2 \times \frac{1}{2} = \frac{1}{2} \times r^2 \times \frac{\pi}{6} \times \frac{1}{2}</math></p> <p><math>\frac{a^2}{4} = \frac{\pi r^2}{24}</math> o.e. and completion to given answer</p>	<p><b>M1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>soi</p> <p>soi</p> <p>soi</p>	<p>allow sin30</p> <p>no follow through marks available</p> <p>at least one correct intermediate step required, and no wrong working to obtain given answer</p>
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Section A Total: 36

## SECTION B

10	<p>(i) eqn of AB is <math>y = 3x + 1</math> o.e.</p> <p>their "<math>3x + 1</math>" = <math>4x^2</math></p> <p><math>(4x + 1)(x - 1) = 0</math> o.e. so <math>x = -1/4</math></p> <p>at C, <math>x = -1/4</math>, <math>y = 4 \times (-1/4)^2</math> or <math>3 \times (-1/4) + 1 [=1/4</math> as required]</p>	<p><b>M1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>or equiv in <math>y: y = 4\left(\frac{y-1}{3}\right)^2</math></p> <p>or rearranging and deriving roots <math>y = 4</math> or <math>1/4</math></p> <p>condone verification by showing lhs = rhs o.e.</p> <p>or <math>y = 1/4</math> implies <math>x = \pm 1/4</math> so at C <math>x = -1/4</math></p>	<p><b>SC3</b> for verifying that A, B and C are collinear and that C also lies on the curve</p> <p><b>SC2</b> for verifying that A, B and C are collinear by showing that gradient of AB = AC (for example) or showing C lies on AB</p> <p>solely verifying that C lies on the curve scores 0</p>
10	<p>(ii) <math>y' = 8x</math></p> <p>at A <math>y' = 8</math></p> <p>eqn of tgt at A</p> <p><math>y - 4 = \text{their "8"}(x - 1)</math></p> <p><math>y = 8x - 4</math></p> <p>at C <math>y' = 8 \times -1/4 [= -2]</math></p> <p><math>y - 1/4 = -2(x - (-1/4))</math> or other unsimplified equivalent to obtain given result.</p> <p>allow correct verification that <math>(-1/4, 1/4)</math> lies on given line</p>	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>ft their gradient</p> <p>NB if <math>m = -2</math> obtained from given answer or only showing that <math>(-1/4, 1/4)</math> lies on given line <math>y = -2x - 1/4</math> then 0 marks.</p>	<p>gradient must follow from evaluation of </p> <p>condone unsimplified versions of <math>y = 8x - 4</math></p> <p>dependent on award of first <b>M1</b></p> <p><b>SC2</b> if equation of tangent and curve solved simultaneously to correctly show repeated root</p>
10	<p>(iii) their "<math>8x - 4</math>" = <math>-2x - 1/4</math></p> <p><math>y = -1</math> www</p>	<p><b>M1</b></p> <p><b>A1</b></p>	<p>or <math>\frac{y+4}{8} = \frac{y+1/4}{-2}</math></p>	<p>o.e.</p> <p><math>[x = 3/8]</math></p>

11	<p>(i) <math>\frac{x^4}{4} - x^3 - \frac{x^2}{2} + 3x</math>  their integral at 3 – their integral at 1  <math>[= -2.25 - 1.75]</math>   <math>= -4</math> isw  represents area between curve and <math>x</math>  axis between <math>x = 1</math> and 3   negative since below <math>x</math>-axis</p>	<p><b>M2</b>  <b>M1</b>  <b>A1</b>  <b>B1</b>  <b>B1</b></p>	<p><b>M1</b> if at least two terms correct   dependent on integration attempted   or <math>3(x - 1)^2 - 4 [= 0]</math> or better   eg <b>A1</b> for <math>1 \pm \frac{2}{3}\sqrt{3}</math>   allow <math>\leq</math> instead of <math>&lt;</math></p>	<p>ignore <math>+ c</math>   <b>M0</b> for evaluation of <math>x^3 - 3x^2 - x + 3</math> or of  differentiated version   <b>B0</b> for area <i>under</i> or above curve between <math>x = 1</math> and 3</p>
11	<p>(ii) <math>y' = 3x^2 - 6x - 1</math>  their <math>y' = 0</math> so  <math>x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}</math> with <math>a = 3</math>, <math>b = -6</math>  and <math>c = -1</math> isw  <math>x = \frac{6 \pm \sqrt{48}}{6}</math> or better as final answer   <math>\frac{6 - \sqrt{48}}{6} &lt; x &lt; \frac{6 + \sqrt{48}}{6}</math> or ft their  final answer</p>	<p><b>M1</b> <b>M1</b>  <b>M1</b>  <b>A1</b>  <b>B1</b></p>	<p>dependent on differentiation attempted   or <math>3(x - 1)^2 - 4 [= 0]</math> or better   eg <b>A1</b> for <math>1 \pm \frac{2}{3}\sqrt{3}</math>   allow <math>\leq</math> instead of <math>&lt;</math></p>	<p>no follow through; NB <math>\frac{6 \pm \sqrt{48}}{6}</math> or better stated without  working implies use of correct method   <b>A0</b> for incorrect simplification, eg <math>1 \pm \sqrt{48}</math>   allow <b>B1</b> if <i>both</i> inequalities are stated separately and  it's clear that both apply  allow <b>B1</b> if the terms and the signs are in reverse order</p>
12	<p>(i) 50% of 25 000 is 12 500 and the  population [in 2005] is 12 000 [so  consistent]</p>	<p><b>B1</b></p>	<p>or 12 000 is 48% of 25 000 so less than  50% [ so consistent]</p>	
12	<p>(ii) <math>\log_{10} P = \log_{10} a - kt</math> or  <math>\log_{10} \frac{P}{a} = -kt</math> o.e. www</p>	<p><b>B2</b></p>	<p>condone omission of base; <b>M1</b> for  <math>\log_{10} P = \log_{10} a + \log_{10} 10^{-kt}</math> or better  www</p>	

12	(iii) 4.27, 4.21, 4.13, 4.08  plots ruled line of best fit drawn	<b>B1</b> <b>B1</b> <b>B1</b>	accept 4.273..., 4.2108..., 4.130..., 4.079... rounded to 2 or more dp 1 mm tolerance fit their values if at least 4 correct values are correctly plotted	f.t. if at least two calculated values correct must have at least one point on or above and at least one point on or below the line and must cover $0 \leq t \leq 25$
12	(iv) $a = 25000$ to 25400  $0.01 \leq k \leq 0.014$  $P = a \times 10^{-kt}$ or $P = 10^{\log a - kt}$ with values in acceptable ranges	<b>B1</b> <b>B2</b> <b>B1</b>	allow $10^{4.4..}$  <b>M1</b> for $-k = \frac{\Delta y}{\Delta x}$ using values from table or graph; condone $+k$  <b>B0</b> if left in logarithmic form	<b>M1</b> for a correct first step in solving a pair of valid equations in either form <b>A1</b> for $k$ <b>A1</b> for $a$ <b>A1</b> for $P = a \times 10^{-kt}$
12	(v) $P = a \times 10^{-35k}$  8600 to 9000  comparing their value with 9375 o.e. and reaching the correct conclusion for their value	<b>M1</b> <b>A1</b> <b>A1</b>	Their $a$ and $k$  f.t.	allow $\log P = \log a - 35k$

Section B Total: 36