Mark Scheme (Results)
January 2011

GCE Core Mathematics C3 (6665) Paper 1

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General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ✳ The answer is printed on the paper
   - ⌐ The second mark is dependent on gaining the first mark

Question Number	Scheme	Marks
1. (a)	$7\cos x - 24\sin x \ = \ R\cos(x+\alpha)$	
	$7\cos x - 24\sin x \ = \ R\cos x\cos\alpha - R\sin x\sin\alpha$	
	Equate $\cos x$: $\quad 7 = R\cos\alpha$	
	Equate $\sin x$: $\quad 24 = R\sin\alpha$	
	$R = \sqrt{7^2 + 24^2}\ ; = 25$ $\hfill R = 25$	B1
	$\tan\alpha = \frac{24}{7} \Rightarrow \alpha = 1.287002218...^c$ $\hfill \tan\alpha = \frac{24}{7}$ or $\tan\alpha = \frac{7}{24}$ $\hfill$ awrt 1.287	M1 A1
	Hence, $7\cos x - 24\sin x \ = \ 25\cos(x+1.287)$	(3)
(b)	Minimum value $= \underline{-25}$ $\hfill -25$ or $-R$	B1ft (1)
(c)	$7\cos x - 24\sin x \ = \ 10$	
	$25\cos(x+1.287) = 10$	
	$\cos(x+1.287) = \dfrac{10}{25}$ $\hfill \cos(x \pm \text{their } \alpha) \ = \ \dfrac{10}{(\text{their } R)}$	M1
	PV $= 1.159279481...^c$ or $66.42182152...^{\circ}$ $\hfill$ For applying $\cos^{-1}\!\left(\dfrac{10}{\text{their } R}\right)$	M1
	So, $x + 1.287 = \{1.159279...^c,\ 5.123906...^c,\ 7.442465...^c\}$ $\hfill$ either $2\pi +$ or $-$ their PV^c or $360^{\circ} +$ or $-$ their PV$^{\circ}$	M1
	gives, $x = \{3.836906...,\ 6.155465...\}$ $\hfill$ awrt 3.84 OR 6.16 $\hfill$ awrt 3.84 AND 6.16	A1 A1 (5) [9]

Question Number	Scheme		Marks
2. (a)	$\dfrac{4x-1}{2(x-1)} - \dfrac{3}{2(x-1)(2x-1)}$		
	$= \dfrac{(4x-1)(2x-1) - 3}{2(x-1)(2x-1)}$	An attempt to form a single fraction	M1
	$= \dfrac{8x^2 - 6x - 2}{\{2(x-1)(2x-1)\}}$	Simplifies to give a correct quadratic numerator over a correct quadratic denominator	A1 aef
	$= \dfrac{2(x-1)(4x+1)}{\{2(x-1)(2x-1)\}}$	An attempt to factorise a 3 term quadratic numerator	M1
	$= \dfrac{4x+1}{2x-1}$		A1 (4)
(b)	$f(x) = \dfrac{4x-1}{2(x-1)} - \dfrac{3}{2(x-1)(2x-1)} - 2 , \quad x > 1$		
	$f(x) = \dfrac{(4x+1)}{(2x-1)} - 2$		
	$= \dfrac{(4x+1) - 2(2x-1)}{(2x-1)}$	An attempt to form a single fraction	M1
	$= \dfrac{4x+1 - 4x + 2}{(2x-1)}$		
	$= \dfrac{3}{(2x-1)}$	Correct result	A1 * (2)
(c)	$f(x) = \dfrac{3}{(2x-1)} = 3(2x-1)^{-1}$		
	$f'(x) = 3(-1)(2x-1)^{-2}(2)$	$\pm k(2x-1)^{-2}$	M1
			A1 aef
	$f'(2) = \dfrac{-6}{9} = -\dfrac{2}{3}$	Either $\dfrac{-6}{9}$ or $-\dfrac{2}{3}$	A1 (3) [9]
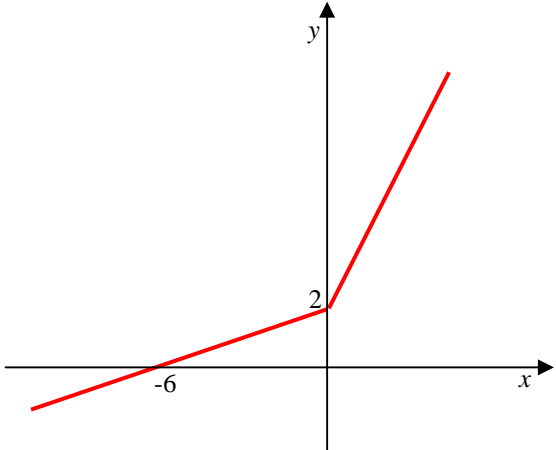
Question Number	Scheme	Marks
3.	$2\cos 2\theta = 1 - 2\sin\theta$	
	$2\left(1 - 2\sin^2\theta\right) = 1 - 2\sin\theta$ Substitutes either $1 - 2\sin^2\theta$ or $2\cos^2\theta - 1$ or $\cos^2\theta - \sin^2\theta$ for $\cos 2\theta$.	M1
	$2 - 4\sin^2\theta = 1 - 2\sin\theta$	
	$4\sin^2\theta - 2\sin\theta - 1 = 0$ Forms a "quadratic in sine" $= 0$	M1(*)
	$\sin\theta = \dfrac{2 \pm \sqrt{4 - 4(4)(-1)}}{8}$ Applies the quadratic formula See notes for alternative methods.	M1
	PVs: $\alpha_1 = 54^{\circ}$ or $\alpha_2 = -18^{\circ}$	
	$\theta = \{54, 126, 198, 342\}$ Any one correct answer 180-their pv All four solutions correct.	A1 dM1(*) A1 [6]

Question Number	Scheme	Marks
4.		
(a)	$\theta = 20 + Ae^{-kt}$ (eqn *)	
	$\{t = 0, \theta = 90 \Rightarrow\}$ $90 = 20 + Ae^{-k(0)}$ Substitutes $t = 0$ and $\theta = 90$ into eqn *	M1
	$90 = 20 + A \implies \underline{A = 70}$ $\underline{A = 70}$	A1
		(2)
(b)	$\theta = 20 + 70e^{-kt}$	
	$\{t = 5, \theta = 55 \Rightarrow\}$ $55 = 20 + 70e^{-k(5)}$ Substitutes $t = 5$ and $\theta = 55$ into eqn * and rearranges eqn * to make $e^{\pm 5k}$ the subject.	M1
	$\dfrac{35}{70} = e^{-5k}$	
	$\ln\left(\frac{35}{70}\right) = -5k$ Takes 'lns' and proceeds to make '$\pm 5k$' the subject.	dM1
	$-5k = \ln\left(\frac{1}{2}\right)$	
	$-5k = \ln 1 - \ln 2 \implies -5k = -\ln 2 \implies \underline{k = \frac{1}{5}\ln 2}$ Convincing proof that $k = \frac{1}{5}\ln 2$	A1 *
		(3)
(c)	$\theta = 20 + 70e^{-\frac{1}{5}t\ln 2}$	
	$\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = -\dfrac{1}{5}\ln 2 \cdot (70)e^{-\frac{1}{5}t\ln 2}$ $\pm\alpha\, e^{-kt}$ where $k = \frac{1}{5}\ln 2$	M1
	$-14\ln 2\, e^{-\frac{1}{5}t\ln 2}$	A1 oe
	When $t = 10$, $\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = -14\ln 2\, e^{-2\ln 2}$	
	$\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = -\dfrac{7}{2}\ln 2 = -2.426015132\ldots$	
	Rate of decrease of $\theta = 2.426\ ^{\circ}C/\min$ (3 dp.) awrt ± 2.426	A1
		(3)
		[8]

Question Number	Scheme	Marks
5. (a)	Crosses x-axis $\Rightarrow f(x) = 0 \Rightarrow (8-x)\ln x = 0$ Either $(8-x) = 0$ or $\ln x = 0 \Rightarrow x = 8, 1$ Either one of $\{x\}=1$ OR x$=\{8\}$ Coordinates are $A(1, 0)$ and $B(8, 0)$. Both $A(1, \{0\})$ and $B(8, \{0\})$	B1 B1 (2)
(b)	Apply product rule: $\begin{cases} u = (8-x) & v = \ln x \\ \dfrac{du}{dx} = -1 & \dfrac{dv}{dx} = \dfrac{1}{x} \end{cases}$ $vu' + uv'$ $f'(x) = -\ln x + \dfrac{8-x}{x}$ Any one term correct Both terms correct	M1 A1 A1 (3)
(c)	$f'(3.5) = 0.032951317...$ $f'(3.6) = -0.058711623...$ Sign change (and as $f'(x)$ is continuous) therefore the x-coordinate of Q lies between 3.5 and 3.6. Attempts to evaluate **both** $f'(3.5)$ and $f'(3.6)$ both values correct to at least 1 sf, sign change and conclusion	M1 A1 (2)
(d)	At Q, $f'(x) = 0 \Rightarrow -\ln x + \dfrac{8-x}{x} = 0$ Setting $f'(x) = 0$. $\Rightarrow -\ln x + \dfrac{8}{x} - 1 = 0$ Splitting up the numerator and proceeding to x= $\Rightarrow \dfrac{8}{x} = \ln x + 1 \Rightarrow 8 = x(\ln x + 1)$ $\Rightarrow x = \dfrac{8}{\ln x + 1}$ (as required) For correct proof. No errors seen in working.	M1 M1 A1 (3)

Question Number	Scheme	Marks
(e)	Iterative formula: $x_{n+1} = \dfrac{8}{\ln x_n + 1}$	
	$x_1 = \dfrac{8}{\ln(3.55) + 1}$ An attempt to substitute $x_0 = 3.55$ into the iterative formula. Can be implied by $x_1 = 3.528(97)...$	M1
	$x_1 = 3.528974374...$ $x_2 = 3.538246011...$ $x_3 = 3.534144722...$ Both $x_1 =$ awrt 3.529 and $x_2 =$ awrt 3.538	A1
	$x_1 = 3.529,\ x_2 = 3.538,\ x_3 = 3.534,$ to 3 dp. x_1, x_2, x_3 all stated correctly to 3 dp	A1
		(3)
		[13]

Question Number	Scheme		Marks
6.			
(a)	$y = \dfrac{3-2x}{x-5} \Rightarrow y(x-5) = 3-2x$	Attempt to make x (or swapped y) the subject	M1
	$xy - 5y = 3 - 2x$		
	$\Rightarrow xy + 2x = 3 + 5y \Rightarrow x(y+2) = 3+5y$	Collect x terms together and factorise.	M1
	$\Rightarrow x = \dfrac{3+5y}{y+2} \qquad \therefore \mathrm{f}^{-1}(x) = \underline{\dfrac{3+5x}{x+2}}$	$\dfrac{3+5x}{x+2}$	A1 oe
			(3)
(b)	Range of g is $-9 \le \mathrm{g}(x) \le 4$ or $-9 \le y \le 4$	Correct Range	B1
			(1)
(c)		Deduces that $\mathrm{g}(2)$ is 0. Seen or implied.	M1
	$\mathrm{g\,g}(2) = \mathrm{g}(0) = -6$, from sketch.	-6	A1
			(2)
(d)	$\mathrm{fg}(8) = \mathrm{f}(4)$	Correct order g followed by f	M1
	$= \dfrac{3 - 4(2)}{4-5} = \dfrac{-5}{-1} = \underline{5}$	5	A1
			(2)

Question Number	Scheme	Marks
(e)(ii)	Correct shape Graph goes through $\big(\{0\},\,2\big)$ and $\big(-6,\,\{0\}\big)$ which are marked.	B1 B1 (4)
(f)	Domain of g^{-1} is $-9 \le x \le 4$ Either correct answer or a follow through from part (b) answer	B1 $\sqrt{}$ (1) [13]

Question Number	Scheme	Marks
7		

(a) $y = \dfrac{3 + \sin 2x}{2 + \cos 2x}$

Apply quotient rule:

$$\left\{ \begin{array}{ll} u = 3 + \sin 2x & v = 2 + \cos 2x \\ \dfrac{du}{dx} = 2\cos 2x & \dfrac{dv}{dx} = -2\sin 2x \end{array} \right\}$$

$\dfrac{dy}{dx} = \dfrac{2\cos 2x(2 + \cos 2x) - -2\sin 2x(3 + \sin 2x)}{(2 + \cos 2x)^2}$

Applying $\dfrac{vu' - uv'}{v^2}$ **M1**

Any one term correct on the numerator   **A1**

Fully correct (unsimplified).   **A1**

$= \dfrac{4\cos 2x + 2\cos^2 2x + 6\sin 2x + 2\sin^2 2x}{(2 + \cos 2x)^2}$

$= \dfrac{4\cos 2x + 6\sin 2x + 2(\cos^2 2x + \sin^2 2x)}{(2 + \cos 2x)^2}$

$= \dfrac{4\cos 2x + 6\sin 2x + 2}{(2 + \cos 2x)^2}$ (as required)

For correct proof with an understanding that $\cos^2 2x + \sin^2 2x = 1$. No errors seen in working.   **A1***

(4)

(b) When $x = \frac{\pi}{2}$, $y = \dfrac{3 + \sin\pi}{2 + \cos\pi} = \dfrac{3}{1} = 3$ $y = 3$ **B1**

At
$(\frac{\pi}{2}, 3)$, $m(\mathbf{T}) = \dfrac{6\sin\pi + 4\cos\pi + 2}{(2 + \cos\pi)^2} = \dfrac{-4 + 2}{1^2} = -2$ $m(\mathbf{T}) = -2$ **B1**

Either **T**: $y - 3 = -2(x - \frac{\pi}{2})$
or $y = -2x + c$ and
$3 = -2(\frac{\pi}{2}) + c \Rightarrow c = 3 + \pi$;

$y - y_1 = m(x - \frac{\pi}{2})$ with 'their TANGENT gradient' and their y_1; or uses $y = mx + c$ with 'their TANGENT gradient';   **M1**

T: $y = -2x + (\pi + 3)$ $y = -2x + \pi + 3$ **A1**

(4)

[8]

Question Number	Scheme	Marks
8.		
(a)	$y = \sec x \;=\; \dfrac{1}{\cos x} \;=\; (\cos x)^{-1}$	
	$\dfrac{dy}{dx} \;=\; -1(\cos x)^{-2}(-\sin x)$ Writes $\sec x$ as $(\cos x)^{-1}$ and gives $\dfrac{dy}{dx} = \pm\big((\cos x)^{-2}(\sin x)\big)$	M1
	$-1(\cos x)^{-2}(-\sin x)$ or $(\cos x)^{-2}(\sin x)$	A1
	$\dfrac{dy}{dx} \;=\; \left\{\dfrac{\sin x}{\cos^2 x}\right\} = \left(\dfrac{1}{\cos x}\right)\left(\dfrac{\sin x}{\cos x}\right) = \underline{\underline{\sec x \tan x}}$ Convincing proof. Must see both underlined steps.	A1 **AG**
		(3)
(b)	$x = \sec 2y, \quad y \neq (2n+1)\frac{\pi}{4}\,,\; n\in\mathbb{Z}.$	
	$\dfrac{dx}{dy} \;=\; 2\sec 2y \tan 2y$ $K\sec 2y \tan 2y$	M1
	$2\sec 2y \tan 2y$	A1
		(2)
(c)	$\dfrac{dy}{dx} \;=\; \dfrac{1}{2\sec 2y \tan 2y}$ Applies $\dfrac{dy}{dx} = \dfrac{1}{\left(\frac{dx}{dy}\right)}$	M1
	$\dfrac{dy}{dx} \;=\; \dfrac{1}{2x\tan 2y}$ Substitutes x for $\sec 2y$.	M1
	$1 + \tan^2 A = \sec^2 A \;\Rightarrow\; \tan^2 2y = \sec^2 2y - 1$ Attempts to use the identity $1 + \tan^2 A = \sec^2 A$	M1
	So $\tan^2 2y = x^2 - 1$	
	$\dfrac{dy}{dx} \;=\; \dfrac{1}{2x\sqrt{(x^2-1)}}$ $\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{(x^2-1)}}$	A1
		(4)
		[9]