

Mathematics

Advanced GCE

Unit **4724**: Core Mathematics 4

Mark Scheme for January 2011

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- 1 (i) First two terms are $1 - \frac{1}{2}x$ B1
- Third term = $\frac{\frac{1}{2} \cdot -\frac{1}{2}}{2} [(-x)^2 \text{ or } x^2 \text{ or } -x^2]$ M1
- = $-\frac{1}{8}x^2$ A1 3 $-\frac{1}{8}x^2$ without work → M1 A1
- (ii) Attempt to replace x by $2y - 4y^2$ or $2y + 4y^2$ M1 or write as $1 - (2y - 4y^2 \text{ or } 2y + 4y^2)$
- First two terms are $1 - y$ B1
- Third term = $+\frac{3}{2}y^2$ or $\sqrt{(4b+2)}y^2$ A1√ 3 where $b = cf(x^2)$ in part (i)

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- 2 (i) $A(x-2) + B = 7 - 2x$ M1 or $A(x-2)^2 + B(x-2) = (7-2x)(x-2)$
- $A = -2$ A1
- $B = 3$ A1 3
- (ii) $\int \frac{A}{x-2} dx = \left(A \text{ or } \frac{1}{A} \right) \ln(x-2)$ B1 Accept $\ln|x-2|, \ln|2-x|, \ln(2-x)$
- $\int \frac{B}{(x-2)^2} dx = -\left(B \text{ or } \frac{1}{B} \right) \cdot \frac{1}{x-2}$ B1 Negative sign is required
- Correct f.t. of A & B; $A \ln(x-2) - \frac{B}{x-2}$ B1√ Still accept lns as before
- Using limits = $-2 \ln 3 + 2 \ln 2 + \frac{1}{2}$ ISW B1 4 No indication of $\ln(\text{negative})$

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- 3 (i) State/imply $\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$ or $\frac{d}{dx}(\cos x)^{-1}$ B1 Not just $\sec x = \frac{1}{\cos x}$
- Attempt quotient rule or chain rule to power -1 M1 Allow $\frac{u dv - v du}{v^2}$ & wrong trig signs
- Obtain $\frac{\sin x}{\cos^2 x}$ or $-(\sin x)(\cos x)^{-2}$ A1 No inaccuracy allowed here
- Simplify with suff evid to **AG** e.g. $\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$ A1 4 Or vice versa. Not just = $\sec x \tan x$
- (ii) Use $\cos 2x = +/-1 +/- 2 \cos^2 x$ or $+/-1 +/- 2 \sin^2 x$ M1 or $\pm(\cos^2 x - \sin^2 x)$
- Correct denominator = $\sqrt{2 \cos^2 x}$ A1 $\sqrt{2 - 2 \sin^2 x}$ needs simplifying
- Evidence that $\frac{\tan x}{\cos x} = \sec x \tan x$ or $\int \frac{\tan x}{\cos x} dx = \sec x$ B1 irrespective of any const multiples
- $\frac{1}{\sqrt{2}} \sec x$ (+ c) A1 4 Condone θ for x except final line

8

- 4 (i) Attempt to use $\frac{dy}{dx} \cdot \frac{dx}{dt}$ or $\frac{dy}{dt} \cdot \frac{dt}{dx}$ M1 Not just quote formula
- $\frac{4}{2t}$ or $\frac{2}{t}$ A1 2
- (ii) Subst $t = 4$ into their (i), invert & change sign M1
- Subst $t = 4$ into (x,y) & use num grad for tgt/normal M1
- $y = -2x + 52$ AEF CAO (no f.t.) A1 3 Only the eqn of normal accepted
- (iii) Attempt to eliminate t from the 2 given equations M1
- $x = 2 + \frac{y^2}{16}$ or $y^2 = 16(x-2)$ AEF ISW A1 2 Mark at earliest acceptable form.
- 7**
- 5 (i) Attempt to connect dx and du M1 Including $\frac{du}{dx} =$ or $du = \dots dx$; not $dx = du$
- $5 - x = 4 - u^2$ B1 perhaps in conjunction with next line
- Show $\int \frac{4-u^2}{2+u} \cdot 2u \, du$ reduced to $\int 4u - 2u^2 \, du$ AG A1 In a fully satisfactory & acceptable manner
- Clear explanation of why limits change B1 e.g. when $x = 2$, $u = 1$ and when $x = 5$, $u = 2$
- $\frac{4}{3}$ B1 5 not dependent on any of first 4 marks
- (ii)(a) $5 - x$ *B1 1 Accept $4 - x - 1 = 5 - x$ (this is not AG)
- (b) Show reduction to $2 - \sqrt{x-1}$ dep*B1
- $\int \sqrt{x-1} \, dx = \frac{2}{3}(x-1)^{\frac{3}{2}}$ B1 Indep of other marks, seen anywhere in (b)
- $\left(10 - \frac{2}{3} \cdot 8\right) - \left(4 - \frac{2}{3}\right) = \frac{4}{3}$ or $4\frac{2}{3} - 3\frac{1}{3} = \frac{4}{3}$ B1 3 Working must be shown
- 9**
- 6 (i) Work with correct pair of direction vectors M1
- Demonstrate correct method for finding scalar product M1 Of any two 3x3 vectors rel to question
- Demonstrate correct method for finding modulus M1 Of any vector relevant to question
- 24, 24.0 (24.006363..) (degrees) 0.419 (0.41899..) (rad) A1 4 Mark earliest value, allow trunc/rounding
- (ii) Attempt to set up 3 equations M1 Of type $3 + 2s = 5, 3s = 3 + t, -2 - 4s = 2 - 2t$
- Find correct values of $(s, t) = (1, 0)$ or $(1, 4)$ or $(5, 12)$ A1 Or 2 diff values of s (or of t)
- Substitute their (s, t) into equation not used M1 and make a relevant deduction
- Correctly demonstrate failure A1 4 dep on all 3 prev marks
- (iii) Subst their (s, t) from first 2 eqns into new 3rd eqn M1 New 3rd eqn of type $a - 4s = 2 - 2t$
- $a = 6$ A1 2
- 10**

7	Attempt parts with $u = x^2 + 5x + 7$, $dv = \sin x$	M1	as far as $f(x) + / - \int g(x) dx$
	1 st stage = $-(x^2 + 5x + 7)\cos x + \int (2x + 5)\cos x dx$	A1	signs need not be amalgamated at this stage
	$\int (2x + 5)\cos x dx = (2x + 5)\sin x - \int 2 \sin x dx$	B1	indep of previous A1 being awarded
	$= (2x + 5)\sin x + 2 \cos x$	B1	
	$I = -(x^2 + 5x + 7)\cos x + (2x + 5)\sin x + 2 \cos x$	A1	WWW
	(Substitute $x = \pi$) $-(\text{Substitute } x = 0)$	M1	An attempt at subst $x = 0$ must be seen
	$\pi^2 + 5\pi + 10$ WWW AG	A1 7	
			7
8 (i)	$\frac{d}{dx}(y^2) = 2y \frac{dy}{dx}$	B1	
	$\frac{d}{dx}(-5xy) = (-)(5)x \frac{dy}{dx} + (-)(5)y$	M1	i.e. reasonably clear use of product rule
	LHS completely correct $4x - 5x \frac{dy}{dx} - 5y + 2y \frac{dy}{dx} (= 0)$	A1	Accept “ $\frac{dy}{dx} =$ ” provided it is not used
	Substitute $\frac{dy}{dx} = \frac{3}{8}$ or solve for $\frac{dy}{dx}$ & then equate to $\frac{3}{8}$	M1	Accuracy not required for “solve for $\frac{dy}{dx}$ ”
	Produce $x = 2y$ WWW AG (Converse acceptable)	A1 5	Expect $17x = 34y$ and/or $\frac{dy}{dx} = \frac{5y - 4x}{2y - 5x}$
(ii)	Substitute $2y$ for x or $\frac{1}{2}x$ for y in curve equation	M1	
	Produce either $x^2 = 36$ or $y^2 = 9$	A1	
	AEF of $(\pm 6, \pm 3)$	A1 3	ISW Any correct format acceptable
			8
9 (i)	Attempt to sep variables in the form $\int \frac{P}{(x-8)^{1/3}} dx = \int q dt$	M1	Or invert as $\frac{dt}{dx} = \frac{r}{(x-8)^{1/3}}$; p, q, r const
	$\int \frac{1}{(x-8)^{1/3}} dx = k(x-8)^{2/3}$	A1	k const
	All correct (+ c)	A1	
	For equation containing ‘c’; substitute $t = 0$, $x = 72$	M1	M2 for $\int_{72}^{35} = \int_0^t$ or $\int_{35}^{72} = \int_0^t$
	Correct corresponding value of c from correct eqn	A1	
	Subst their c & $x = 35$ back into eqn	M1	
	$t = \frac{21}{8}$ or 2.63 / 2.625 [C.A.O]	A1 7	A2: $t = \frac{21}{8}$ or 2.63 / 2.625 WWW
(ii)	State/imply in some way that $x = 8$ when flow stops	B1	
	Substitute $x = 8$ back into equation containing numeric ‘c’	M1	
	$t = 6$	A1 3	

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