

$$f(x) = \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} - 2, \quad x > 1,$$
This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 2 continued

[illegible]

where A and k are positive constants.

(a) find the value of A .

The tea takes 5 minutes to decrease in temperature from 90°C to 55°C.

(b) Show that $k = \frac{1}{5} \ln 2$.

(c) Find the rate at which the temperature of the tea is decreasing at the instant when $t = 10$. Give your answer, in $^{\circ}\text{C}$ per minute, to 3 decimal places.

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This image shows a full page of blank, lined paper. It features approximately 20 horizontal blue or grey lines spaced evenly apart, typical of notebook paper. The lines extend across the entire width of the page, leaving small margins at the top and bottom. There are no vertical lines, text, or other markings on the page.



5.

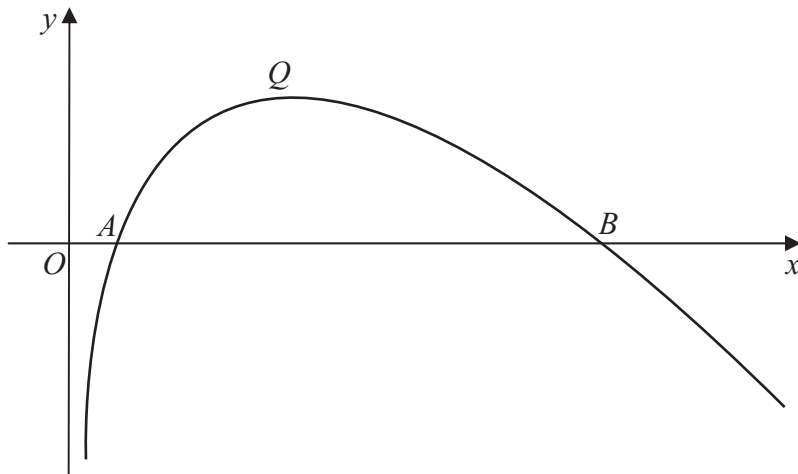
**Figure 1**

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (8-x)\ln x, \quad x > 0$$

The curve cuts the x -axis at the points A and B and has a maximum turning point at Q , as shown in Figure 1.

(a) Write down the coordinates of A and the coordinates of B . (2)

(b) Find $f'(x)$. (3)

(c) Show that the x -coordinate of Q lies between 3.5 and 3.6 (2)

(d) Show that the x -coordinate of Q is the solution of

$$x = \frac{8}{1 + \ln x} \quad (3)$$

To find an approximation for the x -coordinate of Q , the iteration formula

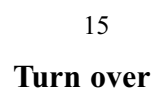
$$x_{n+1} = \frac{8}{1 + \ln x_n}$$

is used.

(e) Taking $x_0 = 3.55$, find the values of x_1 , x_2 and x_3 .
Give your answers to 3 decimal places. (3)



Question 5 continued



6. The function f is defined by

$$f: x \mapsto \frac{3-2x}{x-5}, \quad x \in \mathbb{R}, \quad x \neq 5$$

(a) Find $f^{-1}(x)$.

(3)

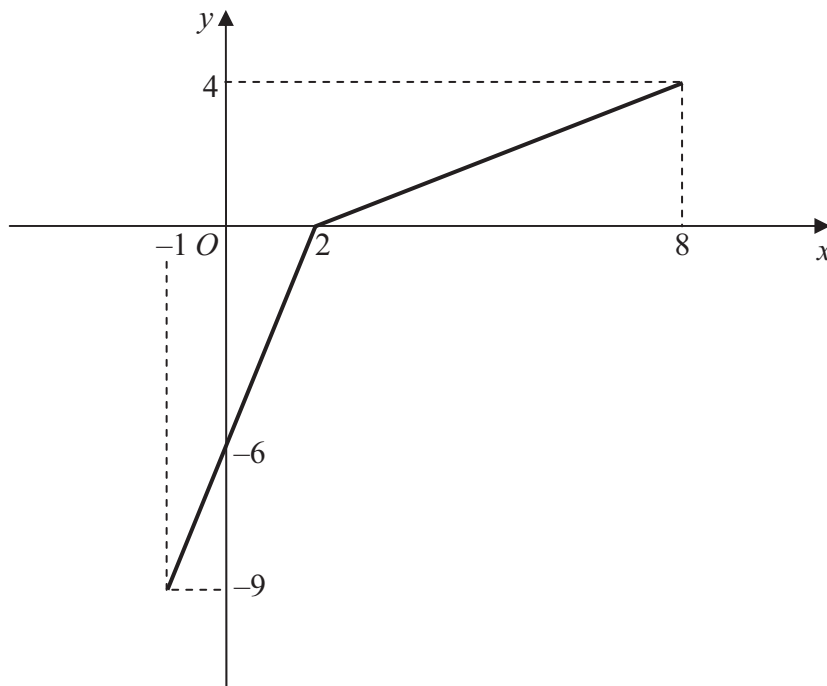


Figure 2

The function g has domain $-1 \leq x \leq 8$, and is linear from $(-1, -9)$ to $(2, 0)$ and from $(2, 0)$ to $(8, 4)$. Figure 2 shows a sketch of the graph of $y = g(x)$.

(b) Write down the range of g .

(1)

(c) Find $gg(2)$.

(2)

(d) Find $fg(8)$.

(2)

(e) On separate diagrams, sketch the graph with equation

(i) $y = |g(x)|$,

(ii) $y = g^{-1}(x)$.

Show on each sketch the coordinates of each point at which the graph meets or cuts the axes.

(4)

(f) State the domain of the inverse function g^{-1} .

(1)



Question 6 continued

[illegible]

$$y = \frac{3 + \sin 2x}{2 + \cos 2x}$$

$$\frac{dy}{dx} = \frac{6 \sin 2x + 4 \cos 2x + 2}{(2 + \cos 2x)^2}$$

(4)

(b) Find an equation of the tangent to C at the point on C where $x = \frac{\pi}{2}$.

Write your answer in the form $y = ax + b$, where a and b are exact constants.

(4)

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show that $\frac{d}{dx}(\sec x) = \sec x \tan x$.

$$x = \sec 2y$$

(b) find $\frac{dx}{dy}$ in terms of y .

(c) Hence find $\frac{dy}{dx}$ in terms of x .

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Q8

(Total 9 marks)

TOTAL FOR PAPER: 75 MARKS

END