OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

Core Mathematics 3

4723

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4723
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Wednesday 19 January 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

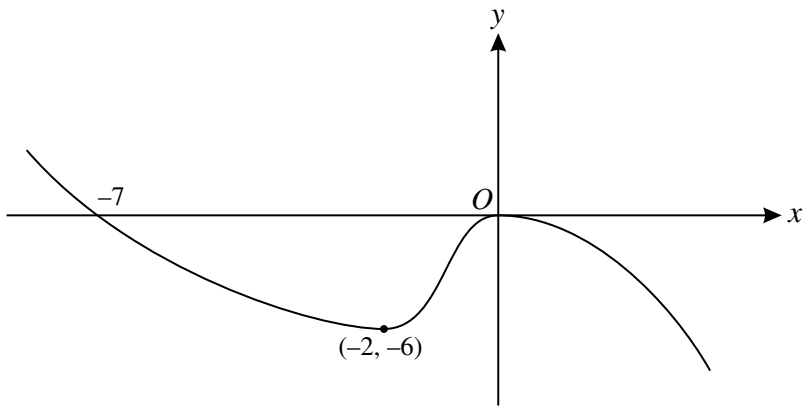
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

1 Solve the equation $|3x + 4a| = 5a$, where a is a positive constant. **[3]**

2



The diagram shows the curve with equation $y = f(x)$. It is given that $f(-7) = 0$ and that there are stationary points at $(-2, -6)$ and $(0, 0)$. Sketch the curve with equation $y = -4f(x + 3)$, indicating the coordinates of the stationary points. **[4]**

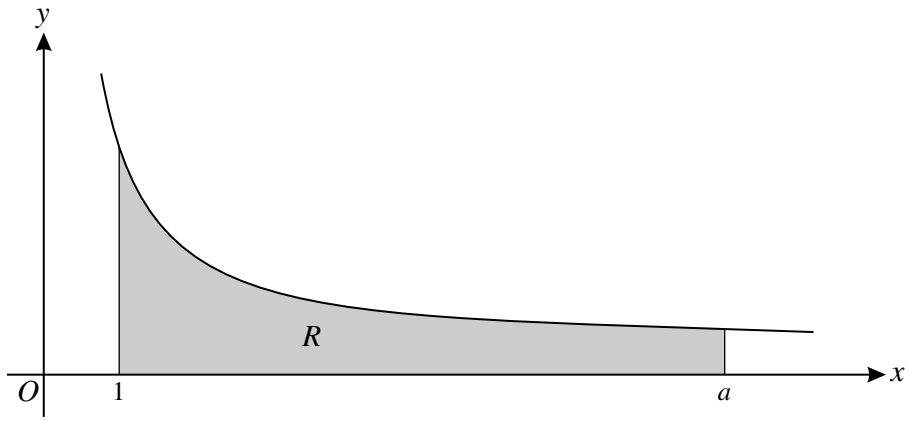
3 A giant spherical balloon is being inflated in a theme park. The radius of the balloon is increasing at a rate of 12 cm per hour. Find the rate at which the surface area of the balloon is increasing at the instant when the radius is 150 cm. Give your answer in cm^2 per hour correct to 2 significant figures.

[Surface area of sphere $= 4\pi r^2$.] **[3]**

4 **(i)** Express $24 \sin\theta + 7 \cos\theta$ in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. **[3]**

(ii) Hence solve the equation $24 \sin\theta + 7 \cos\theta = 12$ for $0^\circ < \theta < 360^\circ$. **[4]**

5



The diagram shows the curve with equation $y = \dfrac{6}{\sqrt{3x - 2}}$. The region R, shaded in the diagram, is bounded by the curve and the lines $x = 1$, $x = a$ and $y = 0$, where a is a constant greater than 1. It is given that the area of R is 16 square units. Find the value of a and hence find the exact volume of the solid formed when R is rotated completely about the x-axis. **[9]**

6 The curve with equation $y = \dfrac{3x+4}{x^3 - 4x^2 + 2}$ has a stationary point at P. It is given that P is close to the point with coordinates $(2.4, -1.6)$.

(i) Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and show that the x-coordinate of P satisfies the equation

$$x = \sqrt[3]{\tfrac{16}{3}x + 1}.$$ **[4]**

(ii) By first using an iterative process based on the equation in part **(i)**, find the coordinates of P, giving each coordinate correct to 3 decimal places. **[5]**

7 The function f is defined for $x > 0$ by $\mathrm{f}(x) = \ln x$ and the function g is defined for all real values of x by $\mathrm{g}(x) = x^2 + 8$.

(i) Find the exact, positive value of x which satisfies the equation $\mathrm{fg}(x) = 8$. **[3]**

(ii) State which one of f and g has an inverse and define that inverse function. **[3]**

(iii) Find the exact value of the gradient of the curve $y = \mathrm{gf}(x)$ at the point with x-coordinate e^3. **[3]**

(iv) Use Simpson's rule with four strips to find an approximate value of

$$\int_{-4}^{4} \mathrm{fg}(x)\,\mathrm{d}x,$$

giving your answer correct to 3 significant figures. **[3]**

8 **(a)** **(i)** Sketch the graph of $y = \operatorname{cosec} x$ for $0 < x < 4\pi$. **[3]**

(ii) It is given that $\operatorname{cosec}\alpha = \operatorname{cosec}\beta$, where $\frac{1}{2}\pi < \alpha < \pi$ and $2\pi < \beta < \frac{5}{2}\pi$. By using your sketch, or otherwise, express β in terms of α. **[2]**

(b) **(i)** Write down the identity giving $\tan 2\theta$ in terms of $\tan\theta$. **[1]**

(ii) Given that $\cot\phi = 4$, find the exact value of $\tan\phi \cot 2\phi \tan 4\phi$, showing all your working. **[6]**

[Question 9 is printed overleaf.]

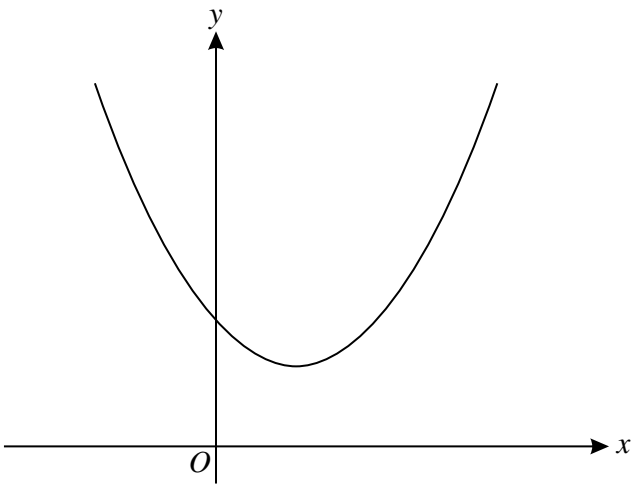
9 **(i)** The function f is defined for all real values of x by

$$f(x) = e^{2x} - 3e^{-2x}.$$

(a) Show that $f'(x) > 0$ for all x. **[3]**

(b) Show that the set of values of x for which $f''(x) > 0$ is the same as the set of values of x for which $f(x) > 0$, and state what this set of values is. **[5]**

(ii)



The function g is defined for all real values of x by

$$g(x) = e^{2x} + ke^{-2x},$$

where k is a constant greater than 1. The graph of $y = g(x)$ is shown above. Find the range of g, giving your answer in simplified form. **[5]**

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PRINTED ANSWER BOOK

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* 4 7 2 3 *

Candidate forename		Candidate surname	

Centre number						Candidate number				

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1	
2	

3	

4 (i)	
4 (ii)	

5

6 (i)

6 (ii)

7 (i)	
7 (ii)	
7 (iii)	

7 (iv)	

8 (a) (i)

8 (a) (ii)

8 (b) (i)	
8 (b) (ii)	

9 (i) (a)

9 (i) (b)

9 (ii)	