OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4751:** Introduction to Advanced Mathematics

Mark Scheme for January 2012

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

Annotations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			grad = –1/5 oe	M1		allow embedded eg $5 \times -\frac{1}{5} = -1$
			y – 6 = their m (x – 1) or 6 = their m [× 1] + c	M1		if first M1 not earned, allow second M1 for y – 6 = k(x – 1) oe, k any number except 0 and 1
			y = –0.2x + 6.2 oe isw	A1	terms collected, with y as subject or for a = –0.2, b = 6.2 oe	allow A1 for c = 6.2 oe if y = –0.2x + c oe already seen condone $y = \frac{-x+31}{5}$ for A1
				[3]		
2	(i)		$\frac{1}{3}$ as final answer	2	allow $\pm\frac{1}{3}$ M1 for $\frac{1}{9^{\frac{1}{2}}}$ or for $9^{\frac{1}{2}} = \sqrt{9}$ or 3 soi	eg M1 for 3^{-1}
				[2]		
2	(ii)		$32x^{10}y^{-3}$ or $\frac{32x^{10}}{y^3}$ oe as final answer	3	B1 for each element if B0, allow M1 for $(4x^4)^3 = 64x^{12}$	allow 2^5 instead of 32
				[3]		
3			$6n^2 + 12n + 8$ or $2(3n^2 + 6n + 4)$ oe as final answer	3	B2 for 2 terms correct in final answer or for $(n + 2)^3 = n^3 + 6n^2 + 12n + 8$ or B1 for 1, 3, 3, 1 soi or SC2 for final answer of $3n^2 + 6n + 4$	B1 for $n^3 + 4n^2 + 4n + 2n^2 + 8n + 8\left[-n^3\right]$, condoning one error
				[3]		

Question			Answer	Marks	Guidance	
4	(i)		$23+\sqrt{2}$ as final answer	3 **[3]**	B2 for 23 and B1 for $\sqrt{2}$ or $1\sqrt{2}$ <u>or</u> M2 for 3 or more terms correct of $35-14\sqrt{2}+15\sqrt{2}-12$ or M1 for 2 terms correct	mark one scheme or other, but not a mixture, to advantage of candidate eg M2 for 35 + $\sqrt{2}$ + 24
4	(ii)		$5\sqrt{6}$ isw	2 **[2]**	condone $\frac{30}{\sqrt{6}}$ for 2 marks M1 for $\left[\sqrt{54}=\right]3\sqrt{6}$ or $\left[\frac{12}{\sqrt{6}}=\right]2\sqrt{6}$	eg 2 isw for $5\sqrt{6}=\sqrt{150}$

Question			Answer	Marks	Guidance	
5			$6(2x + 1) < 5(3x + 4)$	M1	for multiplying up correctly or for correct use of a common denominator	first three Ms may be earned with an equality condone omission of brackets only if then expanded as if brackets present
			$12x + 6 < 15x + 20$ or ft	M1	for expanding brackets correctly; for combined first two steps with one error, such as $12x + 6 < 15x + 4$, allow M1M0	eg $\frac{12x+6}{30} < \frac{15x+20}{30}$ oe earns M1M1
			$-14 < 3x$ or $-3x < 14$ or ft	M1	for collecting terms correctly	ft from two x terms and two constants
			$x > -\frac{14}{3}$ oe or ft isw	M1	for final division of their inequality with ax on one side, $a \neq 1$ or 0, and non-zero number on the other allow SC3 for $-14/3$ found without correct inequality symbol(s)	allow working with equality and making correct decision at end eg allow last M1 for $x > \frac{14}{-3}$ or $\frac{-14}{3} < x$ isw reminder : $(-14/3, \infty)$ is acceptable notation
			<u>or</u> $\frac{1}{5} - \frac{4}{6} < \frac{3x}{6} - \frac{2x}{5}$ oe	<u>or</u> M1		
			$\frac{-7}{15} < \frac{3x}{30}$ oe or ft	M2	M1 for one side correct ft	
			$x > -\frac{14}{3}$ oe or ft isw	M1	as in previous method	
				[4]		

Question			Answer	Marks	Guidance	
6			$4h + ha^2 = 9a - 5$	M1	correctly collecting h terms on one side, remaining terms on other	
			$h(4 + a^2) = 9a - 5$	M1	for factorising, ft eg sign error	
			$[h =]\dfrac{9a-5}{4+a^2}$ oe as final answer	M1	for division by their factor; ft only for equiv difficulty	M0 if seen and spoilt, eg by incorrect 'cancelling'
				[3]		
7	(i)		'tick' at (2,4)(3,1)(5,6)	2	mark intent M1 for two points correct or for 'tick' at (2,–2) (3,–5) and (5,0)	overlay to be provided condone tick unruled; allow M1 for points not joined but all correct:
				[2]		
7	(ii)		'tick' at (0,1)(1,–2)(3,3)	2	mark intent M1 for two points correct or for 'tick' at (4,1) (5,–2) and (7,3)	overlay to be provided condone tick unruled; allow M1 for points not joined but all correct:
				[2]		
8			$5(x + 1.5)^2 + 0.75$ oe www	4	B1 for $a = 5$ and B1 for $b = 3/2$ oe	condone omission of square symbol
					and B2 for $c = 3/4$ oe or M1 for $12 - 5 \times (\text{their } 3/2)^2$ oe soi or for $2.4 - (\text{their } 3/2)^2$ oe [eg 0.15] soi	eg $5[(x + 7.5)^2 - 7.5^2] + 12$ oe earns B1B0M1ft
			0.75 oe or ft their c	1	0 for (–1.5, 0.75)	condone found independently eg by differentiation
				[5]		

Question			Answer	Marks	Guidance	
9	(i)		'if n even then n^3 even, so $n^3 + 1$ odd' oe $\Leftarrow$ with if $n^3 + 1$ odd then n^3 even but if n^3 is even, n is not necessarily an integer <u>or</u> $\Leftrightarrow$ with '$n^3 + 1$ odd then n^3 even so n even', [assuming n is an integer]	B1 B1 **[2]**	must mention n^3 is even or even^3 is even or even × even = even or '$\Leftrightarrow$ with if n is odd, n^3 is odd, so $n^3 + 1$ is even' if 0 in question, allow SC1 for $\Leftrightarrow$ or $\Leftarrow$ and attempt at using general odd/even in explanation	0 for just 'if n is even, $n^3 + 1$ is odd' 0 if just examples of numbers used condone $\leftrightarrow$ instead of $\Leftrightarrow$ etc in both parts must go further than restating the info in the qn; please annotate as SC
9	(ii)		showing $\Leftarrow$ is true $\Leftarrow$ chosen and showing that $\Rightarrow$ [and therefore $\Leftrightarrow$] is/ are not true	B1 B1 **[2]**	eg when $x > 3$, +ve × +ve > 0 stating that true when $x < 2$ or giving a counterexample such as 1, 0 or a negative number [to show quadratic inequality also true for this number] allow B2 for $\Leftarrow$ and $x > 3$ and $x < 2$ shown/stated as soln or sketch showing two solns of $x^2 - 5x + 6 > 0$	0 for just example(s) or for simply stating it is true 0 for saying another solution $x > 2$ or B1 for this argument with another symbol

Question			Answer	Marks	Guidance	
10	(i)		grad AB = $\frac{7-1}{4-2}$ oe or 3	M1		
			$y - 7$ = their m $(x - 4)$ or $y - 1$ = their m $(x - 2)$	M1	or use of y = their gradient $x + c$ with coords of A or B or M2 for $\frac{y-1}{7-1} = \frac{x-2}{4-2}$ o.e.	allow step methods used or eg M1 for $7 = 4m + c$ and $1 = 2m + c$ then M1 for correctly finding one of m and c
			$y = 3x - 5$ oe	A1 **[3]**	accept equivalents if simplified eg $3x - y = 5$ allow B3 for correct eqn www	allow A1 for $c = -5$ oe if $y = 3x + c$ oe already seen B2 for eg $y - 1 = 3(x - 2)$
10	(ii)		showing grad BC = $\frac{2-1}{-1-2} = -\frac{1}{3}$ oe	B1	may be calculation or showing on diagram	
			and $-1/3 \times 3 = -1$ or grad BC is neg reciprocal of grad AB, [so 90°]	B1	may be earned for statement / use of $m_1m_2 = -1$ oe, even if first B1 not earned for B1+B1, must be fully correct, with 3 as gradient in (i)	eg allow 2nd B1 for statement grad BC = $-1/3$ with no working if first B1 not earned
			<u>or</u> for finding AC or AC^2 independently of AB and BC	<u>or</u> B1	working needed such as $AC^2 = 5^2 + 5^2 = 50$	condone any confusion between squares and square roots etc for first B1 and for two M1s eg AC = 25 + 25 = $\sqrt{50}$
			for correctly showing $AC^2 = BC^2 + AB^2$ oe	B1	working needed using correct notation such as $BC^2 = 3^2 + 1^2 = 10$; $AB^2 = 6^2 + 2^2 = 40$, 40 + 10 = 50 [hence $AC^2 = BC^2 + AB^2$]	accept eg 3 and 1 shown on diagram and $BC^2 = 10$ etc 0 for eg $\sqrt{40} + \sqrt{10} = \sqrt{50}$

Question			Answer	Marks	Guidance	
			or finding equation of line through C perpendicular to AB ($y = -\frac{1}{3}x + \frac{5}{3}$ oe)	or B1		eg B1 for $x + 3y = 5$
			showing B is on this line either by substitution or finding intersection of this line with AB	B1		or B1 for finding the equation of the line through B and C as $y = -\frac{1}{3}x + \frac{5}{3}$ oe and B1 for using condition for perp lines and showing true
			BC = $\sqrt{3^2 + 1^2}$ or $\sqrt{10}$	M1		
			AB= $\sqrt{6^2 + 2^2}$ or $\sqrt{40}$ or $2\sqrt{10}$	M1	both these Ms may be earned earlier if Pythag used to show angle ABC = 90°, but are for BC and AB, not BC^2 and AB^2	for both M1s accept unsimplified equivs
			Area = 10 [square units]	A1	must be simplified to 10	
			or area under AC – area under AB – area under BC	or M1		mark equivalently for other valid methods, eg trapezium – 2 triangles method, omitting below $y = 1$: $\frac{1}{2} \times 7 \times 5 - (\frac{1}{2} \times 3 \times 1 + \frac{1}{2} \times 2 \times 6)$ $= 17.5 - (1.5 + 6)$
			at least two of 22.5, 8 and 4.5 oe	M1		
			Area = 10 [square units]	A1	must be simplified to 10	
				[5]		

Question			Answer	Marks	Guidance	
10	(iii)		(1.5, 4.5) oe	2	B1 each coordinate	
			angle in semicircle oe is a right-angle [so B is on circle] and must mention AC as diameter or D as centre [hence A, B, C all same distance from D]	E1 **[3]**	or '[since b = 90°,] ABC are three vertices of a rectangle. D is the midpoint of one diagonal <u>and</u> so D is the centre of the rectangle <u>or</u> the diagonals of a rectangle are equal and bisect each other, [hence DA=DB=DC] or condone showing that line from D to mid point of AB is perp to AB, so DBA is isos [hence DB = DA = DC] [or equiv using DBC]	E0 for just stating 'D is midpt of the hypotenuse of a rt angled triangle ABC so DAB is isos' without showing that it is isw eg wrong calcn of radius NB some wrongly asserting that ABC is isos

Question			Answer	Marks	Guidance	
11	(i)		f(–3) used	M1		
			$-54 - 27 + 69 + 12$ [= 0] isw	A1	or M1 for correct division by $(x + 3)$ or for the quadratic factor found by inspection and A1 for concluding that $x = -3$ [is a root] (may be earned later)	A0 for concluding that $x = -3$ is a factor
			attempt at division by $(x + 3)$ as far as $2x^3 + 6x^2$ in working	M1	or inspection with at least two terms of three–term quadratic factor correct; or at least one further root found using remainder theorem	
			correctly obtaining $2x^2 - 9x + 4$	A1	or stating further factor, found from using remainder theorem again	
			factorising the correct quadratic factor	M1	for factors giving two terms of quadratic correct or for factors ft one error in quadratic formula or completing square; M0 for formula etc without factors found	allow for $(x - 4)$ and $(x - ½)$ given as factors eg after using remainder theorem again or quadratic formula etc
			$(2x - 1)(x - 4)[(x + 3)]$ isw	A1	allow $2(x - ½)$ instead of $(2x - 1)$, oe condone inclusion of '= 0'	isw $(x - ½)$ as factor and/or roots found, even if stated as factors
				[6]		

Question			Answer	Marks	Guidance	
11	(ii)		sketch of cubic right way up, with two turning points	B1	0 if stops at x-axis ignore graph of $y = 4x + 12$	must not be ruled; no curving back (except condone between $x = 0$ and $x = 0.5$); condone some 'flicking out' at ends but not approaching more turning points; must continue beyond axes; allow max on y axis or in 1st or 2nd quadrants condone some doubling / feathering
			values of intns on x axis shown, correct (–3, 0.5 and 4) or ft from their factors or roots in (i)	B1	on graph or nearby in this part mark intent for intersections with both axes	allow if no graph condone 3 on neg x axis as slip for –3; condone eg 0.5 roughly halfway between their 0 and 1 marked on x axis
			12 marked on y-axis	B1 **[3]**	or $x = 0$, $y = 12$ seen in this part if consistent with graph drawn	allow if no graph, but eg B0 for graph with intn on –ve y-axis or nowhere near their indicated 12
11	(iii)		$2x^3 - 3x^2 - 23x + 12 = 4x + 12$ oe	M1	or ft their factorised f(x)	
			$2x^3 - 3x^2 - 27x$ [= 0]	A1	after equating, allow A1 for cancelling ($x + 3$) factor on both sides and obtaining $2x^2 - 9x$ [= 0]	condone slip of '= y' instead of '= 0'
			[x]($2x - 9$)($x + 3$) [= 0]	M1	for linear factors of correct cubic, giving two terms correct or for quadratic formula or completing square used on correct quadratic $2x^2 - 3x - 27 = 0$, condoning one error in formula etc;	or after cancelling ($x + 3$) factor allow M1 for $x(2x - 9)$ oe or obtaining $x = 0$ or 9/2 oe M0 for eg quadratic formula used on cubic, unless recovery and all 3 roots given
			[x =] 0, –3 and 9/2 oe	A1 **[4]**	need not be all stated together	eg $x = 0$ may be earlier

Question			Answer	Marks	Guidance	
12	(i)		$\sqrt{20}$ isw or $2\sqrt{5}$	B1	0 for $\pm\sqrt{20}$	
			(2, 0)	B1 **[2]**		
12	(ii)		subst of $x = 0$ into circle eqn soi	M1	or Pythag used on sketch of circle: $2^2 + y^2 = 20$ oe	M0 for just $y^2 = 20$; M1 for $y^2 = 16$ or for $y = 4$ ignore intns with x-axis also found
			$y = \pm 4$ oe	A1	or B2 for just $y = \pm 4$ seen oe; accept both 4 and –4 shown on y axis on sketch if both values not stated	
			sketch of circle with centre (2, 0) or ft their centre from (i)	B1 **[3]**	if the centre is not marked, it should look roughly correct by eye – coords need not be given on sketch; condone intersections with axes not marked	circle should intersect both +ve and neg x- and y-axes; must be clear attempt at circle; ignore any tangents drawn
12	(iii)		$(x - 2)^2 + (2x + k)^2 = 20$	M1	for attempt to subst $2x + k$ for y	allow for attempt to subst $k = y - 2x$ into given eqn
			$x^2 - 4x + 4 + 4x^2 + 4kx + k^2 = 20$	M1 dep	for correct expansion of at least one set of brackets, dependent on first M1	similarly for those working backwards
			$5x^2 + (4k - 4)x + k^2 - 16 = 0$	A1 **[3]**	correct completion to given answer; dependent on both Ms	condone omission of further interim step if both sets of brackets expanded correctly, but for cands working backwards, at least one interim step is needed; if cands have made an error and tried to correct it, corrections must be complete to award this A mark

Question			Answer	Marks	Guidance	
12	(iv)		$b^2 - 4ac = 0$ seen or used	M1	need not be substituted into; may be stated after formula used or argument towards expressing eqn as a perfect square	eg M1 for $(4k - 4)^2 - 4 \times 5 \times (k^2 - 16) = 0$
			$4k^2 + 32k - 336$ [= 0] or $k^2 + 8k - 84$ [= 0]	M1	expansion and collection of terms, condoning one error ft their $b^2 - 4ac$	dep on an attempt at $b^2 - 4ac$ with at least two of a, b and c correct; may be earned with < 0 etc; may be in formula
			use of factorising or quadratic formula or completing square	M1	condone one error ft	dep on attempt at obtaining required quadratic equation in k, not for use with any eqn/inequality they have tried
			$k = 6$ or -14 <u>or</u> Grad of tgt is 2, and normal passes through centre, hence finding equation of normal as $y = -\frac{1}{2}x + 1$ oe	A1 <u>or</u> M1		
			finding x values where diameter $y = -x/2 + 1$ intersects circle as $x = 6$ or -2 (condone one error in method)	M1	oe for y values; condone one error in method	or finding intn of tgt and normal as $\left(\frac{2-2k}{5}, \frac{k+4}{5}\right)$
			finding corresponding y values on circle and subst into $y = 2x + k$ or subst their x values into $5x^2 + (4k - 4)x + k^2 - 16 = 0$	M1	intns are (6, –2) and (–2, 2), M0 for just (6, 2) and (–2, –2) used but condone used as well as correct intns this last method gives extra values for k, for the non-tangent lines $y =$ through (6, 2) and (–2, –2), but allow for the M mark	or subst their intn of tgt and normal into eqn of circle: $\left(\frac{2-2k}{5} - 2\right)^2 + \left(\frac{k+4}{5}\right)^2 = 20$ or ft
			$k = 6$ or -14	A1 **[4]**	and no other values	

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553