RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced Subsidiary GCE

Unit **4721:** Core Mathematics 1

Mark Scheme for January 2012

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

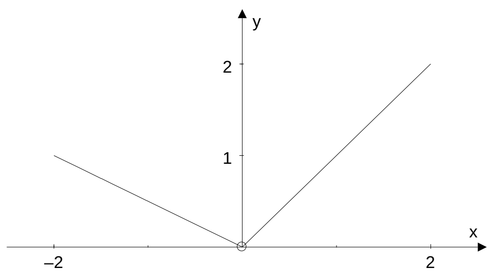
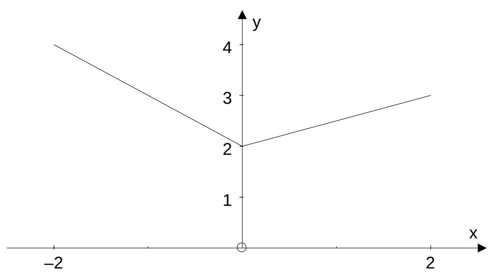
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

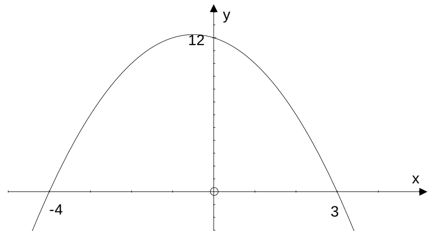
Question			Answer	Marks	Guidance	
1			$\dfrac{15+\sqrt{3}}{3-\sqrt{3}} \times \dfrac{3+\sqrt{3}}{3+\sqrt{3}}$	M1	Multiply top and bottom by $\pm(3+\sqrt{3})$	**SC** If A0A0A0 scored, both parts correct but unsimplified **B1** i.e. $\dfrac{45+15\sqrt{3}+3\sqrt{3}+3}{9+3\sqrt{3}-3\sqrt{3}-3}$ o.e.
			$=\dfrac{48+18\sqrt{3}}{9-3}$	A1	Numerator correct and simplified	<u>Alternative method:</u> Equates expression to $a + b\sqrt{3}$ and forms simultaneous equations in a and b **M1**
				A1	Denominator correct and simplified to 6	Correct method to solve simultaneous equations **M1**
			$=8+3\sqrt{3}$	A1	cao	a = 8 found **A1**
				[4]		b = 3 found **A1**
2	(i)			M1	Reflection of given graph in either axis	
				A1	Correct reflection in y-axis	Clear intention to show (-2, 1), (0,0), (2,2) by numbers, dashes or co-ordinates **A0** If significantly short or long
				[2]		
2	(ii)			M1	Translation of **given** graph vertically (up or down)	
				A1	Correct translation of two units vertically	Clear intention to show (-2, 4), (0,2), (2,3) by numbers, dashes or co-ordinates **A0** If significantly short or long
				[2]		

Question			Answer	Marks	Guidance	
3			$5x^2 + px - 8 = 5(x-1)^2 + r$ $= 5(x^2 - 2x + 1) + r$ $= 5x^2 - 10x + 5 + r$ $p = -10$ $r = -13$	B1 B1 M1 A1 **[4]**	$q = 5$ (may be embedded on RHS) $p = -10$ $-8 = \pm q + r$ or $\frac{-p^2}{20} - 8 = r$ $r = -13$	Allow from $p = 10$
4	(i)		$\frac{1}{9}$	B1 **[1]**		
4	(ii)		$\left(\sqrt[4]{16}\right)^3$ $= 8$	M1 A1 **[2]**	Interprets the power $\frac{3}{4}$ correctly ± 8 is **A0**	$(\sqrt[4]{16})^3$ or $(\sqrt[4]{16^3})$ or $\left(16^{\frac{1}{4}}\right)^3$ or $\left(16^3\right)^{\frac{1}{4}}$
4	(iii)		$5\sqrt{8} \div \sqrt{8}$ $= 5$	M1 A1 **[2]**	$\sqrt{100}\ \sqrt{2} \div \sqrt{4}\ \sqrt{2}$ or $\sqrt{\frac{200}{8}}$ or $\sqrt{25}\ \sqrt{8} \div \sqrt{8}$ or $\sqrt{1600} \div 8$ soi Condone ± 5	

Question			Answer	Marks	Guidance	
5			$k=\frac{1}{y^2}$	M1*	Use a correct substitution or pair of substitutions to obtain a quadratic or factorise into 2 brackets each containing $\frac{1}{y^2}$	**No marks** if straight to quadratic formula to get y = "$-\frac{2}{3}$", y = "4" unless correct substitution applied later i.e. reciprocal and square root
			$3k^2-10k-8=0$ $(3k+2)(k-4)=0$	M1dep	Correct method to solve a quadratic	
			$k=-\frac{2}{3}$ or $k=4$	A1	k = 4 from correct method. If other root stated it must be correct.	**No marks** if quadratic found from incorrect substitution
			$y^2=-\frac{3}{2}$ or $y^2=\frac{1}{4}$	M1	Attempt to reciprocal and square root to obtain y (either term)	**SC** If **M0** Spotted solutions **www B1** each Justifies 2 solutions exactly **B3**
			$y=\pm\frac{1}{2}$	A1 **[5]**	No other roots given. Must be from k = 4 from correct method.	
			Alternative method below: $3-10y^2-8y^4=0$ $k=y^2$ $8k^2+10k-3=0$ M1* $(4k-1)(2k+3)=0$ M1 dep $k=\frac{1}{4}$ or $k=-\frac{3}{2}$ A1 $y=\pm\frac{1}{2}$ M1 A1		$k=\frac{1}{4}$ from correct method. If other root stated it must be correct.	

Question			Answer	Marks	Guidance	
6	(i)		$f'(x) = -4x^{-2} - 3$	M1 A1 A1 **[3]**	Attempt to differentiate $-4x^{-2}$ Fully correct derivative (no "+ c")	kx^{-2} or -3 correctly obtained
6	(ii)		$f''(x) = 8x^{-3}$ $f''\left(\frac{1}{2}\right) = \frac{8}{\left(\frac{1}{2}\right)^3}$ $= 64$	M1* A1 M1dep A1 **[4]**	Attempts to differentiate their **(i)** Correct derivative Substitutes $x = \frac{1}{2}$ correctly into their $f''(x)$ e.g. $8\left(\frac{1}{2}\right)^{-3}$ (allow "invisible brackets") **www**	Must involve reducing power of an x term by 1 $f''(x)$ must involve x.
7	(i)		$x^3 - 3x^2 + 5x + 2x^2 - 6x + 10$ $= x^3 - x^2 - x + 10$ $\frac{dy}{dx} = 3x^2 - 2x - 1$ $(3x + 1)(x - 1) = 0$ $x = -\frac{1}{3}$ or $x = 1$ $\frac{d^2y}{dx^2} = 6x - 2$, $x = 1$ gives +ve (4) Min point at $x = 1$ $y = 9$ found	M1 M1 M1* M1 A1 M1dep A1 A1 **[8]**	Attempt to multiply out brackets Attempt to differentiate their cubic Sets their $\frac{dy}{dx} = 0$ Correct method to solve quadratic Correct x values of turning points found **www** Valid method to establish which is min point with a conclusion Correct conclusion for $x = 1$ found from correct factorisation (even if other root incorrect) **www** for (1, 9) given as minimum point (ignore other point here)	<u>Alternative for product rule</u> Attempt to use product rule **M1** Expand brackets of both parts **M1** Then as main scheme Any extra values for turning points loses all three **A** marks (eg by sketching positive cubic, second diff method for either of their x values, y co-ords etc.) If constant incorrect in initial expansion, max **5/8**

Question			Answer	Marks	Guidance	
7	(ii)		$(-3)^2 - 4 \times 1 \times 5$ $= -11$	M1 A1 **[2]**	Uses $b^2 - 4ac$	$\sqrt{b^2 - 4ac}$ is **M0**
7	(iii)			B2 **[2]**	Fully correct argument - no extra incorrect statements e.g. 1) Justifying the quadratic factor having no roots so only intersection with x-axis is at $x = -2$ and stating it's a positive cubic 2) Sketch of positive cubic with one root at (–2, 0) and a min point at (1, 9) (f/t positive $y(1)$ from (i))	Award **B1** for either of: 1) Justifying the quadratic factor having no roots so only intersection with x-axis is at $x = -2$ 2) Sketch of positive cubic with one root at (–2, 0) and a min point with y coordinate positive or 0
8			B lies on l so has coordinates (x, 11 - 2x) $(x-3)^2 + (11-2x-5)^2 = (6\sqrt{5})^2$ $5x^2 - 30x - 135 = 0$ $5(x+3)(x-9) = 0$ $x = -3$, $x = 9$ $y = 17$, $y = -7$	M1 M1 M1* M1dep A1 A1 **[6]**	Attempt to find equation of l with gradient -2 $(x-3)^2 + (y - 5)^2 = (6\sqrt{5})^2$ o.e. seen Attempts to solve the equations simultaneously to get a quadratic Correct method to solve their quadratic Both x values Both y values	e.g. by substitution as shown **SC** If A0 A0, one correct pair of values from correct factorisation **www B1**
			Alternative method: Use of (1, 2, $\sqrt{5}$) triangle with –ve gradient M1 Scaling to $6\sqrt{5}$ M1 (3, 5) + (6, -12) M1 (9, -7) A1 (3, 5) - (6, -12) M1 (-3, 17) A1		**SC** Spotted solutions Each correct pair **www B1** (May also earn first two Ms as in main scheme)* **-1** for one or two extra incorrect solutions **-2** for three or more extra incorrect solutions Checks solutions and justifies only two solutions **B2** *** NB –** First M1 may also be awarded for establishing gradient between (3,5) and their solution(s) is **–** 2	

Question			Answer	Marks	Guidance	
9	(i)		$(x-3)(x+4)=0$ $x=3$ or $x=-4$  	M1 A1 B1 B1 B1 **[5]**	Correct method to find roots Correct roots Negative quadratic curve y intercept (0, 12) Good curve, with correct roots 3 and –4 indicated and max point in 2nd quadrant	 i.e. max at (0, 12) **B0** Curve must go below x-axis for final mark
9	(ii)		$-4 < x < 3$	M1 A1 **[2]**	Correct method to solve quadratic inequality Allow $\le$ for the method mark but not the accuracy mark	their lower root < x < their higher root Allow "$x > -4, x < 3$" Allow "$x > -4$ and $x < 3$" Do not allow "$x > -4$ **or** $x < 3$"
9	(iii)		$y = 4 - 3x$ $12 - x - x^2 = 4 - 3x$ $x^2 - 2x - 8 = 0$ $(x-4)(x+2) = 0$ $x = 4$ or $x = -2$ $y = -8$ or $y = 10$	 M1 A1 M1 A1 A1 **[5]**	 substitute for x/y or attempt to get an equation in 1 variable only obtain correct 3 term quadratic correct method to solve 3 term quadratic	e.g. for first mark $3x + 12 - x - x^2 = 4$, or $y = 12 - \left(\frac{4-y}{3}\right) - \left(\frac{4-y}{3}\right)^2$ (this leads to $y^2 - 2y - 80 = 0$). Condone poor algebra for this mark. **SC** If **A0 A0**, give **B1** for one correct pair of values spotted or from correct factorisation **www**

Question			Answer	Marks	Guidance	
10	(i)		$(x+2)^2+(y-4)^2=25$	M1	$(x+2)^2$ and $(y-4)^2$ seen (or implied by x^2+4x+y^2-8y)	<u>Alternative markscheme for *f*, *g*, *c* method</u>:
			$x^2+4x+4+y^2-8y+16-25=0$	M1	$(x\pm 2)^2+(y\pm 4)^2 = 25$	x^2+4x+y^2-8y **B1**
			$x^2+y^2+4x-8y-5=0$	A1	Correct equation in correct form (terms can be in any order but must have "=0")	$c = 2^2 + (\pm 4)^2 - 25$ **M1** Correct equation in correct form **A1**
				[3]		
10	(ii)		gradient of radius = $\frac{8-4}{-5+2}$	M1	uses $\frac{y_2-y_1}{x_2-x_1}$ (3/4 substitutions correct)	
			$= -\frac{4}{3}$	A1	Allow $\frac{4}{-3}$	
			gradient of tangent = $\frac{3}{4}$	B1FT		
			$y-8=\frac{3}{4}(x+5)$	M1	correct equation of straight line through (– 5, 8), any non-zero gradient	
			$3x-4y+47=0$	A1 **[5]**	Shows rearrangement to given equation **AG** **CWO** throughout for A1	
			<u>Alternative by rearrangement</u> Gradient of radius = $\frac{8-4}{-5+2} = \frac{-4}{3}$ **M1* A1** Attempts to rearrange equation of line to find gradient of line = $\frac{3}{4}$ **M1dep** Multiply gradients to get -1 **B1** Check (-5, 8) lies on line **B1 (dep on both M1s)**		<u>Alternative for equating given line to circle</u> Substitute for *x*/*y* or attempt to get an equation in 1 variable only **M1** $k(x^2 + 10x + 25) = 0$ or $k(y^2 - 16y + 64) = 0$ **A1** Correct method to solve quadratic **M1** x = -5, y = 8 found **A1** States one root implies tangent **B1**	<u>Alternative markscheme for implicit differentiation</u>: **M1** Attempt at implicit diff as evidenced by $2y\frac{dy}{dx}$ term **A1ft** $2x + 2y\frac{dy}{dx} + 4 - 8\frac{dy}{dx} = 0$ ft from their equation in (i) **A1** Substitution of (-5, 8) to obtain $\frac{3}{4}$ then final 2 marks as main scheme

Question			Answer	Marks	Guidance	
10	(iii)		(3 × 3) – (4 × 14) + 47 = 0	B1 **[1]**	Sufficient correct working to verify statement e.g. verifying co-ordinate as shown	Alt: showing line joining (-5, 8) to (3, 14) has same gradient etc.
10	(iv)		$\sqrt{(3-{}^-5)^2+(14-8)^2}$ $=10$ Area of triangle $= \frac{1}{2}\times 10\times 5$ $= 25$	M1 A1 M1 A1 **[4]**	Use of $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ for *TP* Must use their *TP* and their *CP*	<u>Alternative method:</u> Attempt to find area of enclosing rectangle and subtract areas of other three triangles **M1*** Correct use area of triangle formula **M1 dep** All four values correct **A1** Final answer correct **A1** (Use the same principle for any enclosing shape)

Solving a quadratic

This is particularly important to mark correctly as it can sometimes feature several times on a single examination paper. An example is usually included with the markscheme each session; this has varied slightly over the years and should be referred to every session. Consider the equation $3x^2 - 10x - 8 = 0$.

1) If the candidate attempts to solve by factorisation, their attempt when expanded must produce the **correct quadratic term** and **one other correct term** (with correct sign):

$(3x+1)(x-8)=0$	**M1**	$3x^2$ and -8 obtained from expansion
$(3x-1)(x-3)=0$	**M1**	$3x^2$ and $-10x$ obtained from expansion
$(3x-2)(x-4)=0$	**M0**	only $3x^2$ term correct

2) If the candidate attempts to solve by using the formula

a) If the formula is quoted incorrectly then **M0**.
b) If the formula is quoted correctly then one **sign** slip is permitted. Substituting the wrong numerical value for *a* or *b* or *c* scores **M0**.

$\dfrac{-10 \pm \sqrt{(-10)^2 - 4\times3\times-8}}{6}$	earns **M1** (minus sign incorrect at start of formula)
$\dfrac{10 \pm \sqrt{(-10)^2 - 4\times3\times-8}}{2\times3}$	earns **M1** (8 for *c* instead of −8)
$\dfrac{-10 \pm \sqrt{(-10)^2 - 4\times3\times8}}{6}$	**M0** (2 sign errors: initial sign and *c* incorrect)
$\dfrac{10 \pm \sqrt{(-10)^2 - 4\times3\times-8}}{2\times-10}$	**M0** (2*b* on the denominator)

Notes – for equations such as $3x^2 - 10x - 8 = 0$, then $b^2 = 10^2$ would be condoned in the discriminant and would not be counted as a sign error. Repeating the sign error for *a* both occurrences in the formula would be two sign errors and score **M0**.

c) If the formula is not quoted at all, substitution must be completely correct to earn the **M1**

3) If the candidate attempts to complete the square, they must get to the "square root stage" involving ±; we are looking for evidence that the candidate knows a quadratic has two solutions.

$$3x^2 - 10x - 8 = 0$$

$$3\left(x^2 - \frac{10}{3}x\right) - 8 = 0$$

$$3\left[\left(x - \frac{5}{3}\right)^2 - \frac{25}{9}\right] - 8 = 0$$

$$\left(x - \frac{5}{3}\right)^2 = \frac{49}{9}$$

$$x - \frac{5}{3} = \pm\sqrt{\frac{49}{9}}$$

This is where the **M1** is awarded – arithmetical errors may be condoned provided $x - \frac{5}{3}$ (or equivalent) seen or implied

If a candidate makes repeated attempts (e.g. fails to factorise and then tries the formula), mark only what you consider to be their last full attempt – see guidance later in this document.

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