GCE

Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4766:** Statistics 1

Mark Scheme for January 2012

Annotations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions

a   Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c   The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d　　When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e　　The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f　　Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g　　Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h   For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	Additional Guidance
1	(i)		0 | 8 8 10 | 5 5 20 | 5 6 9 30 | 1 1 4 4 6 Key 20 | 9 represents 29 degrees Celsius	G1 G1 G1 G1 **[4]**	Stem **(in either order)** Leaves Sorted and aligned **(use paper test if unsure)** Key	Do not allow leaves 25 ,26, 29 etc Ignore commas between leaves (indep). Condone 1 error or omission Allow errors in leaves if sorted Condone missing units (Celsius) Allow stem 0, 1, 2, 3
	(ii)		Median = 27.5	B1 **[1]**		**CAO**
1	(iii)		The median since the mean is affected by the skewness of the distribution	B1 E1 **[2]**	For median Allow **E2 for** mean if supported by very convincing reason **EG takes all values into account and no extreme values**	Do not allow 'less affected by extremes or outliers' unless also mention (**positive or negative**) skewness. Condone 'bottom half more spread' or similar
2	(i)		Mean $= \dfrac{759.00}{60} = £12.65$ Sxx $= 11736.59 - \dfrac{759^2}{60} = 2135.24$ $s = \sqrt{\dfrac{2135.24}{59}} = £6.02$	B1 M1 A1 **[3]**	Ignore units For Sxx CAO ignore units Allow more accurate answers	**CAO** Do not allow 759/60 as final answer but allow $12\,{}^{13}/_{20}$ M1 for 11736.59 - 60 × their mean^2 BUT NOTE M0 if their $S_{xx} < 0$ For s^2 of 36.2 (or better) allow M1A0 with or without working For RMSD of 5.97 or 5.96 (or better) allow M1A0 provided working seen For RMSD^2 of 35.6 (or better) allow M1A0 provided working seen
2	(ii)		New mean = 12.65 × 1.02 = £12.90 New sd = 6.02 × 1.02 = £6.14	B1 B1 **[2]**	FT their mean Awrt 12.90 Allow 12.9 FT their sd	If candidate 'starts again' only award marks for CAO **Deduct at most 1 mark overall in whole question for overspecification of Mean and 1mark overall for SD**

Question			Answer	Marks	Guidance	Additional Guidance
2	(iii)		New mean = 12.65 + 0.25 = £12.90	B1	FT their mean Awrt 12.90	If candidate 'starts again' only award marks for CAO
			New sd = £6.02	B1 **[2]**	FT their sd (unless negative) Awrt 6.02	Allow sd unchanged (or similar)
3	(i)			G1 G1 G1 **[3]**	Do a vertical scan and give: First column Second column Final column	All indep All probs must be correct Without extra branches in final column Ignore anything before third set Allow labels 'win' and 'lose' in place of Jimmy and Alan respectively **but if no labels, no marks**
3	(ii)		P(Alan wins) = (0.4×0.3×0.6) + (0.6×0.4×0.3) + (0.6×0.6) = 0.504	M1 M1 A1 **[3]**	For any one '**correct**' product For all three '**correct**' products **and no extras** CAO	FT their tree for both M marks Provided **correct number of terms in product(s) for both M1's**
3	(iii)		P(Ends after 4) = (0.4×0.7) + (0.6×0.6) = 0.28 + 0.36 = 0.64	M1 A1 **[2]**	For both products CAO	FT their tree for M mark but not for A mark Provided **two terms in each product**
4	(i)		Because P(T / M) ≠ P(T)	E1 **[1]**	Or 0.8 ≠ 0.55	Or P($T \cap M$) (= 0.264) ≠ P(T) × P(M), **provided 0.264 in (ii)** Or 0.264 ≠ 0.55 × 0.33 (=0.1815) Look out for complement methods, etc
4	(ii)		P($T \cap M$) = P(T / M) × P(M) = 0.80 × 0.33 = 0.264	M1 A1 **[2]**	For product CAO	**A0 for 0.26**

Question			Answer	Marks	Guidance	Additional Guidance
4	(iii)			G1	For two labelled intersecting circles	Allow labels such as P(T) etc
				G1	For at least 2 correct probabilities. FT their P($T \cap M$)	Allow other shapes in place of circles
						No need for 'box'
				G1	For remaining probabilities. FT their P($T \cap M$), **providing probabilities between 0 and 1**	**FT from 0.1815 in (ii) gives 0.3685, 0.1815, 0.1485, 0.3015**
				[3]		
5	(i)		P(*X*=1) = P(g,b)+P(b,g)+P(b,b,g)+P(b,b,b,g)	M1	For any two correct fractions	Must have correct ref to numbers of boys and girls, not just fractions
			$= \dfrac{1}{4}+\dfrac{1}{4}+\dfrac{1}{8}+\dfrac{1}{16}=\dfrac{11}{16}$	M1	For all four correct fractions	With no extras Accept 0.6875, not 0.688.
			OR P(*X*=1) =1- P(*X*≠1) =1 - (P(bbbb)+P(ggb)+P(gggb)+P(gggg)) $=1-\left(\dfrac{1}{16}+\dfrac{1}{8}+\dfrac{1}{16}+\dfrac{1}{16}\right)=\dfrac{11}{16}$	A1	*NB Answer given*	Watch for use of B(4, 0.5) P(*X*≤2)=0.6875 which gets M0M0A0.
				[3]		

Question			Answer	Marks	Guidance	Additional Guidance
5	(ii)		$E(X) = (0 \times \frac{1}{16}) + (1 \times \frac{11}{16}) + (2 \times \frac{1}{8}) + (3 \times \frac{1}{16}) + (4 \times \frac{1}{16})$ $= 1\frac{3}{8} = 1.375$ $E(X^2) = (0 \times \frac{1}{16}) + (1 \times \frac{11}{16}) + (4 \times \frac{1}{8}) + (9 \times \frac{1}{16}) + (16 \times \frac{1}{16})$ $= 2\frac{3}{4} = 2.75$ $Var(X) = 2\frac{3}{4} - \left(1\frac{3}{8}\right)^2 = \frac{55}{64} = 0.859$	M1 A1 M1 M1 A1 **[5]**	For Σrp (at least 3 terms correct) A1 CAO **Allow 1.38, not 1.4** For $\Sigma r^2 p$ (at least 3 terms correct) M1dep for – their E(X)² A1 FT their E(X) provided Var(X) > 0 **0.86, not 0.9**	Allow 22/16 Use of $E(X-\mu)^2$ gets M1 for attempt at $(x-\mu)^2$ should see $(-1.375)^2$, $(-0.375)^2$, $(0.625)^2$, 1.625^2, 2.625^2 (if E(X) correct but FT their E(X)) (all 5 correct for M1), then M1 for $\Sigma p(x-\mu)^2$ (at least 3 terms correct) Division by 5 or other spurious value at end gives max M1A1M1M1A0, or M1A0M1M1A0 if E(X) also divided by 5. Unsupported correct answers get 5 marks. **Using 1.38 gets Var of 0.8456 gets A1**
6	(i)	(A)	$X \sim B(20, 0.25)$ $P(4 \text{ smokers}) = \binom{20}{4} \times 0.25^4 \times 0.75^{16} = 0.1897$ **OR** Or from tables = 0.4148 – 0.2252 = 0.1896	M1 M1 A1 M2 A1 **[3]**	For $0.25^4 \times 0.75^{16}$ For $\binom{20}{4} \times p^4 \times q^{16}$ CAO For 0.4148 – 0.2252 CAO	With $p + q = 1$ Also for 4845×0.00003915 Allow 0.19 or better See tables at the website http://www.mei.org.uk/files/pdf/formula_book_mf2.pdf **0.189 gets A0**
6	(i)	(B)	$P(3 \le X \le 6) = 0.7858 – 0.0913 = 0.6945$	M1 M1 A1 **[3]**	For (P(X≤6) =) 0.7858 seen For **their** 0.7858 – 0.0913 CAO	Or P(X=3) + P(X=4) + P(X=5) + P(X=6) = 0.1339 + 0.1897 + 0.2023 + 0.1686 = 0.6945. M1 for three correct terms (to 2sf). Accept 0.69 or better **P(X≥3) - P(X>6) = 0.9087-0.2142 = 0.6945 Gets M1 M1 A1**

Question			Answer	Marks	Guidance	Additional Guidance
6	(i)	(*C*)	$E(X) = np = 20 \times 0.25 = 5$	B1 **[1]**	CAO	
6	(ii)	(*A*)	Let p = probability that a randomly selected student is a smoker H_0: $p = 0.25$ H_1: $p < 0.25$	B1 B1 B1 **[3]**	For definition of p in context For H_0 For H_1 **Allow complementary probabilities. Mark as per scheme.** **ie H_0:$p = 0.75$ etc**	Minimum needed for B1 is p = probability that student is a smoker. Allow p = P(student smokes) for B1 Definition of p must include word probability (or chance or proportion or percentage or likelihood but NOT possibility). Preferably as a separate comment. However can be at end of H_0 as long as it is a clear definition ' p = the probability that student is a smoker.,NOT just a sentence 'probability is 0.25' H_0 : p(student is a smoker) = 0.25, H_1 : p(student is a smoker) < 0.25 gets B0B1B1 Allow p=25%, allow θ or π and ρ but not x. However allow any single symbol if defined Allow $H_0 = p$=0.25, Do not allow H_0 : P(X=x) = 0.25, H_1 : P(X=x) < 0.25 Do not allow H_0: =0.25, =25%, P(0.25), p(0.25), p(x)=0.25, x=0.25 (unless x correctly defined as a probability) Do not allow H_1:$p \leq 0.25$, Do not allow H_0 and H_1 reversed for B marks but can still get E1 below Allow NH and AH in place of H_0 and H_1 For hypotheses given in words allow Maximum B0B1B1 and E1 below. Hypotheses in words must include probability (or chance or proportion or percentage) and the figure 0.25 oe.

Question			Answer	Marks	Guidance	Additional Guidance
6	(ii)	(B)	H_1 has this form as the programme aims to reduce the **proportion** of smokers.	E1 [1]	**Allow 'number' Allow 'aims for a reduction' or similar**	E0 if H_1 **upper tail or two tailed**
6	(iii)		$P(X \leq 1) = 0.0243 < 5\%$ $P(X \leq 2) = 0.0913 > 5\%$ So critical region is {0,1}	B1 B1 M1 A1 [4]	For **P(X ≤ 1) =** 0.0243 For **P(X ≤ 2) =** 0.0913 For at least one comparison with 5% CAO for critical region *dep* on M1 and at least one B1	**With full correct notation. Penalise once for eg P(X=1), P(X=2)** Allow any form of statement of CR eg $X \leq 1$, $X < 2$, annotated number line, etc but not $P(X \leq 1)$ NB USE OF POINT PROBABILITIES gets B0B0M0A0 If no working but correct CR, no marks See additional notes below the scheme for other possibilities
6	(iv)		3 does not lie in the critical region, so not significant, So there is not enough evidence to reject the null hypothesis and we conclude that there is not enough evidence to suggest that the percentage of smokers has decreased.	E1**dep** E1**dep** [2]	For 3 not in CR or for not significant **or reject H_1** For conclusion in context **Condone omission of 'not enough evidence' in this case**	Dep on correct CR, (**correctly obtained**) E0E0 for P(X=3) not in CR **E0E0 if wrong working after 3 not in CR** Alternative scheme P(X≤3) = 0.2252 > 5% so not sig etc. gets E2 for complete method but E0 otherwise.
7	(i)		Percentage $= \dfrac{40}{200} \times 100 = 20$	M1 A1 [2]	For 40 **seen or implied** CAO	
7	(ii)		Median = 5.2 kg Q1 = 4.2 Q3 = 5.8 Inter-quartile range $= 5.8 - 4.2 = 1.6$	B1 B1 B1 [3]	For Q1 or Q3 For IQR	Allow 4.2 to **4.3** for Q1 **Dep on both quartiles correct**

Question			Answer	Marks	Guidance	Additional Guidance
7	(iii)		Lower limit $4.2 - (1.5 \times 1.6) = 1.8$	B1	For 1.8 ft	Any use of <u>median</u> ± 1.5 IQR scores B0 B0 E0
			Upper limit $5.8 + (1.5 \times 1.6) = 8.2$	B1	For 8.2 ft	**E0 if say some outliers at bottom end, unless lower limit > 2.0**
			So there are one or more outliers (if any lamb weighs more than 8.2 kg)	E1	Dep on their 1.8 and 8.2 **Allow any number of outliers ≤ 5**	If FT leads to limits above 9.0 <u>and</u> below 2.0 then E0
						No marks for ± 2 or 3 IQR
			Should not be disregarded because:	E1	Indep **Must give reason.**	**With 4.3 and 5.8 lower = 2.05 and upper = 8.05**
			'Nothing to suggest they are not genuine items of data'			In this part FT their values from (ii) if sensibly obtained but not from location ie 12.5, 37.5
			Allow other convincing reasons such as very few so will not make much difference			
				[4]		No marks for use of mean $\pm 2s$
7	(iv)		Median for Welsh Mountain = 3.6	B1		
			IQR for Welsh Mountain = 0.8	B1		
			Welsh Mountain lambs have lower average weight than crossbred	E1 **indep**	Must imply average or CT, not just median. **Allow generally lighter**	FT their medians FT their IQRs Can get max B1B0E1E1 for use of range
			Welsh Mountain lambs also have lower variation in weight than crossbred	E1 **indep**	**Must imply spread or variation, not just IQR or range** **Allow correct comment on consistency**	
				[4]		
7	(v)		**Median unchanged**	E1		
			IQR unchanged OR range or spread increased	E1	**even if used IQR in (iv)**	**E2 for 'Both comparisons remain the same'**
				[2]		**E1 for 'the range remains smaller'**

Question			Answer	Marks	Guidance	Additional Guidance
7	(vi)		$P(\text{Crossbred} > 3.9) = \dfrac{165}{200}$	B1		Allow 162 to 165 out of 200
			$P(\text{Welsh Mountain} > 3.9) = \dfrac{1}{4}$	B1	For product of their probabilities , provided one is correct	
			$P(\text{Both} > 3.9) = \dfrac{165}{200} \times \dfrac{1}{4} = \dfrac{165}{800} = \dfrac{33}{160} = 0.206$	M1	CAO	**Allow answers in range 0.2025 to 0.20625 with correct working**
				A1 **[4]**		

NOTE RE OVER-SPECIFICATION OF ANSWERS

If answers are grossly over-specified , deduct the final answer mark in every case. Probabilities should also be rounded to a sensible degree of accuracy. In general final non probability answers should not be given to more than 4 significant figures. Allow probabilities given to 5 sig fig.

Additional notes re Q6 parts iii, iv:

Smallest critical region method for part (iii):

Smallest critical region that 1 could fall into has size 0.0243 gets B1,
Smallest critical region that 2 could fall has size 0.0913 gets B1,   This is > 5% or above < 5% gets M1, A1 as per scheme

Use of k method with no probabilities quoted:

$P(X \leq k) > 5\%$ and $P(X \leq k - 1) < 5\%$ followed by $k = 2$ gets SC1
so CR is {0, 1} gets another SC1 dep on first SC1

Use of k method with one probability quoted:

Mark as per scheme – max B0B1M1A1

Two tailed test with H$_1$: $p \neq 0.25$
Gets SC2 for fully correct FT with working as follows $P(X \leq 1) = 0.0243 < 0.025$ and $P(X \geq 10) = 0.0139 > 0.025$ B1 CR is {0,1, 10, 11, ..., 20}
 (iv) Final 2 marks Max M1A1.

Two tailed test done but with correct H_1: $p<0.25$
(ii) gets max B1B1B1E1
(iii) if compare with 5% ignore work on upper tail and mark lower tail as per scheme but if include upper tail in CR then A0
if compare with 2.5% no marks B0B0M0A0
(iv) Final 2 marks can get M1A1 if correct CR, or SC2 if they start again, provided that they compare with 5%, not 2.5%.

Lower **or upper** tailed test with H_1: $p>0.25$ **and 6(ii)B wrong way around**
(ii) gets max B1B1B0E0
(iii) no marks B0B0M0A0
(iv) Final 2 marks get M0A0

Lower tailed test with H_1: $p>0.25$ and 6(ii)B right way around
(ii) gets max B1B1B0E0 , note E0, not E1
(iii) and (iv) Mark as per scheme, so full marks possible

Line diagram method for (iii)
No marks unless some 0.0243 shown on diagram, then B1 for squiggly line between 1and 2 or on 1, B1*dep* for arrow pointing to left, M1 0.0243 seen on diagram from squiggly line or from 1, A1 for CR written down in words/symbols. . **If 0.0243 and 0.0913 both seen and no other marks earned give B1.**

(iv) M1A1 as per scheme

Bar chart method for (iii)
No marks unless 0.0243 shown on diagram, then B1 for line clearly on boundary between 1 and 2 or within 1 block, B1*dep* for arrow pointing to left, M1 0.0243 seen on diagram from boundary line or from 1, A1 for CR written down in words/ symbols. **If 0.0243 and 0.0913 both seen and no other marks earned give B1.**
(iv) M1A1 as per scheme.

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