

Tuesday 17 January 2012 – Morning

**A2 GCE MATHEMATICS**

**4724** Core Mathematics 4

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4724
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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1 When the polynomial  $f(x)$  is divided by  $x^2 + 1$ , the quotient is  $x^2 + 4x + 2$  and the remainder is  $x - 1$ . Find  $f(x)$ , simplifying your answer. [3]

2 (i) Find, in the form  $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ , an equation of the line  $l$  through the points  $(4, 2, 7)$  and  $(5, -4, -1)$ . [3]

(ii) Find the acute angle between the line  $l$  and a line in the direction of the vector  $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ . [4]

3 The equation of a curve  $C$  is  $(x + 3)(y + 4) = x^2 + y^2$ .

(i) Find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ . [4]

(ii) The line  $2y = x + 3$  meets  $C$  at two points. What can be said about the tangents to  $C$  at these points? Justify your answer. [2]

(iii) Find the equation of the tangent at the point  $(6, 0)$ , giving your answer in the form  $ax + by = c$ , where  $a$ ,  $b$  and  $c$  are integers. [2]

4 (i) Expand  $(1 - 4x)^{\frac{1}{4}}$  in ascending powers of  $x$ , up to and including the term in  $x^3$ . [5]

(ii) The term of lowest degree in the expansion of

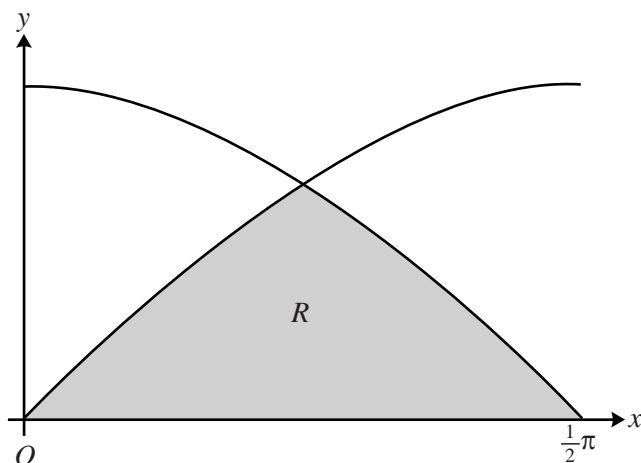
$$(1 + ax)(1 + bx^2)^7 - (1 - 4x)^{\frac{1}{4}}$$

in ascending powers of  $x$  is the term in  $x^3$ . Find the values of the constants  $a$  and  $b$ . [4]

5 Use the substitution  $u = \cos x$  to find the exact value of

$$\int_0^{\frac{1}{3}\pi} \sin^3 x \cos^2 x \, dx. \quad [6]$$

6



The diagram shows the curves  $y = \cos x$  and  $y = \sin x$ , for  $0 \leq x \leq \frac{1}{2}\pi$ . The region  $R$  is bounded by the curves and the  $x$ -axis. Find the volume of the solid of revolution formed when  $R$  is rotated completely about the  $x$ -axis, giving your answer in terms of  $\pi$ . [7]

7 The equation of a straight line  $l$  is

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}.$$

$O$  is the origin.

- (i) Find the position vector of the point  $P$  on  $l$  such that  $OP$  is perpendicular to  $l$ . [3]
- (ii) A point  $Q$  on  $l$  is such that the length of  $OQ$  is 3 units. Find the two possible position vectors of  $Q$ . [3]

8 A curve is defined by the parametric equations

$$x = \sin^2 \theta, \quad y = 4 \sin \theta - \sin^3 \theta,$$

where  $-\frac{1}{2}\pi \leq \theta \leq \frac{1}{2}\pi$ .

- (i) Show that  $\frac{dy}{dx} = \frac{4 - 3 \sin^2 \theta}{2 \sin \theta}$ . [3]
- (ii) Find the coordinates of the point on the curve at which the gradient is 2. [3]
- (iii) Show that the curve has no stationary points. [2]
- (iv) Find a cartesian equation of the curve, giving your answer in the form  $y^2 = f(x)$ . [2]

[Questions 9 and 10 are printed overleaf.]

9 Find the exact value of  $\int_0^1 (x^2 + 1)e^{2x} dx$ . [7]

10 (i) Write down the derivative of  $\sqrt{y^2 + 1}$  with respect to  $y$ . [1]

(ii) Given that  $\frac{dy}{dx} = \frac{(x-1)\sqrt{y^2+1}}{xy}$  and that  $y = \sqrt{e^2 - 2e}$  when  $x = e$ ,  
find a relationship between  $x$  and  $y$ . [8]

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