Mark Scheme (Results)

January 2013

GCE Core Mathematics – C3 (6665/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- Unless indicated in the mark scheme a correct answer with no working should gain full marks for that part of the question.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎣ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but incorrect answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of ‘0’ or ‘1’ for each mark, or “trait”, as shown:

	0	1
aM		●
aA	●	
bM1		●
bA1	●	
bB	●	
bM2		●
bA2		●

January 2013
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $-32 = (2w-3)^5 \Rightarrow w = \frac{1}{2}$ oe	M1A1
		(2)
	(b) $\frac{dy}{dx} = 5\times(2x-3)^4\times 2$ or $10(2x-3)^4$	M1A1
	When $x=\frac{1}{2}$, Gradient = 160	M1
	Equation of tangent is $'160' = \frac{y-(-32)}{x-'\frac{1}{2}'}$ oe	dM1
	$y = 160x - 112$ cso	A1
		(5)
		(7 marks)

(a) M1 Substitute ***y***=-32 into $y=(2w-3)^5$ **and** proceed to *w*=….. [Accept positive sign used of *y*, ie *y*=+32]

A1 Obtains w or $x=\frac{1}{2}$ oe with no incorrect working seen. Accept alternatives such as 0.5.

Sight of just the answer would score both marks as long as no incorrect working is seen.

(b) M1 Attempts to differentiate $y=(2x-3)^5$ using the chain rule.

Sight of $\pm A(2x-3)^4$ where *A* is a non- zero constant is sufficient for the method mark.

A1 A correct (un simplified) form of the differential.

Accept $\frac{dy}{dx} = 5\times(2x-3)^4\times 2$ or $\frac{dy}{dx} = 10(2x-3)^4$

M1 This is awarded for an attempt to find the gradient of the tangent to the curve at *P*
Award for substituting their numerical value to part (a) into their differential to find the numerical gradient of the tangent

dM1 Award for a correct method to find an equation of the tangent to the curve at *P*. It is dependent upon the previous M mark being awarded.

Award for $'their\ 160' = \frac{y-(-32)}{x - their\ '\frac{1}{2}'}$

If they use $y=mx+c$ it must be a full method, using m= 'their 160', their '$\frac{1}{2}$' and -32.

An attempt must be seen to find *c*=…

A1 cso $y = 160x - 112$. The question is specific and requires the answer in this form.
You may isw in this question after a correct answer.

Question Number	Scheme	Marks
2.	(a) $0=e^{x-1}+x-6 \Rightarrow x=\ln(6-x)+1$	M1A1* (2)
	(b) Sub $x_0=2$ into $x_{n+1}=\ln(6-x_n)+1 \Rightarrow x_1=2.3863$	M1, A1
	AWRT 4 dp. $x_2=2.2847$ $x_3=2.3125$	A1 (3)
	(c) Chooses interval [2.3065,2.3075]	M1
	g(2.3065)=-0.0002(7), g(2.3075)=0.004(4)	dM1
	Sign change , hence root (correct to 3dp)	A1 (3)
		(8 marks)

(a) M1 Sets g(x)=0,and using correct *ln* work, makes the x of the e^{x-1} term the subject of the formula.
Look for $e^{x-1}+x-6=0 \Rightarrow e^{x-1}=\pm 6\pm x \Rightarrow x=\ln(\pm 6\pm x)\pm 1$
Do not accept $e^{x-1}=6-x$ **without firstly seeing** $e^{x-1}+x-6=0$ **or a statement that g(x)=0 ⇒**

A1* cso. $x=\ln(6-x)+1$ **Note that this is a given answer (and a proof).**
'Invisible' brackets are allowed for the M but not the A
Do not accept recovery from earlier errors for the A mark. The solution below scores 0 marks.
$0=e^{x-1}+x-6 \Rightarrow \quad 0=x-1+\ln(x-6) \Rightarrow x=\ln(6-x)+1$

(b) M1 Sub $x_0=2$ into $x_{n+1}=\ln(6-x_n)+1$ to produce a numerical value for x_1.
Evidence for the award could be any of $\ln(6-2)+1$, $\ln 4+1$, 2.3….. or awrt 2.4

A1 Answer correct to 4 dp $x_1=2.3863$.
The subscript is not important. Mark as the first value given/found.

A1 Awrt 4 dp. $x_2=2.2847$ *and* $x_3=2.3125$
The subscripts are not important. Mark as the second and third values given/found

(c) M1 Chooses the interval [2.3065,2.3075] or smaller containing the root 2.306558641

dM1 Calculates g(2.3065) and g(2.3075) with at least one of these correct to 1sf.
The answers can be rounded or truncated
g(2.3065) = -0.0003 rounded, g(2.3065)= -0.0002 truncated
g(2.3075) = (+) 0.004 rounded and truncated

A1 Both values correct (rounded or truncated),
A reason which could include change of sign, >0 <0, g(2.3065)× g(2.3075) <0
AND a minimal conclusion such as hence root, α=2.307 or □
Do not accept continued iteration as question demands an interval to be chosen.

Alternative solution to (a) working backwards

M1 Proceeds from $x=\ln(6-x)+1$ using correct exp work to …….=0

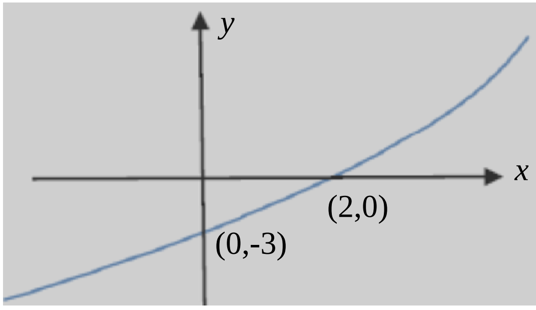
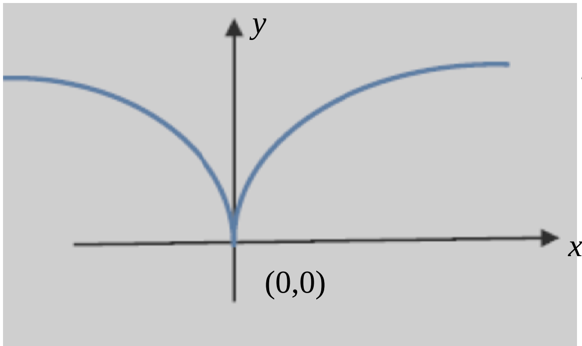
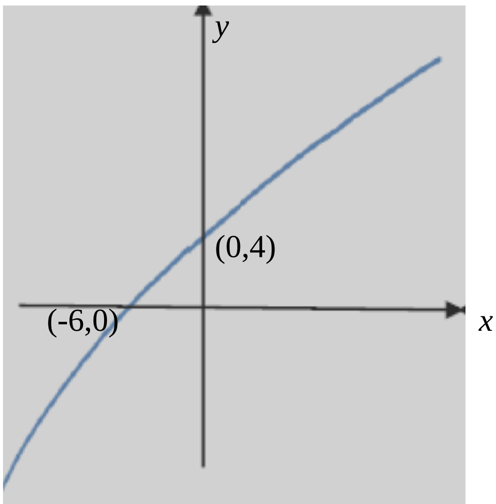
A1 **Arrives correctly** at $e^{x-1}+x-6=0$ **and** makes a statement to the effect that this is g(x)=0

Alternative solution to (c) using $\mathrm{f}(x)=\ln(6-x)+1-x$ **{Similarly** $\mathrm{h}(x)=x-1-\ln(6-x)$ **}**

M1 Chooses the interval [2.3065,2.3075] or smaller containing the root 2.306558641

dM1 Calculates f(2.3065) and f(2.3075) with at least 1 correct rounded or truncated
f(2.3065) = 0.000074. Accept 0.00007 rounded or truncated. Also accept 0.0001

$f(2.3075) = -\ 0.0011..$ Accept -0.001 rounded or truncated

Question Number	Scheme		Marks
3.	(a) ff(-3)= f(0),=2		M1,A1 (2)
	(b)  $y = f^{-1}(x)$	Shape (0,-3) and (2,0)	B1 B1 (2)
	(c)  y=f(|x|)-2	Shape (0,0)	B1 B1 (2)
	(d) 	Shape (-6,0) or (0,4) (-6,0) and (0,4)	B1 B1 B1 (3)
			(9 marks)

(a) M1 A full method of finding ff(-3). f(0) is acceptable but f(-3)=0 is not.

Accept a solution obtained from two substitutions into the equation $y = \frac{2}{3}x + 2$ as the line passes through both points. Do not allow for $y = \ln(x+4)$, which only passes through one of the points.

A1 Cao ff(-3)=2. Writing down 2 on its own is enough for both marks provided no incorrect working is seen.

(b)

B1 For the correct shape. Award this mark for an increasing function in quadrants 3, 4 and 1 only. Do not award if the curve bends back on itself or has a clear minimum

B1 This is independent to the first mark and for the graph passing through (0,-3) and (2, 0)

Accept -3 and 2 marked on the correct axes.
Accept (-3,0) and (0,2) instead of (0,-3) and (2,0) as long as they are on the correct axes
Accept P'=(0,-3), Q'=(2,0) stated elsewhere as long as P'and Q' are marked in the correct place on the graph
There must be a graph for this to be awarded

(c)

B1 Award for a correct shape 'roughly' symmetrical about the *y*- axis. It must have a cusp and a gradient that 'decreases' either side of the cusp. Do not award if the graph has a clear maximum

B1 (0,0) lies on their graph. Accept the graph passing through the origin without seeing (0, 0) marked

(d) B1 Shape. The position is not important. The gradient should be always positive but decreasing There should not be a clear maximum point.

B1 The graph passes through (0,4) **or** (-6,0). See part (b) for allowed variations

B1 The graph passes through (0,4) **and** (-6,0). See part (b) for allowed variations

Question Number	Scheme	Marks
4.	(a) $R^2 = 6^2 + 8^2 \Rightarrow R = 10$	M1A1
	$\tan\alpha = \frac{8}{6} \Rightarrow \alpha = \text{awrt } 0.927$	M1A1
		(4)
	(b)(i) $\mathrm{p}(x) = \frac{4}{12 + 10\cos(\theta - 0.927)}$	
	$\mathrm{p}(x) = \frac{4}{12 - 10}$	M1
	Maximum = 2	A1
		(2)
	(b)(ii) $\theta - '\mathit{their}\,\alpha' = \pi$	M1
	$\theta = \text{awrt } 4.07$	A1
		(2)
		(8 marks)

(a) M1 Using Pythagoras' Theorem with 6 and 8 to find *R*. Accept $R^2 = 6^2 + 8^2$

If α has been found first accept $R = \pm\frac{8}{\sin'\alpha'}$ or $R = \pm\frac{6}{\cos'\alpha'}$

A1 $R = 10$. Many candidates will just write this down which is fine for the 2 marks. Accept ± 10 but not -10

M1 For $\tan\alpha = \pm\frac{8}{6}$ or $\tan\alpha = \pm\frac{6}{8}$

If *R* is used then only accept $\sin\alpha = \pm\frac{8}{R}$ or $\cos\alpha = \pm\frac{6}{R}$

A1 $\alpha = \text{awrt } 0.927$. Note that 53.1^0 is A0

(b) Note that (b)(i) and (b)(ii) can be marked together

(i) M1 Award for $\mathrm{p}(x) = \frac{4}{12 - 'R'}$.

A1 Cao $\mathrm{p}(x)_{\max} = 2$.

The answer is acceptable for both marks as long as no incorrect working is seen

(ii) M1 For setting $\theta - '\mathit{their}\,\alpha' = \pi$ and proceeding to θ=..

If working exclusively in degrees accept $\theta - '\mathit{their}\,\alpha' = 180$

Do not accept mixed units

A1 $\theta = \text{awrt } 4.07$. If the final A mark in part (a) is lost for 53.1, then accept awrt 233.1

Question Number	Scheme	Marks
5.	(i)(a) $\frac{dy}{dx} = 3x^2 \times \ln 2x + x^3 \times \frac{1}{2x} \times 2$ $= 3x^2 \ln 2x + x^2$	M1A1A1 (3)
	(i)(b) $\frac{dy}{dx} = 3\boldsymbol{(x + \sin 2x)}^2 \times (1 + 2\cos 2x)$	**B1**M1A1 (3)
	(ii) $\frac{dx}{dy} = -\text{cosec}^2 y$	M1A1
	$\frac{dy}{dx} = -\frac{1}{\text{cosec}^2 y}$	M1
	Uses $\text{cosec}^2 y = 1 + \cot^2 y$ and $x = \cot y$ in $\frac{dy}{dx}$ or $\frac{dx}{dy}$ to get an expression in x	
	$\frac{dy}{dx} = -\frac{1}{\text{cosec}^2 y} = -\frac{1}{1 + \cot^2 y} = -\frac{1}{1 + x^2}$ cso	M1, A1* (5) **(11 marks)**

(i)(a) M1 Applies the product rule vu'+uv' to $x^3 \ln 2x$.

If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, with terms written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') then only accept answers of the form

$$Ax^2 \times \ln 2x + x^3 \times \frac{B}{x} \qquad \text{where } A, B \text{ are constants} \neq 0$$

A1 One term correct, either $3x^2 \times \ln 2x$ or $x^3 \times \frac{1}{2x} \times 2$

A1 Cao. $\frac{dy}{dx} = 3x^2 \times \ln 2x + x^3 \times \frac{1}{2x} \times 2$. The answer does not need to be simplified.

For reference the simplified answer is $\frac{dy}{dx} = 3x^2 \ln 2x + x^2 = x^2(3\ln 2x + 1)$

(i)(b) B1 Sight of $(x + \sin 2x)^2$

M1 For applying the chain rule to $(x + \sin 2x)^3$. If the rule is quoted it must be correct. If it is not quoted possible forms of evidence could be sight of $C(x + \sin 2x)^2 \times (1 \pm D\cos 2x)$ where C and D are non- zero constants.

Alternatively accept $u = x + \sin 2x$, u'= followed by $Cu^2 \times$ their u'

Do not accept $C(x + \sin 2x)^2 \times 2\cos 2x$ unless you have evidence that this is their u'

Allow 'invisible' brackets for this mark, ie. $C(x + \sin 2x)^2 \times 1 \pm D\cos 2x$

A1 Cao $\frac{dy}{dx} = 3(x + \sin 2x)^2 \times (1 + 2\cos 2x)$. There is no requirement to simplify this.

You may ignore subsequent working (isw) after a correct answer in part (i)(a) and (b)

(ii) M1 Writing the derivative of cot**y** as -cosec^2**y**. It must be in terms of y

A1 $\frac{dx}{dy} = -\text{cosec}^2 y$ or $1 = -\text{cosec}^2 y \frac{dy}{dx}$. Both lhs and rhs must be correct.

M1 Using $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$

M1 Using $\text{cosec}^2 y = 1 + \cot^2 y$ **and** $x = \cot y$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ just in terms of x.

A1 cso $\frac{dy}{dx} = -\frac{1}{1+x^2}$

Alternative to (a)(i) when ln(2x) is written lnx+ln2

M1 Writes $x^3 \ln 2x$ as $x^3 \ln 2 + x^3 \ln x$.
Achieves Ax^2 for differential of $x^3 \ln 2$ and applies the product rule vu'+uv' to $x^3 \ln x$.

A1 Either $3x^2 \times \ln 2 + 3x^2 \ln x$ or $x^3 \times \frac{1}{x}$

A1 A correct (un simplified) answer. Eg $3x^2 \times \ln 2 + 3x^2 \ln x + x^3 \times \frac{1}{x}$

Alternative to 5(ii) using quotient rule

M1 Writes $\cot y$ as $\frac{\cos y}{\sin y}$ and applies the quotient rule, a form of which appears in the formula book. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their $\frac{vu'-uv'}{v^2}$) only accept answers of the form $\frac{\sin y \times \pm \sin y - \cos y \times \pm \cos y}{(\sin y)^2}$

A1 Correct un simplified answer with both lhs and rhs correct.

$$\frac{dx}{dy} = \frac{\sin y \times -\sin y - \cos y \times \cos y}{(\sin y)^2} = \left\{-1 - \cot^2 y\right\}$$

M1 Using $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$

M1 Using $\sin^2 y + \cos^2 y = 1$, $\frac{1}{\sin^2 y} = \text{cosec}^2 y$ and $\text{cosec}^2 y = 1 + \cot^2 y$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in x

A1 cso $\frac{dy}{dx} = -\frac{1}{1+x^2}$

Alternative to 5(ii) using the chain rule, first two marks

M1 Writes $\cot y$ as $(\tan y)^{-1}$ and applies the chain rule (or quotient rule).

Accept answers of the form $-(\tan y)^{-2} \times \sec^2 y$

A1 Correct un simplified answer with both lhs and rhs correct.

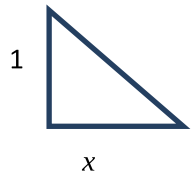
$$\frac{\mathrm{d}x}{\mathrm{d}y} = -(\tan y)^{-2} \times \sec^2 y$$

Alternative to 5(ii) using a triangle – last M1

M1 Uses triangle with $\tan y = \frac{1}{x}$ to find siny

and get $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ just in terms of x

$$x = \cot y \Rightarrow \tan y = \frac{1}{x}$$



$$\sin y = \frac{1}{\sqrt{1+x^2}}$$

Question Number	Scheme	Marks
6.	(i) $(\sin 22.5+\cos 22.5)^2=\sin^2 22.5+\cos^2 22.5+......$	M1
	$=\sin^2 22.5+\cos^2 22.5+2\sin 22.5\cos 22.5$	
	States or uses $\sin^2 22.5+\cos^2 22.5=1$	B1
	Uses $2\sin x\cos x=\sin 2x \Rightarrow 2\sin 22.5\cos 22.5=\sin 45$	M1
	$(\sin 22.5+\cos 22.5)^2=1+\sin 45$	A1
	$=1+\dfrac{\sqrt{2}}{2}$ or $1+\dfrac{1}{\sqrt{2}}$ cso	A1
		(5)
	(ii) (a) $\cos 2\theta+\sin\theta=1 \Rightarrow 1-2\sin^2\theta+\sin\theta=1$	M1
	$\sin\theta-2\sin^2\theta=0$	
	$2\sin^2\theta-\sin\theta=0$ or $k=2$	A1*
		(2)
	(b) $\sin\theta(2\sin\theta-1)=0$	M1
	$\sin\theta=0,\quad \sin\theta=\dfrac{1}{2}$	A1
	Any two of 0,30,150,180	B1
	All four answers 0,30,150,180	A1
		(4)
		(11 marks)

(i) M1 Attempts to expand $(\sin 22.5+\cos 22.5)^2$. Award if you see $\sin^2 22.5+\cos^2 22.5+......$

There must be > two terms. Condone missing brackets ie $\sin 22.5^2+\cos 22.5^2+......$

B1 Stating or using $\sin^2 22.5+\cos^2 22.5=1$. Accept $\sin 22.5^2+\cos 22.5^2=1$ as the intention is clear. Note that this may also come from using the double angle formula

$$\sin^2 22.5+\cos^2 22.5=\left(\frac{1-\cos 45}{2}\right)+\left(\frac{1+\cos 45}{2}\right)=1$$

M1 Uses $2\sin x\cos x=\sin 2x$ to write $2\sin 22.5\cos 22.5$ as sin 45 or sin(2×22.5)

A1 Reaching the intermediate answer $1+\sin 45$

A1 Cso $1+\dfrac{\sqrt{2}}{2}$ or $1+\dfrac{1}{\sqrt{2}}$. Be aware that both 1.707 and $\dfrac{2+\sqrt{2}}{2}$ can be found by using a calculator for 1+sin45. Neither can be accepted on their own without firstly seeing one of the two answers given above. **Each stage should be shown as required by the mark scheme.**

Note that if the candidates use $(\sin\theta+\cos\theta)^2$ they can pick up the first M and B marks, but no others until they use $\theta=22.5$. All other marks then become available.

(iia) M1 Substitutes $\cos 2\theta=1-2\sin^2\theta$ in $\cos 2\theta+\sin\theta=1$ to produce an equation in $\sin\theta$ only.

It is acceptable to use $\cos 2\theta=2\cos^2\theta-1$ *or* $\cos^2\theta-\sin^2\theta$ as long as the $\cos^2\theta$ is subsequently replaced by $1-\sin^2\theta$

A1* Obtains the correct simplified equation in $\sin\theta$.

$\sin\theta-2\sin^2\theta=0$ or $\sin\theta=2\sin^2\theta$ must be written in the form $2\sin^2\theta-\sin\theta=0$ as required by the question. Also accept $k=2$ as long as no incorrect working is seen.

(iib) M1 Factorises or divides by $\sin\theta$. For this mark $1='k'\sin\theta$ is acceptable. If they have a 3 TQ in $\sin\theta$ this can be scored for correct factorisation

A1 **Both** $\sin\theta=0$, and $\sin\theta=\dfrac{1}{2}$

B1 Any two answers from 0, 30, 150, 180.

A1 All four answers 0, 30, 150, 180 with no extra solutions inside the range. Ignore solutions outside the range.

Question Number	Scheme	Marks
6.alt 1	(i) $(\sin 22.5+\cos 22.5)^2=\sin^2 22.5+\cos^2 22.5+......$	M1
	$=\sin^2 22.5+\cos^2 22.5+2\sin 22.5\cos 22.5$	
	States or uses $\sin^2 22.5+\cos^2 22.5=1$	B1
	Uses $2\sin x\cos x=2\sqrt{\frac{1-\cos 2x}{2}}\sqrt{\frac{\cos 2x+1}{2}}\Rightarrow\sqrt{1-\cos 45}\sqrt{1+\cos 45}$	M1
	$=\sqrt{1-\cos^2 45}$	A1
	Hence $(\sin 22.5+\cos 22.5)^2=1+\frac{\sqrt{2}}{2}$ or $1+\frac{1}{\sqrt{2}}$	A1
		(5)

Question Number	Scheme	Marks
6.alt 2	(i) Uses Factor Formula $(\sin 22.5+\sin 67.5)^2=\left(2\sin 45\cos 22.5\right)^2$	M1,A1
	Reaching the stage $=2\cos^2 22.5$	B1
	Uses the double angle formula $=2\cos^2 22.5=1+\cos 45$	M1
	$=1+\frac{\sqrt{2}}{2}$ or $1+\frac{1}{\sqrt{2}}$	A1
		(5)

Question Number	Scheme	Marks
6.alt 3	(i) Uses Factor Formula $(\cos 67.5+\cos 22.5)^2=\left(2\cos 45\cos 22.5\right)^2$	M1,A1
	Reaching the stage $=2\cos^2 22.5$	B1
	Uses the double angle formula $=2\cos^2 22.5=1+\cos 45$	M1
	$=1+\frac{\sqrt{2}}{2}$ or $1+\frac{1}{\sqrt{2}}$	A1
		(5)

Question Number	Scheme	Marks
7.	(a) $\frac{2}{x+2}+\frac{4}{x^2+5}-\frac{18}{(x+2)(x^2+5)}=\frac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$	M1A1
	$=\frac{2x(x+2)}{(x+2)(x^2+5)}$	M1
	$=\frac{2x}{(x^2+5)}$	A1*
		(4)
	(b) $\mathrm{h}'(x)=\frac{(x^2+5)\times 2-2x\times 2x}{(x^2+5)^2}$	M1A1
	$\mathrm{h}'(x)=\frac{10-2x^2}{(x^2+5)^2}$ cso	A1
		(3)
	(c) Maximum occurs when $\mathrm{h}'(x)=0\Rightarrow 10-2x^2=0\Rightarrow x=..$	M1
	$\Rightarrow x=\sqrt{5}$	A1
	When $x=\sqrt{5}\Rightarrow \mathrm{h}(x)=\frac{\sqrt{5}}{5}$	M1,A1
	Range of h(x) is $0\le h(x)\le\frac{\sqrt{5}}{5}$	A1ft
		(5) **(12 marks)**

(a) M1 Combines the three fractions to form a single fraction with a common denominator.
Allow errors on the numerator but at least one must have been adapted.
Condone 'invisible' brackets for this mark.
Accept three separate fractions with the same denominator.
Amongst possible options allowed for this method are

$\frac{2x^2+5+4x+2-18}{(x+2)(x^2+5)}$ Eg 1 An example of 'invisible' brackets

$\frac{2(x^2+5)}{(x+2)(x^2+5)}+\frac{4}{(x+2)(x^2+5)}-\frac{18}{(x+2)(x^2+5)}$ Eg 2An example of an error (on middle term), 1st term has been adapted

$\frac{2(x^2+5)^2(x+2)+4(x+2)^2(x^2+5)-18(x^2+5)(x+2)}{(x+2)^2(x^2+5)^2}$ Eg 3 An example of a correct fraction with a different denominator

A1 Award for a correct un simplified fraction with the correct (lowest) common denominator.

$\frac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$

Accept if there are three separate fractions with the correct (lowest) common denominator.

Eg $\frac{2(x^2+5)}{(x+2)(x^2+5)}+\frac{4(x+2)}{(x+2)(x^2+5)}-\frac{18}{(x+2)(x^2+5)}$

Note, Example 3 would score M1A0 as it does not have the correct lowest common denominator

M1 There must be a single denominator. Terms must be collected on the numerator. A factor of (x+2) must be taken out of the numerator and then cancelled with one in the denominator. The cancelling may be assumed if the term 'disappears'

A1* Cso $\frac{2x}{(x^2+5)}$ This is a given solution and this mark should be withheld if there are any errors

(b) M1 Applies the quotient rule to $\frac{2x}{(x^2+5)}$, a form of which appears in the formula book.

If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their $\frac{vu'-uv'}{v^2}$) then only accept answers of the form

$$\frac{(x^2+5)\times A-2x\times Bx}{(x^2+5)^2} \quad \text{where } A,B>0$$

A1 Correct unsimplified answer $\mathrm{h}'(x)=\frac{(x^2+5)\times 2-2x\times 2x}{(x^2+5)^2}$

A1 $\mathrm{h}'(x)=\frac{10-2x^2}{(x^2+5)^2}$ The correct simplified answer. Accept $\frac{2(5-x^2)}{(x^2+5)^2}$ $\frac{-2(x^2-5)}{(x^2+5)^2}$, $\frac{10-2x^2}{(x^4+10x^2+25)}$

DO NOT ISW FOR PART (b). INCORRECT SIMPLIFICATION IS A0

(c) M1 Sets their h'(x)=0 and proceeds with a correct method to find x. There must have been an attempt to differentiate. Allow numerical errors but do not allow solutions from 'unsolvable' equations.

A1 Finds the correct x value of the maximum point $x=\sqrt{5}$.
Ignore the solution $x=-\sqrt{5}$ but withhold this mark if other positive values found.

M1 Substitutes their answer into their h'(x)=0 in h(x) to determine the maximum value

A1 Cso-the maximum value of h(x) = $\frac{\sqrt{5}}{5}$. Accept equivalents such as $\frac{2\sqrt{5}}{10}$ **but not** 0.447

A1ft Range of h(x) is $0\le \mathrm{h}(x)\le \frac{\sqrt{5}}{5}$. Follow through on their maximum value if the M's have been scored. Allow $0\le y\le \frac{\sqrt{5}}{5}$, $0\le Range\le \frac{\sqrt{5}}{5}$, $\left[0,\frac{\sqrt{5}}{5}\right]$ but not $0\le x\le \frac{\sqrt{5}}{5}$, $\left(0,\frac{\sqrt{5}}{5}\right)$

If a candidate attempts to work out $h^{-1}(x)$ in (b) and does all that is required for (b) in (c), then allow. Do not allow $h^{-1}(x)$ to be used for h'(x) in part (c). For this question (b) and (c) can be scored together.

Alternative to (b) using the product rule

M1 Sets $\mathrm{h}(x)=2x(x^2+5)^{-1}$ and applies the product rule vu'+uv' with terms being 2x and $(x^2+5)^{-1}$
If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') then only accept answers of the form

$$(x^2+5)^{-1}\times A+2x\times \pm Bx(x^2+5)^{-2}$$

A1 Correct un simplified answer $(x^2+5)^{-1}\times 2+2x\times -2x(x^2+5)^{-2}$

A1 The question asks for h'(x) to be put in its simplest form. Hence in this method the terms need to be combined to form a single correct expression.

For a correct simplified answer accept

$$h'(x) = \frac{10-2x^2}{(x^2+5)^2} = \frac{2(5-x^2)}{(x^2+5)^2} = \frac{-2(x^2-5)}{(x^2+5)^2} = (10-2x^2)(x^2+5)^{-2}$$

Question Number	Scheme	Marks
8.	(a) (£) 19500	B1 (1)
	(b) $9500 = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$ $17e^{-0.25t} + 2e^{-0.5t} = 9$ $(\times e^{0.5t}) \Rightarrow 17e^{0.25t} + 2 = 9e^{0.5t}$	
	$0 = 9e^{0.5t} - 17e^{0.25t} - 2$	M1
	$0 = (9e^{0.25t} + 1)(e^{0.25t} - 2)$	M1
	$e^{0.25t} = 2$	A1
	$t = 4\ln(2)$ *oe*	A1 (4)
	(c) $(\frac{dV}{dt}) = -4250e^{-0.25t} - 1000e^{-0.5t}$	M1A1
	When t=8 Decrease = 593 (£/year)	M1A1 (4)
		(9 marks)

(a) B1 19500. The £ sign is not important for this mark

(b) M1 Substitute V=9500, collect terms and set on 1 side of an equation =0. Indices must be correct
Accept $17000e^{-0.25t} + 2000e^{-0.5t} - 9000 = 0$ and $17000x + 2000x^2 - 9000 = 0$ where $x = e^{-0.25t}$

M1 Factorise the quadratic in $e^{0.25t}$ or $e^{-0.25t}$
For your information the factorised quadratic in $e^{-0.25t}$ is $(2e^{-0.25t} - 1)(e^{-0.25t} + 9) = 0$
Alternatively let $'x' = e^{0.25t}$ or otherwise and factorise a quadratic equation in x

A1 Correct solution of the quadratic. Either $e^{0.25t} = 2$ or $e^{-0.25t} = \frac{1}{2}$ oe.

A1 Correct exact value of t. Accept variations of $4\ln(2)$, such as $\ln(16)$, $\frac{\ln(\frac{1}{2})}{-0.25}, \frac{\ln(2)}{0.25}, -4\ln(\frac{1}{2})$

.(c) M1 Differentiates $V = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$ by the chain rule.
Accept answers of the form $(\frac{dV}{dt}) = \pm Ae^{-0.25t} \pm Be^{-0.5t}$ A, B are constants $\neq 0$

A1 Correct derivative $(\frac{dV}{dt}) = -4250e^{-0.25t} - 1000e^{-0.5t}$.
There is no need for it to be simplified so accept
$$(\frac{dV}{dt}) = 17000 \times -0.25e^{-0.25t} + 2000 \times -0.5e^{-0.5t} \quad oe$$

M1 Substitute t=8 into their $\frac{dV}{dt}$.
This is not dependent upon the first M1 but there must have been some attempt to differentiate.
Do not accept t=8 in V

A1 ±593. Ignore the sign and the units. If the candidate then divides by 8, withhold this mark. **This would not be isw. Be aware that sub t=8 into V first and then differentiating can achieve 593. This is M0A0M0A0.**

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