

Wednesday 23 January 2013 – Morning

A2 GCE MATHEMATICS

4723/01 Core Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4723/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hours 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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1 For each of the following curves, find the gradient at the point with x -coordinate 2.

(i) $y = \frac{3x}{2x+1}$ [3]

(ii) $y = \sqrt{4x^2+9}$ [3]

2 The acute angle A is such that $\tan A = 2$.

(i) Find the exact value of $\operatorname{cosec} A$. [2]

(ii) The angle B is such that $\tan(A+B) = 3$. Using an appropriate identity, find the exact value of $\tan B$. [3]

3 (a) Given that $|t| = 3$, find the possible values of $|2t - 1|$. [3]

(b) Solve the inequality $|x - \sqrt{2}| > |x + 3\sqrt{2}|$. [4]

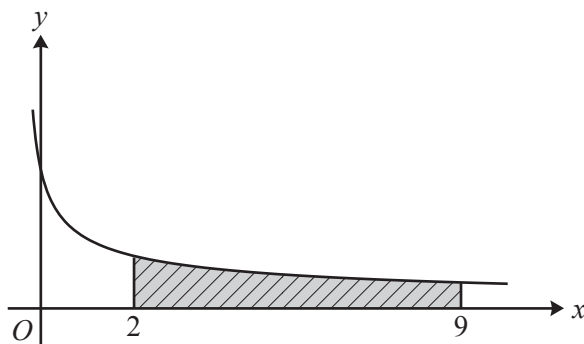
4 The mass, m grams, of a substance is increasing exponentially so that the mass at time t hours is given by

$$m = 250e^{0.021t}.$$

(i) Find the time taken for the mass to increase to twice its initial value, and deduce the time taken for the mass to increase to 8 times its initial value. [3]

(ii) Find the rate at which the mass is increasing at the instant when the mass is 400 grams. [3]

5



The diagram shows the curve $y = \frac{6}{\sqrt{3x+1}}$. The shaded region is bounded by the curve and the lines $x = 2$, $x = 9$ and $y = 0$.

(i) Show that the area of the shaded region is $4\sqrt{7}$ square units. [4]

(ii) The shaded region is rotated completely about the x -axis. Show that the volume of the solid produced can be written in the form $k\ln 2$, where the exact value of the constant k is to be determined. [5]

- 6 (i) By sketching the curves $y = \ln x$ and $y = 8 - 2x^2$ on a single diagram, show that the equation

$$\ln x = 8 - 2x^2$$

has exactly one real root.

[3]

- (ii) Explain how your diagram shows that the root is between 1 and 2.

[1]

- (iii) Use the iterative formula

$$x_{n+1} = \sqrt{4 - \frac{1}{2} \ln x_n},$$

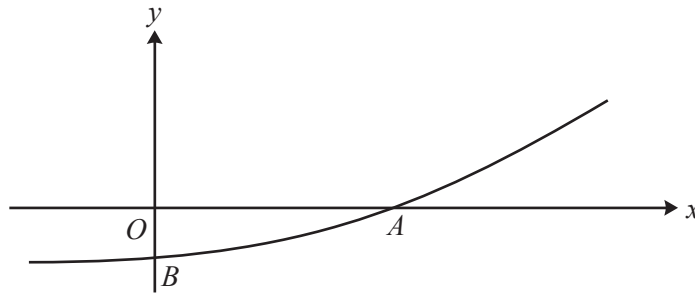
with a suitable starting value, to find the root. Show all your working and give the root correct to 3 decimal places.

[4]

- (iv) The curves $y = \ln x$ and $y = 8 - 2x^2$ are each translated by 2 units in the positive x -direction and then stretched by scale factor 4 in the y -direction. Find the coordinates of the point where the new curves intersect, giving each coordinate correct to 2 decimal places.

[3]

7



The diagram shows the curve with equation

$$x = (y + 4) \ln (2y + 3).$$

The curve crosses the x -axis at A and the y -axis at B .

- (i) Find an expression for $\frac{dx}{dy}$ in terms of y .
- (ii) Find the gradient of the curve at each of the points A and B , giving each answer correct to 2 decimal places.

[3]

[5]

- 8 The functions f and g are defined for all real values of x by

$$f(x) = x^2 + 4ax + a^2 \quad \text{and} \quad g(x) = 4x - 2a,$$

where a is a positive constant.

- (i) Find the range of f in terms of a .
- (ii) Given that $fg(3) = 69$, find the value of a and hence find the value of x such that $g^{-1}(x) = x$.

[4]

[6]

9 (i) Prove that

$$\cos^2(\theta + 45^\circ) - \frac{1}{2}(\cos 2\theta - \sin 2\theta) \equiv \sin^2 \theta. \quad [4]$$

(ii) Hence solve the equation

$$6 \cos^2\left(\frac{1}{2}\theta + 45^\circ\right) - 3(\cos \theta - \sin \theta) = 2$$

$$\text{for } -90^\circ < \theta < 90^\circ. \quad [3]$$

(iii) It is given that there are two values of θ , where $-90^\circ < \theta < 90^\circ$, satisfying the equation

$$6 \cos^2\left(\frac{1}{3}\theta + 45^\circ\right) - 3\left(\cos \frac{2}{3}\theta - \sin \frac{2}{3}\theta\right) = k,$$

$$\text{where } k \text{ is a constant. Find the set of possible values of } k. \quad [3]$$

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