M1 IAL JAN 16 (Kprime 2)

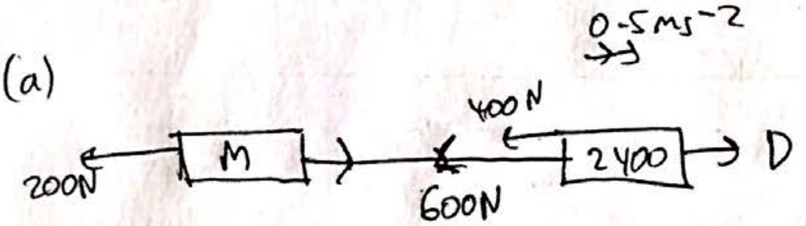
1. A truck of mass 2400 kg is pulling a trailer of mass M kg along a straight horizontal road. The tow bar, connecting the truck to the trailer, is horizontal and parallel to the direction of motion. The tow bar is modelled as being light and inextensible. The resistance forces acting on the truck and the trailer are constant and of magnitude 400 N and 200 N respectively. The acceleration of the truck is 0.5 m s^{-2} and the tension in the tow bar is 600 N.

(a) Find the magnitude of the driving force of the truck.
(3)

(b) Find the value of M.
(3)

(c) Explain how you have used the fact that the tow bar is inextensible in your calculations.
(1)

(a)



Consider truck: Let D = driving force

$\rightarrow$: $F = ma$

~~R+60~~ $D - 600 - 400 = 2400 \times 0.5$

$\Rightarrow D = 2200 \text{ N}$

(b) Consider Trailer:

$\rightarrow F = ma$

$600 - 200 = M \times 0.5$

$\Rightarrow M = 800 \text{ kg}$

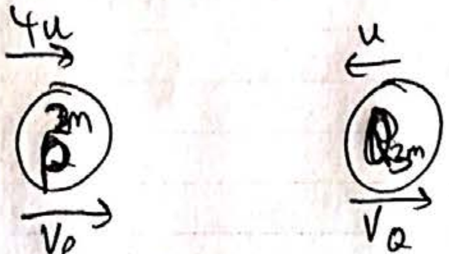
(C) The acceleration remains the same for both vehicles.

Two particles P and Q are moving in opposite directions along the same horizontal straight line. Particle P is moving due east and particle Q is moving due west. Particle P has mass $2m$ and particle Q has mass $3m$. The particles collide directly. Immediately before the collision, the speed of P is $4u$ and the speed of Q is u. The magnitude of the impulse in the collision is $\frac{33}{5}mu$.

(a) Find the speed and direction of motion of P immediately after the collision. **(4)**

(b) Find the speed and direction of motion of Q immediately after the collision. **(4)**

2.)



(a). $\overset{+}{\leftarrow}$ $I = m(v-u)$

$\therefore$ Consider I exerted on P:

$\frac{33}{5}mu = 2m(-v_P + 4u)$

$\frac{33}{10}u = 4u - v_P \Rightarrow \Rightarrow v_P = \frac{7}{10}u\ ms^{-1}$ due east

~~$\overset{+}{\leftarrow}$ $I = m(v-u)$~~

~~Consider P:~~

~~$\frac{33}{5}mu = 2m(-v_P + 4u)$~~

~~(b) Consider Q:~~

~~$\frac{33}{5}mu = 3m(-v_Q - u)$~~

~~(b) $\overset{+}{\leftarrow}$ Consider Q~~

~~$\frac{33}{5}mu = 3m($~~

(b) Consider Q:

$\overset{+}{\rightarrow}$ $\frac{33}{5}mu = 3m(v_Q + u)$

$\therefore \frac{11}{5}u = v_Q + u$

$\Rightarrow v_Q = \frac{6}{5}u\ ms^{-1}$

due east

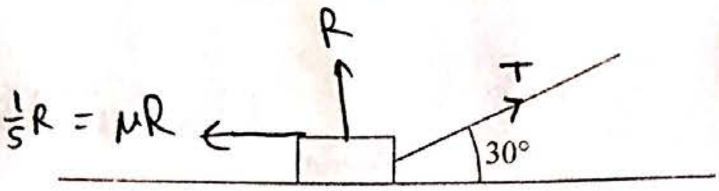
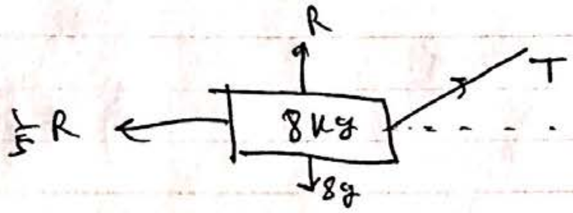


Figure 1

A boy is pulling a sledge of mass 8 kg in a straight line at a constant speed across rough horizontal ground by means of a rope. The rope is inclined at 30° to the ground, as shown in Figure 1. The coefficient of friction between the sledge and the ground is $\frac{1}{5}$. By modelling the sledge as a particle and the rope as a light inextensible string, find the tension in the rope.

(8)

3.



$\uparrow$: $R + T\sin 30 = 8g$

$\Rightarrow R + \frac{1}{2}T = 8g$

$\rightarrow$: $T\cos 30° = \frac{1}{5}R$

$\therefore \frac{\sqrt{3}}{2}T = \frac{1}{5}R \Rightarrow \cancel{T = \frac{2\sqrt{3}}{15}R}$

$R = \frac{5\sqrt{3}}{2}T$ N

$\uparrow$: $\frac{5\sqrt{3}}{2}T + \frac{1}{2}T = 8g$

$\therefore T\left(\frac{1+5\sqrt{3}}{2}\right) = 8g$

$\therefore T = \frac{40\sqrt{3} - 8}{37}g \approx 16$ N (2 sf)

A small stone is projected vertically upwards from the point O and moves freely under gravity. The point A is 3.6 m vertically above O. When the stone first reaches A, the stone is moving upwards with speed 11.2 m s^{-1}. The stone is modelled as a particle.

(a) Find the maximum height above O reached by the stone.

(4)

(b) Find the total time between the instant when the stone was projected from O and the instant when it returns to O.

(5)

(c) Sketch a velocity-time graph to represent the motion of the stone from the instant when it passes through A moving upwards to the instant when it returns to O. Show, on the axes, the coordinates of the points where your graph meets the axes.

(4)

4(a)

h ○ $0\,ms^{-1}$

A ○ $\uparrow 11.2\,ms^{-1}$ $\quad \downarrow 9.8\,ms^{-2}$

3.6m

O ○ $\uparrow u$

Let h = height reached above A.

Consider stone at A:

$s = h$

$u = 11.2$

$v = 0$

$a = -9.8$

$t =$

$v^2 = u^2 + 2as$

$\therefore 0 = 11.2^2 - 19.6h$

$\therefore h = 6.4\text{m}$

$\therefore$ Height above O $= 3.6 + 6.4 = 10$ m

(b) Consider motion from O to max height:

$s = 10$

$u =$

$v = 0$

$a = -9.8$

$t = t_1$

$v^2 = u^2 + 2as$

$\therefore 0 = u^2 - 19.6(10)$

$\Rightarrow u = 14\,ms^{-1}$

Consider motion from O back to O:

$s = 0$

$u = 14$

$v =$

$a = -9.8$

$t = T$

$s = ut + \frac{1}{2}at^2$

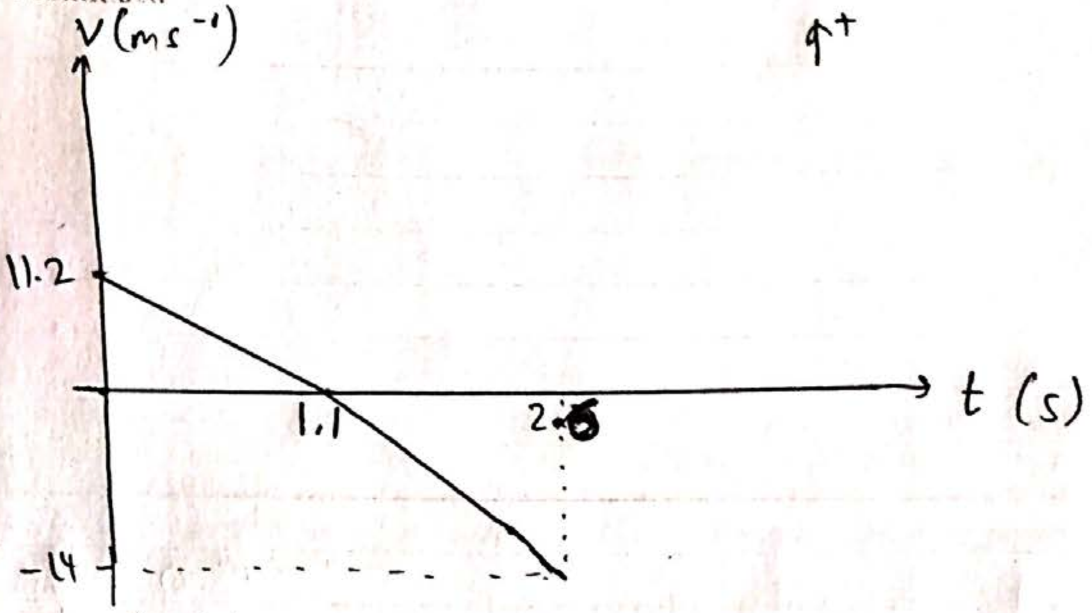
$\therefore 0 = 14T - 4.9T^2$

$\therefore T(14 - 4.9T) = 0$

$T = 0$ at start

$\therefore 14 - 4.9T = 0 \Rightarrow T = 2.9$ seconds

(c)



A to max height:

$V = u + at$

$0 = 11.2 - 9.8t_1' \Rightarrow t_1' = 1.1s$

~~O to max height:~~

~~$v = u + at \Rightarrow 0 = 14 - 9.8t_1 \Rightarrow t_1 = 1.4s$ (2sf)~~

A to O: $\downarrow +$

$s = 10$

$u = 0$

$V = 14$

$a = g = 9.8$

$t = t$

$V = u + at$

$14 = 9.8t$

$t = \frac{10}{7}$

$\frac{10}{7} + \frac{8}{7} \approx 2.6$

5

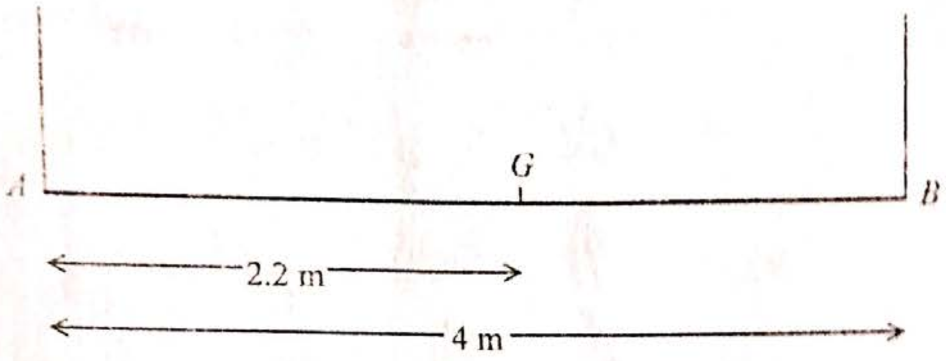


Figure 2

A non-uniform rod AB has length 4 m and weight 120 N. The centre of mass of the rod is at the point G where $AG = 2.2$ m. The rod is suspended in a horizontal position by two vertical light inextensible strings, one at each end, as shown in Figure 2. A particle of weight 40 N is placed on the rod at the point P, where $AP = x$ metres. The rod remains horizontal and in equilibrium.

(a) Find, in terms of x,

(i) the tension in the string at A,

(ii) the tension in the string at B.

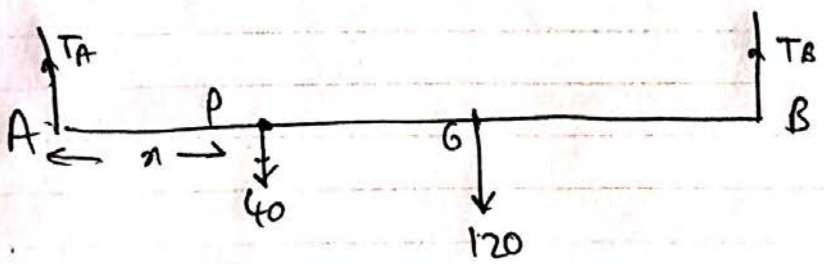
(6)

Either string will break if the tension in it exceeds 84 N.

(b) Find the range of possible values of x.

(4)

5



(a) $\uparrow$: $T_A + T_B = 120 + 40 = 160\text{N}$

(i) $M(B)$: $4T_A = 40(4-x) + 120(1.8)$

$\therefore T_A = 40 - 10x + 54$

$\therefore T_A = (94 - 10x)$ N

(ii) $\uparrow$: $T_A + T_B = 160$ N

$\therefore T_B = 160 - (94 - 10x)$

$\therefore T_B = (66 + 10x)$ N

(b) T_A breaks if $T_A > 84$

$\therefore T_A \leq 84$ for it to not break

$\therefore 94 - 10x \leq 84 \Rightarrow x \geq 1$

$T_B \leq 84$ to not break

$\therefore 66 + 10x \leq 84 \Rightarrow x \leq 1.8$

$\therefore 1 \leq x \leq 1.8$

6 [In this question $\mathbf{i}$ and $\mathbf{j}$ are horizontal unit vectors due east and due north respectively and position vectors are given relative to a fixed origin.]

At 2 pm, the position vector of ship P is $(5\mathbf{i} - 3\mathbf{j})$ km and the position vector of ship Q is $(7\mathbf{i} + 5\mathbf{j})$ km.

(a) Find the distance between P and Q at 2 pm. **(3)**

Ship P is moving with constant velocity $(2\mathbf{i} + 5\mathbf{j})\text{ km h}^{-1}$ and ship Q is moving with constant velocity $(-3\mathbf{i} - 15\mathbf{j})\text{ km h}^{-1}$.

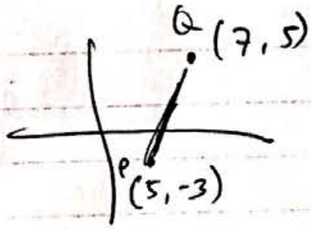
(b) Find the position vector of P at time t hours after 2 pm. **(2)**

(c) Find the position vector of Q at time t hours after 2 pm. **(1)**

(d) Show that Q will meet P and find the time at which they meet. **(5)**

(e) Find the position vector of the point at which they meet. **(2)**

6(a).



$$PQ = \sqrt{(7-5)^2 + (5+3)^2} = 2\sqrt{17}$$

$\therefore$ distance $= \underline{2\sqrt{17}\text{ km}}$

(b) $\underset{\sim}{r}^t = r_0 + \underset{\sim}{v}t$

$$\therefore r = \begin{pmatrix}5\\-3\end{pmatrix} + t\begin{pmatrix}2\\5\end{pmatrix} = \begin{pmatrix}5+2t\\5t-3\end{pmatrix}$$

$\underset{\sim}{p.v}_P = \underline{(5+2t)\underset{\sim}{i} + (5t-3)\underset{\sim}{j}}$

(c) $\underset{\sim}{r}' = \begin{pmatrix}7\\5\end{pmatrix} + t\begin{pmatrix}-3\\-15\end{pmatrix} = \underline{(7-3t)\underset{\sim}{i} + (5-15t)\underset{\sim}{j}}$

(d) Q and P meet if their position vectors are the ~~same~~ same at t hrs after 2pm

$$\Rightarrow \begin{pmatrix} 5+2t \\ 5t-3 \end{pmatrix} = \begin{pmatrix} 7-3t \\ 5-15t \end{pmatrix}$$

$5+2t = 7-3t \Rightarrow 5t = 2$
$\Rightarrow t = \frac{2}{5}$

$t = \frac{2}{5} \Rightarrow 5t-3 = 5-15t = -1$

$\therefore$ P and Q ideed mett $\frac{2}{5}$hrs after 2pm

~~Time is~~ 0.4hrs = 24 mins

$\therefore$ Time is 2:24 pm (14:24)

(e) $\begin{pmatrix} 5+2t \\ 5t-3 \end{pmatrix} = \begin{pmatrix} 29/5 \\ -1 \end{pmatrix} = \begin{pmatrix} 7-3t \\ 5-15t \end{pmatrix}$ for $t = \frac{2}{5}$

$\therefore$ meet at $5.8\underset{\sim}{i} - \underset{\sim}{j}$

7.

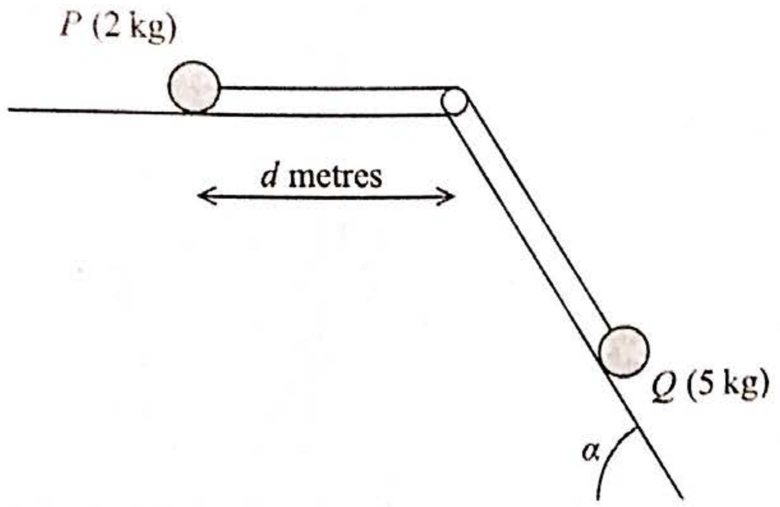


Figure 3

A particle P of mass 2 kg is attached to one end of a light inextensible string. A particle Q of mass 5 kg is attached to the other end of the string. The string passes over a small smooth light pulley. The pulley is fixed at a point on the intersection of a rough horizontal table and a fixed smooth inclined plane. The string lies along the table and also lies in a vertical plane which contains a line of greatest slope of the inclined plane. This plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$. Particle P is at rest on the table, a distance d metres from the pulley. Particle Q is on the inclined plane with the string taut, as shown in Figure 3. The coefficient of friction between P and the table is $\frac{1}{4}$.

The system is released from rest and P slides along the table towards the pulley.

Assuming that P has not reached the pulley and that Q remains on the inclined plane,

(a) write down an equation of motion for P,

(2)

(b) write down an equation of motion for Q,

(2)

(c) (i) find the acceleration of P,

(ii) find the tension in the string.

(5)

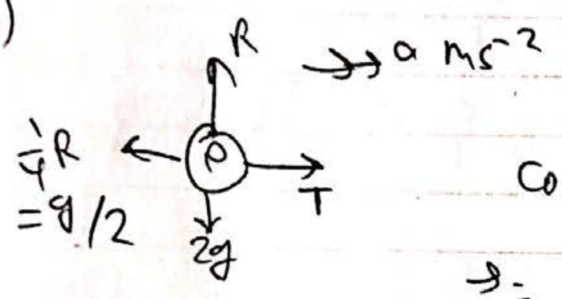
When P has moved a distance 0.5 m from its initial position, the string breaks. Given that P comes to rest just as it reaches the pulley,

(d) find the value of d.

(7)

Question 7 continued

(a)



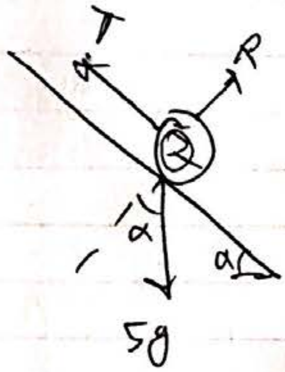
$\uparrow$: $R = 2g$ N

Consider P:

$F = ma$

$\rightarrow$: $\Rightarrow T - \frac{g}{2} = 2a$

(b)



$\tan\alpha = \frac{3}{4} \Rightarrow \sin\alpha = \frac{3}{5}$

$\searrow$: $f = ma$

$5g\sin\alpha - T = 5a$

$\Rightarrow 3g - T = 5a$

(c)(ii) $T - \frac{g}{2} = 2a \therefore \frac{5}{2}(T - g/2) = 5a$

$\therefore \frac{5}{2}T - \frac{5g}{4} = 3g - T$

$\Rightarrow \frac{7}{2}T = \frac{17}{4}g \Rightarrow T = \frac{17g}{14}$ N

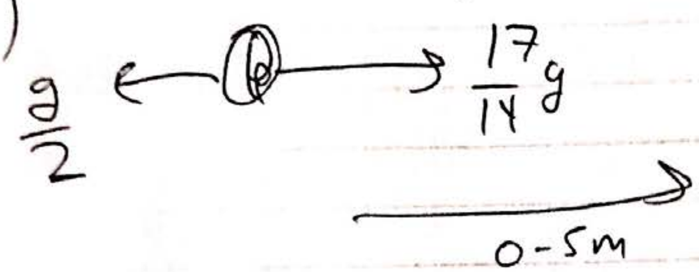
(i) $T - \frac{g}{2} = 2a \Rightarrow \frac{17}{14}g - \frac{g}{2} = 2a$

$\Rightarrow a = \frac{5g}{14}\ ms^{-2}$

Question 7 continued

(d)

$\rightarrow \frac{5g}{14}\ ms^{-2}$



$\rightarrow +$

$s = 0\text{-}5$

$u = 0$

$v =$

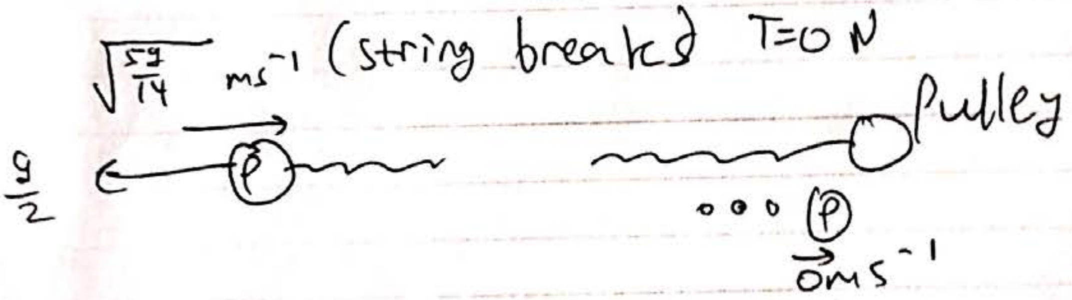
$a = \frac{5g}{14}$

$t =$

$v^2 = u^2 + 2as$

$\therefore v^2 = 2 \times \frac{5g}{14} \times 0\text{-}5$

$\therefore v = \underline{\underline{\sqrt{\frac{5g}{14}}}}\ ms^{-1}$ when string breaks



$\rightarrow +$

$s = d - 0\text{-}5$

$u = \sqrt{\frac{5g}{14}}$

$v = 0$

$a = -g/4$

$t =$

$\rightarrow F = ma$

$\therefore -\frac{g}{2} = 2a$

$\Rightarrow a = -\frac{g}{4}\ ms^{-2}$

$\therefore v^2 = u^2 + 2as$

$\therefore 0 = \frac{5g}{14} - \frac{g}{2}(d - 0\text{-}5)$

$\therefore d - 0\text{-}5 = \frac{5}{7}$

$\Rightarrow d = \frac{17}{14} \approx \underline{\underline{1.2\ m\ (2sf)}}$

Q7

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

END