07 May 2011
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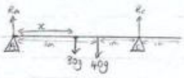
M1 June 03 Model Solutions

① 

$R_A = R_C = R$.

a) ~~Moment~~ $R + R = 80g + 40g \Rightarrow 2R = 120g \Rightarrow \underline{R = 60g\,N}$

b) $M(A)$: $R \times 3 = 80gx + 40g \times 2$

$180g = 80g + 80gx$

$\frac{100}{80} = x \Rightarrow \underline{x = 1.25\,m}$

② 

a) $I = mv - mu$

$= 0.12 \times 3 - 0.12 \times 0$

$= \underline{0.36\ Ns}$

b) $m_1u_1 + m_2u_2 = m_1v_1 + m_2u_2$

$0.12 \times 3 + 0.08 \times 0 = 0.12 \times 1.2 + 0.08v$

$\Rightarrow 0.36 = 0.144 + 0.08v$

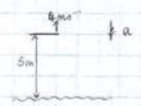
$\Rightarrow \underline{v = 2.7\,ms^{-1}}$

c) Take $\leftarrow\!\rightarrow$ to be positive:

$I = mv - mu$

$= 1.2 \times 0.12 - 3 \times 0.12$

$= \underline{-0.216\ Ns}$ Impulse is $0.216\ Ns \leftarrow$.

③ 

a) $u = -4$

$v = v$

$a = +9.8$

$s = +5$

$t = \backslash$

$v^2 = u^2 + 2as$

$= 16 + 2 \times 9.8 \times 5$

$= 114$

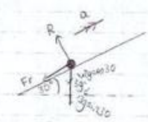
$\Rightarrow v = \underline{10.7\,ms^{-1}}$

b) $s = ut + \frac{1}{2}at^2 \Rightarrow 5 = -4t + \frac{1}{2} \times 9.8t^2$

$\Rightarrow 4.9t^2 - 4t - 5 = 0$

$\Rightarrow t = \frac{4 \pm \sqrt{4^2 - 4 \times 4.9 \times -5}}{9.8} = \frac{4 \pm \sqrt{114}}{9.8} = \underline{1.50\,s}$

c) We have ignored air resistance, the size of the diver, the horizontal component of velocity, spinning in the air, hitting the board on the way down.

③ ⑥ 

a) $\mu = 0.4$

$Fr = 0.4R$

$R = 3g\cos 30 \therefore Fr = 1.2g\cos 30 = \underline{10.2\,N}$

b) $u = 6$

$v = 0$

$a = a$

$s = s$

$t = \backslash$

To find a: $RF = ma$

$-Fr - 3g\sin 30 = 3a$

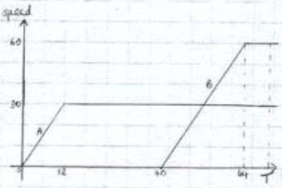
$\Rightarrow \frac{-1.2g\cos 30 - 3g\sin 30}{3} = a$

$\Rightarrow a \approx -8.29\,ms^{-2}$.

$v^2 = u^2 + 2as \Rightarrow 0 = 36 + 2 \times -8.29s$

$\Rightarrow 16.6s = 36$

$\Rightarrow \underline{s = 2.17\,m}$

⑦ a) 

b) At T, both travelled same distance, $\therefore$ area under graphs is equal.

Ⓐ $= \frac{1}{2} \times 30 \times (T + (T - 12))$

$= 15(T + T - 12)$

$= 30T - 180$

Ⓑ $= \frac{1}{2} \times 60 \times ((T - 40) + (T - 64))$

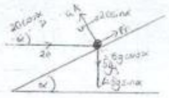
$= 30(2T - 104)$

$= 60T - 3120$

Since Ⓐ = Ⓑ, $30T - 180 = 60T - 3120$

$2940 = 30T$

$\Rightarrow \underline{T = 98\,s}$

④ 

$\alpha = \tan^{-1}(\frac{3}{4}) \Rightarrow \cos\alpha = \frac{4}{5},\ \sin\alpha = \frac{3}{5}$

$Fr = \mu R$

$R = 5g\cos\alpha + 20\sin\alpha = 4g + 12$

$Fr = \mu R = \mu(4g + 12)$

$Fr + 20\cos\alpha = 5g\sin\alpha$

$\mu(4g + 12) + 20\cos\alpha = 5g\sin\alpha$

$\mu = \frac{5g\sin\alpha - 20\cos\alpha}{4g + 12} = \frac{3g - 16}{4g + 12} = \frac{67}{256} \approx \underline{0.262}$

⑤

a) $\underset{\sim}{a} = 2\underset{\sim}{i} - 3\underset{\sim}{j}$ $\quad t = 0,\ \underset{\sim}{v} = -2\underset{\sim}{i} + 7\underset{\sim}{j}$

$\underset{\sim}{u} = -2\underset{\sim}{i} + 7\underset{\sim}{j}$

$\underset{\sim}{v} = \underset{\sim}{v}$

$\underset{\sim}{a} = 2\underset{\sim}{i} - 3\underset{\sim}{j}$

$\underset{\sim}{s} = \backslash$

$t = t$.

$\underset{\sim}{v} = \underset{\sim}{u} + \underset{\sim}{a}t$

$= -2\underset{\sim}{i} + 7\underset{\sim}{j} + (2\underset{\sim}{i} - 3\underset{\sim}{j})t$

$= (-2 + 2t)\underset{\sim}{i} + (7 - 3t)\underset{\sim}{j}$

Parallel to $\underset{\sim}{i}$ means $\underset{\sim}{j}$ component $= 0$:

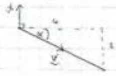
$7 - 3t = 0$

$\Rightarrow t = \underline{\frac{7}{3}\,s}$

b) $t = 3$, $\underset{\sim}{v} = (-2 + 2 \times 3)\underset{\sim}{i} + (7 - 3 \times 3)\underset{\sim}{j}$

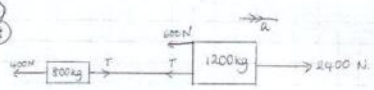
$= 4\underset{\sim}{i} - 2\underset{\sim}{j}$

Speed $= |\underset{\sim}{v}| = \sqrt{4^2 + 2^2} = \sqrt{20} \approx \underline{4.47\,ms^{-1}}$.

c) $t = 3$: 

$\alpha = \tan^{-1}(\frac{2}{4})$

Angle $= 90 + \alpha = \underline{116.6°}$

④ ⑧ 

a) $RF = ma$

Truck: $2400 - 600 - T = 1200a$

$1800 - T = 1200a$

$\Rightarrow T = 1800 - 1200a$ — ①

Car: $T - 400 = 800a$ — ②

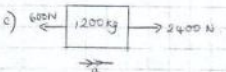
Sub. ① into ②: $(1800 - 1200a) - 400 = 800a$

$1400 = 2000a$

$\underline{a = 0.7\,ms^{-2}}$

b) $T = 1800 - 1200a$

$= 1800 - 1200 \times 0.7 = \underline{960\,N}$

c) 

New acceleration: $RF = ma$

$2400 - 600 = 1200a$

$1800 = 1200a$

$\Rightarrow a = 1.5\,ms^{-2}$

To reach $28\,ms^{-1}$: $u = 20$, $v = 28$, $a = 1.5$, $s = \backslash$, $t = t$

$v = u + at \Rightarrow t = \frac{v - u}{a}$

$= \frac{28 - 20}{1.5}$

$= 5.33\,s$.

If rope had not broken, then time taken to reach $28\,ms^{-1}$:

$u = 20$

$v = 28$

$a = 0.7$

$s = \backslash$

$t = t$

$v = u + at \Rightarrow t = \frac{v - u}{a}$

$= \frac{28 - 20}{0.7}$

$= 11.43\,s$.

Difference between times $= 11.43 - 5.33$

$= 6.1$

$\approx \underline{6\,s}$ $\quad$ QED