

C3 June 2005

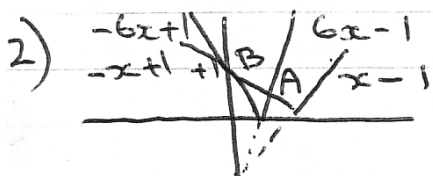
1)  $\rightarrow$  trans  $-3$ ,  $x$  axis reflect  
 $\uparrow$  trans  $+10$

$f(x) \leq 10$



5)  $3\sin\theta + 2\cos\theta = R(\sin\theta\cos\alpha + \cos\theta\sin\alpha)$   
 $R\cos\alpha = 3$   $\tan\alpha = \frac{2}{3}$   
 $R\sin\alpha = 2$   $\alpha = 33.7^\circ$   
 $R = \sqrt{9+4} = \sqrt{13}$

ii)  $f(-1) = 6$   $ff(-1) = -71$



ii)  $\sqrt{13}\sin(\theta + 33.7^\circ) = \frac{7}{2}$

$\theta + 33.7 \leq 76^\circ$  or  $104^\circ$   
 $\theta \leq 42.4$  &  $70.2$

A  $6x-1 = -x+1$   $x = \frac{2}{7}$

B  $-6x+1 = -x+1$   $x = 0$

3) i)  $25 = 180e^{-0.017t}$   
 $\ln\left(\frac{25}{180}\right) = -0.017t$   
 $t = 116$  yrs

6) a)  $y = x \ln x$   
 $y' = x \cdot \frac{1}{x} + \ln x = 1 + \ln x$   
 TP  $1 + \ln x = 0$   
 $\ln x = -1$   
 $x = e^{-1}$

ii)  $\frac{dm}{dt} = -0.017 \cdot 180e^{-0.017t}$

$= -1.2$  gms/yrs if  $t = 55$

v)  $y = \frac{4x+c}{4x-c}$

$y' = \frac{(4x-c)4 - (4x+c)4}{(4x-c)^2}$

f) a)  $V = \int \pi y^2 dx = \int \pi \frac{4}{x} dx = -\frac{8c}{(4x-c)^2} = 0$  for TP

$= 4\pi \ln x$

but  $c \neq 0$  so no T.P.'s

$V_5 = 4\pi \ln 5$   $V_1 = 0$

Vol =  $4\pi \ln 5$

7)  $\cos 2x = 2\cos^2 x - 1$

6)  $I = \frac{1}{3} \left( \overset{y_0}{\sqrt{2}} + 4\overset{y_1}{\sqrt{5}} + 2\overset{y_2}{\sqrt{10}} + 4\overset{y_3}{\sqrt{17}} + \overset{y_4}{\sqrt{26}} \right)$   
 $= 12.758$

LHS =  $\frac{4(2\cos^2 x - 1)}{1 + 2\cos^2 x - 1}$   
 $= \frac{8\cos^2 x - 4}{2\cos^2 x}$

$= 4 - 2\sec^2 x$  power



$$\begin{aligned} \text{iii)} \quad 4 - 2\sec^2 x &= 3\tan x - 7 \\ 4 - 2(1 + \tan^2 x) &= 3\tan x - 7 \\ 2\tan^2 x + 3\tan x - 9 &= 0 \\ \tan x &= -3 \text{ or } \frac{3}{2} \end{aligned}$$

$$x = 0.983, 4.12$$

$$1.89, 5.03 \text{ rads}$$

$$x + m + 7 \sqrt{-4} = f(x)$$

$$f^{-1}(x) = \frac{(x+4)^2 - 7}{m}$$

iii)  $f(x)$  &  $f^{-1}(x)$  are reflections in line  $y=x$   
 so if  $f(x)$  &  $f^{-1}(x)$  do not meet then they do not cross  $y=x$

3) intersection  $e^{\frac{1}{5}x} = (3x+8)^{\frac{1}{3}}$

$$e^{\frac{1}{5}x} - (3x+8)^{\frac{1}{3}} = 0$$

$$f(5.2) = +ve \text{ so root}$$

$$f(5.3) = -ve \text{ lies between}$$

$f(x) = x$  has no roots

ii) b.s  $\frac{1}{5}x = \ln(3x+8)^{\frac{1}{3}}$

$$x = \frac{5}{3} \ln(3x+8)$$

$$\sqrt{mx+7} - 4 = x$$

$$mx+7 = (x+4)^2$$

$$mx+7 = x^2 + 8x + 16$$

$$0 = x^2 + (8-m)x + 9$$

i)  $x = 5.29$

$$A = \int_0^{5.29} (3x+8)^{\frac{1}{3}} - e^{\frac{1}{5}x}$$

$$= \frac{(3x+8)^{\frac{4}{3}}}{\frac{3 \times \frac{4}{3}}{3}} - 5e^{\frac{1}{5}x}$$

$$b^2 - 4ac < 0$$

$$(8-m)^2 - 4 \times 1 \times 9 < 0$$

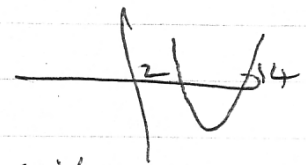
$$64 - 16m + m^2 - 36 < 0$$

$$m^2 - 16m + 28 < 0$$

$$(m-2)(m-14) < 0$$

$A_{5.29} = 2.779$  Area = 3.78

$A_0 = -1$



$m > 2 \text{ \& } m < 14$

$2 < m < 14$

horizontal asymptote - 7  
 horizontal asymptote factor  $\frac{1}{m}$   
 vertical asymptote - 4  
 each x value maps to only 1 y value + vice versa