GCE

Edexcel GCE

Core Mathematics C3 (6665)

Summer 2005

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Mark Scheme (Results)

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June 2005
6665 Core C3
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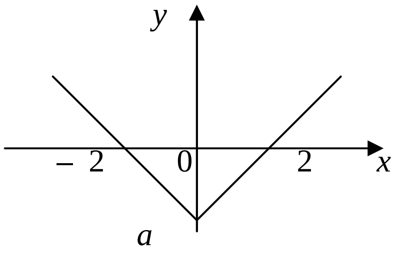
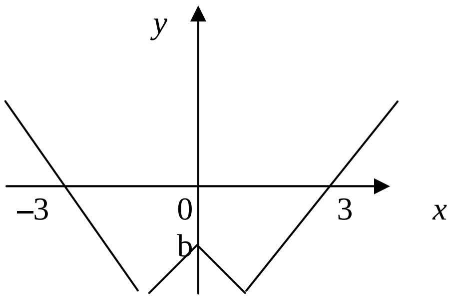
Question Number	Scheme	Marks
1. (a)	Dividing by $\cos^2\theta$: $\dfrac{\sin^2\theta}{\cos^2\theta}+\dfrac{\cos^2\theta}{\cos^2\theta}\equiv\dfrac{1}{\cos^2\theta}$	M1
	Completion **:** $1+\tan^2\theta\equiv\sec^2\theta$ (no errors seen)	A1
		(2)
(b)	Use of $1+\tan^2\theta=\sec^2\theta$: $2(\sec^2\theta-1)+\sec\theta=1$	M1
	$[\ 2\sec^2\theta+\sec\theta-3=0\ ]$	
	Factorising or solving: $(2\sec\theta+3)(\sec\theta-1)=0$	M1
	$[\ \sec\theta=-\frac{3}{2}$ or $\sec\theta=1]$	
	$\theta=0$	B1
	$\cos\theta=-\frac{2}{3}$; $\theta_1=131.8°$	M1 A1
	$\theta_2=228.2°$	A1√
		(6)
	[A1ft for $\theta_2=360°-\theta_1$]	**[8]**

Question Number	Scheme	Marks
2. (a)	(i) $6\sin x\cos x + 2\sec 2x\tan 2x$ **or** $3\ \sin 2x + 2\sec 2x\tan 2x$ [M1 for 6 sinx]	M1A1A1 (3)
	(ii) $3(x + \ln 2x)^2(1 + \frac{1}{x})$ [B1 for $3(x+\ln 2x)^2$]	B1M1A1 (3)
(b)	Differentiating numerator to obtain $10x - 10$ Differentiating denominator to obtain $2(x\text{-}1)$	B1 B1
	Using quotient rule formula correctly: To obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x-1)^2(10x-10) - (5x^2 - 10x + 9)2(x-1)}{(x-1)^4}$	M1 A1
	Simplifying to form $\frac{2(x-1)[5(x-1)^2 - (5x^2 - 10x + 9)}{(x-1)^4}$ $= -\frac{8}{(x-1)^3}$ * (c.s.o.)	M1 A1 (6)
		[12]
	Alternatives for (b) **Either** Using product rule formula correctly: Obtaining 10x – 10 Obtaining $-2(x-1)^{-3}$ To obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = (5x^2 - 10x + 9)\{-2(x-1)^{-3}\} + (10x - 10)(x-1)^{-2}$	M1 B1 B1 A1 cao
	Simplifying to form $\frac{10(x-1)^2 - 2(5x^2 - 10x + 9)}{(x-1)^3}$ $= -\frac{8}{(x-1)^3}$ * (c.s.o.)	M1 A1 (6)
	Or Splitting fraction to give $5 + \frac{4}{(x-1)^2}$ Then differentiating to give answer	M1 B1 B1 M1 A1 A1 (6)

Question Number	Scheme	Marks
3(a)	$\dfrac{5x+1}{(x+2)(x-1)} - \dfrac{3}{x+2}$	B1
	$= \dfrac{5x+1-3(x-1)}{(x+2)(x-1)}$	M1
	M1 for combining fractions even if the denominator is not lowest common	
	$= \dfrac{2x+4}{(x+2)(x-1)} = \dfrac{2(x+2)}{(x+2)(x-1)} = \dfrac{2}{x-1}$ *	M1 A1 cso
	M1 must have linear numerator	(4)
(b)	$y = \dfrac{2}{x-1} \Rightarrow xy - y = 2 \Rightarrow xy = 2 + y$	M1A1
	$f^{-1}(x) = \dfrac{2+x}{x}$ o.e.	A1 (3)
(c)	$fg(x) = \dfrac{2}{x^2+4}$ (attempt) $\left[\dfrac{2}{"g"-1}\right]$	M1
	Setting $\dfrac{2}{x^2+4} = \dfrac{1}{4}$ and finding $x^2 = \ldots$; $x = \pm 2$	M1; A1 (3)
		[10]

Question Number	Scheme	Marks
4 (a)	$f'(x) = 3e^x - \dfrac{1}{2x}$	M1A1A1 (3)
(b)	$3e^x - \dfrac{1}{2x} = 0$	M1
	$\Rightarrow 6\alpha e^{\alpha} = 1 \quad \Rightarrow \alpha = \frac{1}{6}e^{-\alpha}$ (*)	A1 cso (2)
(c)	$x_1 = 0.0613...,\ x_2 = 0.1568..,\ x_3 = 0.1425...,\ x_4 = 0.1445....$	M1 A1 (2)
	[M1 at least x_1 correct , A1 all correct to 4 d.p.]	
	(d) Using $f'(x) = 3e^x - \dfrac{1}{2x}$ with suitable interval e.g. $f'(0.14425) = -0.0007$ $f'(0.14435) = +0.002(1)$	M1
	Accuracy (change of sign and correct values)	A1 (2)
		[9]

Question Number	Scheme	Marks
5. (a)	$\cos 2A = \cos^2 A - \sin^2 A$ (+ use of $\cos^2 A + \sin^2 A \equiv 1$)	M1
	$= (1 - \sin^2 A); - \sin^2 A = 1 - 2\sin^2 A$ (*)	A1 (2)
(b)	$2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3 \equiv 4\sin\theta\cos\theta; -3(1 - 2\sin^2\theta) - 3\sin\theta + 3$	B1; M1
	$\equiv 4\sin\theta\cos\theta + 6\sin^2\theta - 3\sin\theta$	M1
	$\equiv \sin\theta(4\cos\theta + 6\sin\theta - 3)$ (*)	A1 (4)
(c)	$4\cos\theta + 6\sin\theta \equiv R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$	
	Complete method for R (may be implied by correct answer)	
	[$R^2 = 4^2 + 6^2$, $R\sin\alpha = 4$, $R\cos\alpha = 6$]	M1
	$R = \sqrt{52}$ or 7.21	A1
	Complete method for α ; $\alpha = 0.588$ (allow 33.7º)	M1 A1
		(4)
(d)	$\sin\theta\ (4\cos\theta + 6\sin\theta - 3) = 0$	M1
	$\theta = 0$	B1
	$\sin(\theta + 0.588) = \frac{3}{\sqrt{52}} = 0.4160..$ (24.6º)	M1
	$\theta + 0.588 = (0.4291),\ 2.7125$ [or $\theta + 33.7° = (24.6º),\ 155.4°$]	dM1
	$\theta = 2.12$ cao	A1
		(5)
		[15]

Question Number	Scheme		Marks
6. (a)		Translation ← by 1 Intercepts correct	M1 A1 (2)
(b)		$x \geq 0$, correct "shape" provided graph is not original graph Reflection in y-axis Intercepts correct	B1 B1√ B1 (3)
(c)	$a = -2$, $b = -1$		B1B1 (2)
(d)	Intersection of $y = 5x$ with $y = -x - 1$ Solving to give $x = -\frac{1}{6}$		M1A1 M1A1 (4)
	[Notes: (i) If both values found for $5x = -x - 1$ and $5x = x - 3$, or solved algebraically, can score 3 out of 4 for $x = -\frac{1}{6}$ and $x = -¾$; required to eliminate $x = -¾$ for final mark. (ii) Squaring approach: M1 correct method, $24x^2 + 22x + 3 = 0$ (correct 3 term quadratic, any form) A1 Solving M1, Final correct answer A1.]		**[11]**

7. (a)	Setting $p = 300$ at $t = 0 \Rightarrow\ 300 = \dfrac{2800a}{1+a}$	M1
	(300 = 2500a); a = 0.12 (c.s.o) *	dM1A1 (3)
(b)	$1850 = \dfrac{2800(0.12)e^{0.2t}}{1+0.12e^{0.2t}}$; $e^{0.2t} = 16.2...$	M1A1
	Correctly taking logs to $0.2\,t = \ln k$	M1
	$t = 14$ (13.9..)	A1 (4)
(c)	Correct derivation: (Showing division of num. and den. by $e^{0.2t}$; using a)	B1 (1)
(d)	Using $t \to \infty$, $e^{-0.2t} \to 0$,	M1
	$p \to \dfrac{336}{0.12} = 2800$	A1 (2)
		[10]