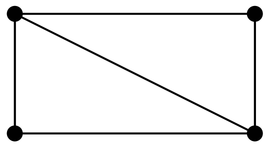
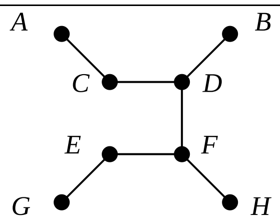
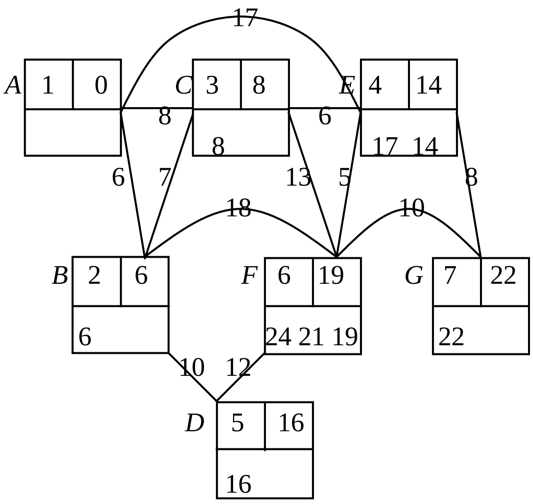
Mark Scheme 4736
June 2005

1	**(a) (i)**	8 7 5 4 3 3 3 3 2 2	M1	For sorting the list into decreasing order
		First bag 8 2 Second bag 7 3 Third bag 5 4 Fourth bag 3 3 3 Fifth bag 2	M1 A1	For trying to apply first-fit to their list For a completely correct solution
	(ii)	A packing that uses fewer bags could be First bag 8 2 Second bag 7 3 Third bag 5 3 2 Fourth bag 4 3 3	B1	For any valid packing into four bags (may be as an incorrect answer to using algorithm, need not be packed in this order)
	(b)	$\left(\frac{500}{100}\right)^3 \times 4$ or $125000000 \times 0.000004$ $= 500$	M1 A1 $\boxed{6}$	For scaling 4 seconds by 5^3 or for an equivalent valid and complete method. Condone minor errors with the number of zeros. For 500 or 500 seconds or 500 s. Accept 8 minutes 20 seconds or 8.3 minutes
2	**(i)**	eg 	B1	For any <u>simple</u> graph with 4 vertices and 5 arcs Vertices need not be labelled Need not be planar
	(ii)	The sum of the orders of the vertices is twice the number of arcs, and hence is even. Hence the sum of the odd orders must be even and so there must be an even number of odd vertices.	M1 A1	<u>Or</u> start from a null graph and successively add in arcs. Each time an arc is added the number of odd vertices is either unchanged or it increases or decreases by 2. So the number of odd nodes is always even
	(iii)	5 arcs $\Rightarrow$ sum of orders of vertices = 10 Simple graph connecting vertices so each vertex has order 1, 2 or 3 1 + 3 + 3 + 3 = 10 or 2 + 2 + 3 + 3 = 10 But 1 + 3 + 3 + 3 is not possible since if three vertices have order 3 they are all connected to the fourth vertex so it also has order 3. With 2 + 2 + 3 + 3 the two vertices of order 2 cannot be adjacent, since otherwise two arcs connect the other two vertices so not simple. Hence only one possible graph.	M1 A1 A1 $\boxed{6}$	 Explaining why 1 + 3 + 3 + 3 is not possible. Explaining why there is only one graph with nodes of orders 2, 2, 3, 3.
3	**(i)**	 Kruskal: *DF*, *CD*, *BD* and *EF*, *FH*, *AC*, *EG* 40	M1 A1 B1	For a correct tree (labels not required) For a valid order (using Prim or Kruskal) For length = 40
	(ii)	*A C D F E G H B A*	M1	At getting at least as far as *A C D F E* (or shown on a diagram)

			A1	For a correct cycle, ending back at *A* (if shown on a diagram, needs direction shown)
	(iii) **(A)**	*ACEG* and *ABGH*	B1	For both, vertices in any order
	(B)	5	B1	For 5
	(C)	*ABCD*	B1 **8**	For *ABCD*, vertices in any order
4	**(i)**			Answer should be on insert sheet
			M1	For using Dijkstra's algorithm – updating at *E* and *F* (even if incomplete)
			A1	For all permanent labels correct
			B1	For valid order of assigning permanent labels
		Vertex *B* *C* *D* *E* *F* *G* Length 6 8 16 14 19 22	B1	For copying their permanent labels, or correct values
		A – *C* – *E* – *G*	B1	Correct answer only
	(ii)	The only odd nodes are *A* and *F* Shortest path from *A* to *F* has length 19 km	M1	For identifying *A* and *F* or value 19 or their 19
		120 + 19	M1	For 120 + their 19
		= 139 km	A1	For 139 (cao)
	(iii)	Need *A* and *G* odd and all other nodes even so need to connect *F* to *G* = 10 km	M1	For identifying *F* and *G* or value 10 as only extra
		120 + 10 = 130 km	A1 **10**	For 130 (cao)

5 (i)

X	N	T	S
	0	0	0
3	1	3	9
6	2	9	45
5	3	14	70
7	4	21	119
3	5	24	128

$M = 4.8$
$D = 1.6$

Mark	Guidance
M1	For initial pass through step 3 correct
M1	For updating each of N, T and S correctly
A1	For final values of N, T and S correct
B1	For 4.8 (ft their $T \div N$)
B1	For 1.6 (ft their $\sqrt{(S \div N) - (M^2)}$)

(ii) 15 additions and 5 multiplications
20 + 5 = 25

Mark	Guidance
B1	
B1	For 'their 20' + 5

(iii) $3n + n + 5$
$= 4n + 5$

Mark	Guidance
M1	For any function of n that gives their answer from (ii) when $n = 5$
A1	For any expression that simplifies to $4n + 5$

(iv) $(5000 \div 1000) \times 2 = 10$ seconds

Mark	Guidance
B1	<u>Or</u> $2 \div 4005 \times 20005 = 9.99 \simeq 10$ seconds

$\boxed{10}$

6 (i)

P	x	y	z	s	t	u	
1	-15	4	4	0	0	0	0
0	10	-4	8	1	0	0	40
0	10	6	9	0	1	0	72
0	-6	4	3	0	0	1	48

Mark	Guidance
M1	For overall structure correct, including three slack variables
A1	For a correct initial tableau, with no extra constraints added. Accept equivalent forms.

(ii) Pivot on 10 in x column 40 row

1	0	-2	16	1.5	0	0	60
0	1	-0.4	0.8	0.1	0	0	4
0	0	10	1	-1	1	0	32
0	0	1.6	7.8	0.6	0	1	72

$x = 4$, $y = 0$, $z = 0$
$P = 60$

Mark	Guidance
M1	For the correct pivot choice for their tableau
A1	For dealing with the pivot row correctly
M1	For dealing with the other rows correctly
A1	For a correct tableau
B1	For reading off x, y and z from their tableau
B1	For reading off P from their tableau

(iii) Pivot on 10 in y column

1	0	0	16.2	1.3	0.2	0	66.4
0	1	0	0.84	0.06	0.04	0	5.28
0	0	1	0.1	-0.1	0.1	0	3.2
0	0	0	7.64	0.76	-0.16	1	66.88

$x = 5.28$, $y = 3.2$, $z = 0$
$P = 66.4$

Mark	Guidance
M1	For the correct pivot choice for their tableau
A1	For dealing with the pivot row correctly
M1	For dealing with the other rows correctly
A1	For a correct tableau
B1	For the correct values of x, y and z at optimum
B1	For the correct value of P at optimum

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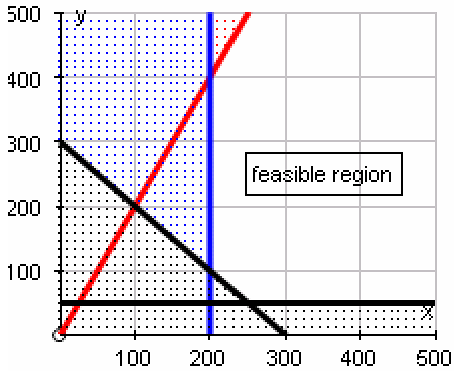
7	(i)	Minimise $70x + 80y + 50z$	B1	For 'minimise' a (non-zero) multiple of $7x+8y+5z$
		'No more than twice as many packs of type Y as packs of type X'	B1	For identifying this constraint from the list, or equivalent
		Other constraints		Ignore extra 'constraints' unless contradictions
		$x \geq 200$, $0 \leq z \leq 50$	B1	For boundary constraints on x and z
		$y \geq z$	B1	For this, or an equivalent correct answer
		$x + z \geq 220$	B1	For this, or an equivalent correct answer
		$x + y \geq 300$	B1	For this, or an equivalent correct answer Use of strict inequalities – penalise first time only
	(ii) (a)	Minimise $70x + 80y$ (+ 2500) (or scaled through) Subject to	M1	For replacing z by 50
		$y \leq 2x$ $x \geq 200$ $y \geq 50$	A1	For their $y \geq 50$
		$x + y \geq 300$		
			M1	For at least two appropriate lines drawn on a graph with plausibly scaled axes.
			M1	For boundary lines drawn correctly (follow through their equations provided there are at least two horizontal or vertical lines and at least two lines that 'slope')
			A1	Feasible region correctly identified (correct answer only, not follow through)
	(b)		M1	For reading off or calculating at least one of their vertices
		(200, 400), (200, 100), (250, 50)	A1	For getting these three vertices correct with no extras
		(200, 100) gives $70x + 80y = 22000$ (£245) (250, 50) gives $70x + 80y = 21500$ (£240)	M1	For calculating their cost at one of their vertices or using an appropriate line of constant cost
		Cost is minimised when $x = 250$, $y = 50$	A1	For identifying vertex (250, 50)
		Cost = £240	B1	For £240 or 24000 p (with units)
	(iii)	eg $x = 300$, $y = 0$, $z = 0$ only costs £210	M1	For finding a feasible point with $z < 50$ Or a written explanation
			A1	For finding such a feasible point with a lower cost than that in (ii)(b) <u>and</u> showing that cost is lower.
			$\boxed{18}$	