OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4724

Core Mathematics 4

Monday **20 JUNE 2005** Morning 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

[Turn over

1 Find the quotient and the remainder when $x^4 + 3x^3 + 5x^2 + 4x - 1$ is divided by $x^2 + x + 1$. [4]

2 Evaluate $\int_0^{\frac{1}{2}\pi} x \cos x \, dx$, giving your answer in an exact form. [5]

3 The line L_1 passes through the points $(2, -3, 1)$ and $(-1, -2, -4)$. The line L_2 passes through the point $(3, 2, -9)$ and is parallel to the vector $4\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$.

(i) Find an equation for L_1 in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. [2]

(ii) Prove that L_1 and L_2 are skew. [5]

4 **(i)** Show that the substitution $x = \tan\theta$ transforms $\int \frac{1}{(1 + x^2)^2} \, dx$ to $\int \cos^2\theta \, d\theta$. [3]

(ii) Hence find the exact value of $\int_0^1 \frac{1}{(1 + x^2)^2} \, dx$. [4]

5 $ABCD$ is a parallelogram. The position vectors of A, B and C are given respectively by

$$\mathbf{a} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}, \qquad \mathbf{b} = 3\mathbf{i} - 2\mathbf{j}, \qquad \mathbf{c} = \mathbf{i} - \mathbf{j} - 2\mathbf{k}.$$

(i) Find the position vector of D. [3]

(ii) Determine, to the nearest degree, the angle ABC. [4]

6 The equation of a curve is $xy^2 = 2x + 3y$.

(i) Show that $\frac{dy}{dx} = \frac{2 - y^2}{2xy - 3}$. [5]

(ii) Show that the curve has no tangents which are parallel to the y-axis. [3]

7 A curve is given parametrically by the equations

$$x = t^2, \qquad y = \frac{1}{t}.$$

(i) Find $\frac{dy}{dx}$ in terms of t, giving your answer in its simplest form. [3]

(ii) Show that the equation of the tangent at the point $P\left(4, -\frac{1}{2}\right)$ is

$$x - 16y = 12.$$ [3]

(iii) Find the value of the parameter at the point where the tangent at P meets the curve again. [4]

8 **(i)** Given that $\dfrac{3x+4}{(1+x)(2+x)^2} \equiv \dfrac{A}{1+x} + \dfrac{B}{2+x} + \dfrac{C}{(2+x)^2}$, find A, B and C. [5]

(ii) Hence or otherwise expand $\dfrac{3x+4}{(1+x)(2+x)^2}$ in ascending powers of x, up to and including the term in x^2. [5]

(iii) State the set of values of x for which the expansion in part **(ii)** is valid. [1]

9 Newton's law of cooling states that the rate at which the temperature of an object is falling at any instant is proportional to the difference between the temperature of the object and the temperature of its surroundings at that instant. A container of hot liquid is placed in a room which has a constant temperature of 20 °C. At time t minutes later, the temperature of the liquid is θ °C.

(i) Explain how the information above leads to the differential equation

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -k(\theta - 20),$$

where k is a positive constant. [2]

(ii) The liquid is initially at a temperature of 100 °C. It takes 5 minutes for the liquid to cool from 100 °C to 68 °C. Show that

$$\theta = 20 + 80\mathrm{e}^{-\left(\frac{1}{5}\ln\frac{5}{3}\right)t}.$$ [8]

(iii) Calculate how much longer it takes for the liquid to cool by a further 32 °C. [3]

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