Core 2 June 2006

1 $(3x-2)^4 = (3x)^4 + \binom{4}{1}(3x)^3(-2) + \binom{4}{2}(3x)^2(-2)^2 + \binom{4}{3}3x(-2)^3 + \binom{4}{4}(-2)^4.$

$= 81x^4 + 4\times27x^3\times-2. + 6\times9x^2\times+4 + 4\times3x\times-8 + 16.$

$= 81x^4 - 216x^3 + 216x^2 - 96x + 16.$

2 $u_1 = 2$ $u_2 = 1-u_1 = 1-2 = -1$ $u_3 = 1-u_2 = 1+1 = 2$ $u_4 = 1-u_3 = -1$

u_1 2

u_2 -1

u_3 2

u_4 -1

$\sum_1^{100} u_n$ $50\times2 + 50\times(-1) = 50.$

3. $\frac{dy}{dx} = 2x^{-1/2}.$ $y = \frac{2x^{1/2}}{\frac{1}{2}} + k = 4x^{1/2} + k$ $y = 4\sqrt{x} + k$ $x=4$ $y=5.$

$5 = 4\times\sqrt{4} + k$ $5 = 8+k$ $k = -3.$ $y = 4\sqrt{x} - 3.$

4. $4-x^2 = x+2.$ $x^2+x-2 = 0$ $(x+2)(x-1) = 0$ $x=-2$ $x=1.$

$\int_{-2}^{1} 4-x^2\,dx = \left[4x - \frac{x^3}{3}\right]_{-2}^{1} = \left(4-\frac{1}{3}\right) - \left(-8+\frac{8}{3}\right) = 12-3 = 9$

area triangle $x=1$ $y=3$, $x=-2$ $y=0.$ $\frac{1}{2}\times3\times3 = 4\frac{1}{2}$

OR $\int_{-2}^{1} x+2\,dx = \left[\frac{x^2}{2}+2x\right]_{-2}^{1} = \left(\frac{1}{2}+2\right) - \left(\frac{4}{2}-4\right) = \frac{3}{2}.+2 = 4\frac{1}{2}$

Area shaded region $9 - 4\frac{1}{2} = 4\frac{1}{2}$ units^2.

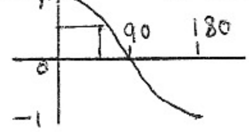
5. $2\sin^2 x = 1+\cos x$ $2(1-\cos^2 x) = 1+\cos x$ $2-2\cos^2 x - 1 - \cos x = 0$

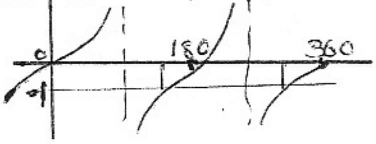
$2\cos^2 x + \cos x - 1 = 0$ $(2\cos x - 1)(\cos x + 1) = 0$

$\cos x = \frac{1}{2}$ $\cos x = -1$ $x = 60°, 180°$

$0<x<180$ $0<2x<360$

$\sin 2x = -\cos 2x$ $\tan 2x = -1$ $2x = 135°, 315°$ $x = 67.5°, 157.5°$





6. 100 105 110 115 240 payments

$a=100$ $d=5$ $n=240.$ 240th $a+(n-1)d$ $100+(239\times5) = 1295$

$S_n = S_{240} = \frac{240}{2}(2\times100 + (239\times5)) = 120(200+1195) = £167\,400$

(ii) $a = 100$ 240th term $ar^{n-1} = 1500$. $100 r^{239} = 1500$

$r = 1.01139519$ $\left(15^{\frac{1}{239}} = r\right)$

$$S_{240} = \frac{a(r^n - 1)}{r-1} = \frac{100(15.170927 - 1)}{.01139519} = £124358.86$$

7. $b^2 = a^2 + c^2 - 2ac\cos B$ $AC^2 = 121 + 64 - 176\cos 0.8^c$

$AC^2 = 62.3796$ $AC = 7.90$ (3sf)

Area segment $= \frac{1}{2}r^2(\theta - \sin\theta) = \frac{1}{2}\, 7.898076\ldots^2 (1.7^c - \sin 1.7^c) = 22.09$ units^2.

arc $DC = 7.898076 \times 1.7 = 13.42673$.

$DC^2 = \overset{AC^2}{62.3796} + \overset{AD^2}{62.3796} - (2 \times 62.3796 \cos 1.7^c) = 132.7965$.

$DC = 11.5237$. Perimeter $13.42673 + 11.5237 = 24.99$ 25.0 cm (3sf)

8. $f(x) = 2x^3 + ax^2 + bx - 10$

$f(2) = 12$ $16 + 4a + 2b - 10 = 12$ $4a + 2b = 6$. $4a + 2b = 6$

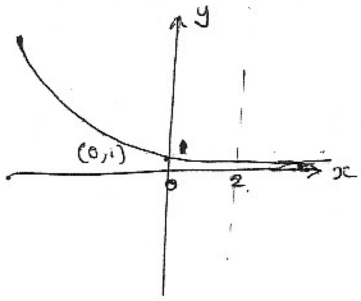
$f(-1) = 0$ $-2 + a - b - 10 = 0$ $a - b = 12$. $2a - 2b = 12$.

$a = 5$ $b = -7$

$$\begin{array}{r} 2x^2 + x - 9 \\ x+2 \overline{\smash{)}\, 2x^3 + 5x^2 - 7x - 10} \\ \underline{2x^3 + 4x^2} \\ +x^2 - 7x \\ \underline{+x^2 + 2x} \\ -9x - 10 \\ \underline{-9x - 18} \\ 8 \end{array}$$

quotient $2x^2 + x - 9$ remainder 8.

9 $y = \left(\frac{1}{2}\right)^x$



x	y
0	1
$\frac{1}{2}$	0.7071
1	0.5
$1\frac{1}{2}$	0.35355
2	0.25

$h = \frac{1}{2}$,

$A = \frac{1}{2} \times \frac{1}{2}\left\{1 + .25 + 2(.7071 + .5 + .3536)\right\}$

$= \frac{1}{4}\left\{1.25 + 2 \times 1.56065\right\}$ $= 1.0928$ $A = 1.09$ (3sf)

(iii) $\frac{1}{6} = \left(\frac{1}{2}\right)^x$ $6 = 2^x$ $\log_{10} 6 = x\log_{10} 2$

$\log_{10} 3 + \log_{10} 2 = x\log_{10} 2$.

$\frac{\log_{10} 3}{\log_{10} 2} + 1 = x$