Core 3 June 2006

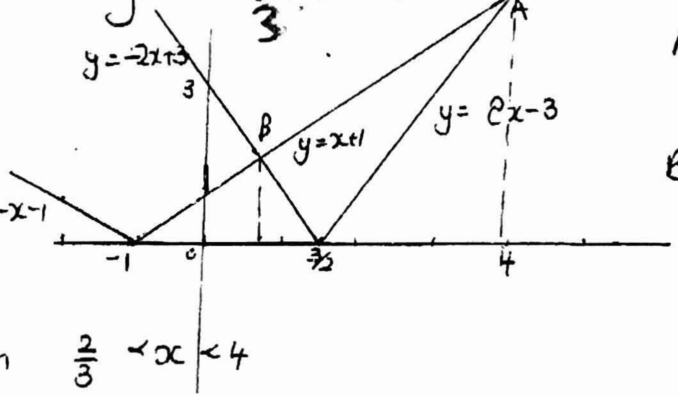
1) $y = \sqrt{4x+1} = (4x+1)^{\frac{1}{2}}$ $\quad \frac{dy}{dx} = \frac{1}{2}(4x+1)^{-1/2} \times 4 = \frac{2}{\sqrt{4x+1}}$

$(2,3)$ $\quad x = 2 \quad \frac{dy}{dx} = \frac{2}{3}$ $\quad y - 3 = \frac{2}{3}(x-2)$

$y = \frac{2}{3}x + \frac{5}{3}$

$3y - 2x - 5 = 0$

2) $|2x-3| = |x+1|$



A $\quad 2x-3 = x+1$

$x = 4$

B $\quad -2x+3 = x+1$

$2 = 3x$

$x = \frac{2}{3}$

$|2x-3| < |x+1|$ when $\frac{2}{3} < x < 4$

3) $f(x) = 2x^3 + 4x - 35$. $\quad f(2) = 16 + 8 - 35 = -11 \quad f(3) = 54 + 12 - 35 = +31$

change of sign -ve to +ve between $x = 2$ and 3.

$x_{n+1} = \sqrt[3]{17.5 - 2x_n} \quad x_1 = 2 \quad x_2 = \sqrt[3]{17.5-4} = 2.3811 \quad x_3 = \sqrt[3]{17.5 - 2 \times 2.3811} = 2.3354$

CALC. 2 ENTER $\quad (17.5-(2\text{ANS}))^{1/3}$ ENTER $\quad x_4 = 2.3410 \quad x_5 = 2.3403 \quad x_6 = 2.3404 \quad x_7 = 2.3404$

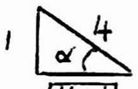
root is 2.34 (2dp).

4) $y = 5^{x-1} \quad \ln y = \ln 5^{x-1} \quad \ln y = (x-1)\ln 5 \quad \ln y + \ln 5 = x\ln 5$

$x = \frac{\ln y}{\ln 5} + 1 \quad \frac{dx}{dy} = \frac{1}{\ln 5} \times \frac{1}{y}$ $\quad$ gradient of curve $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = y\ln 5$

$\frac{dy}{dx} = 5^{x-1}\ln 5$. $\quad$ at $(3, 25)$ $\quad \frac{dy}{dx} = 5^2\ln 5 = 25\ln 5$.

5) ii) $\sin 2\theta = 2\sin\theta\cos\theta \quad \sin\alpha = \frac{1}{4}$



$\cos\alpha = \frac{\sqrt{15}}{4}$.

$\sin 2 = 2 \times \frac{1}{4} \times \frac{\sqrt{15}}{4} = \frac{\sqrt{15}}{8}$

iii) $5 \times 2\sin\beta\cos\beta \frac{1}{\cos\beta} = 3 \quad 10\sin\beta\cos\beta - 3\cos\beta = 0 \quad \cos\beta(10\sin\beta - 3) = 0$

$0 < \beta < 90 \quad \cos\beta = 0$ not required solution $\quad \sin\beta = 0.3 \quad \beta = 17.5°$ (3sf).

6) i) $f(x) = 2 - x^2 \quad x \leq 0 \quad f(-3) = 2 - (-3)^2 = -7 \quad f(-7) = 2 - (-7)^2 = -47$

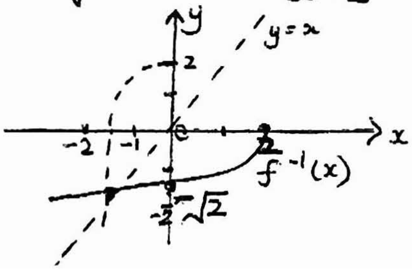
$ff(-3) = -47$.

ii) $x \rightarrow \overset{x^2}{\wedge^2} \longrightarrow \overset{-x^2}{\text{change sign}} \longrightarrow \overset{-x^2+2}{+2} \longrightarrow -x^2+2$

$x \rightarrow \overset{x-2}{-2} \longrightarrow \overset{2-x}{\text{change sign}} \longrightarrow \overset{\sqrt{2-x}}{\pm\sqrt{\ }} \longrightarrow \pm\sqrt{2-x}$. $\quad$ domain $f(x)$ $x \leq 0$

range $f(x)$ $x \leq 2$

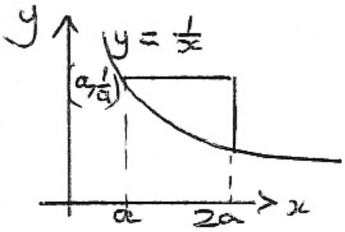
$f^{-1}(x) = -\sqrt{2-x} \quad x \leq 2$ $\quad$ (-ve square root as reflection of $f(x)$ in $y = x$)



graph meets axis $(2,0)$. $(0, -\sqrt{2})$

7a) $\int_1^2 \frac{2}{(4x-1)^2}\,dx = \int_1^2 2(4x-1)^{-2}\,dx = \left[\frac{2(4x-1)^{-1}}{-1\times 4}\right]_1^2 = \left[\frac{-1}{2(4x-1)}\right]_1^2$

$= \left(\frac{-1}{2\times 7}\right) - \left(\frac{-1}{2\times 3}\right) = -\frac{1}{14} + \frac{1}{6} = \frac{-6+14}{84} = \frac{8}{84} = \frac{2}{21}$

b) 

Area rectangle $= a \times \frac{1}{a} = 1$

Area under $y = \frac{1}{x}$ from $x = a$ to $x = 2a$.

$\int_a^{2a} \frac{1}{x}\,dx = [\ln x]_a^{2a} = \ln 2a - \ln a = \ln 2.$

Shaded area $= 1 - \ln 2 = \ln e^1 - \ln 2 = \ln\left(\frac{e}{2}\right)$

8. $5\cos x + 12\sin x \equiv R\cos(x-\alpha)$ $\quad R\cos(x-\alpha) = R\cos x\cos\alpha + R\sin x\sin\alpha$

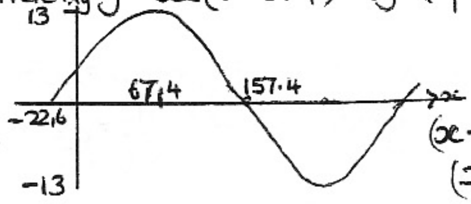
$\frac{R\sin\alpha}{R\cos\alpha} = \frac{12}{5}$ $\quad \tan\alpha = \frac{12}{5}$ $\quad \alpha = 67.38°$. $\quad R^2\sin^2\alpha + R^2\cos^2\alpha = 144 + 25$

$R^2(\cos^2\alpha + \sin^2\alpha) = 169$ $\quad \cos^2\alpha + \sin^2\alpha = 1$ $\quad R = 13$

$5\cos x + 12\sin x = 13\cos(x - 67.4°)$

Translate $y = \cos x$ by $67.4°$ in a positive direction parallel to x axis

Stretch $y = \cos(x - 67.4°)$ by a factor 13 parallel to y axis



$5\cos x + 12\sin x = 2$ $\quad 13\cos(x - 67.4) = 2.$

$(x - 67.3801)$ $\cos(x - 67.4) = \frac{2}{13}$ $\quad -67.4 < x - 67.4 < 292.6$

$(x - 67.4) = 81.15\,,\ 360 - 81.15$

$x = 81.1501 + 67.3801\,,\ 360 - 81.1501 + 67.3801$ $\quad x = 148.5°\,,\ 346.2°$

9. $V = \pi\int x^2\,dy.$ $\quad y = 2\ln(x-1)$ $\quad \frac{y}{2} = \ln(x-1)$ $\quad e^{\frac{y}{2}} = x - 1$ $\quad x = e^{\frac{y}{2}} + 1$

$V = \pi\int_0^p \left(e^{\frac{y}{2}} + 1\right)^2 dy = \pi\int_0^p e^y + 2e^{\frac{y}{2}} + 1\,dy$

$V = \pi\left[e^y + 4e^{\frac{y}{2}} + y\right]_0^p = \pi\left(e^p + 4e^{\frac{p}{2}} + p\right) - \pi(1 + 4 + 0)$

$V = \pi\left(e^p + 4e^{\frac{p}{2}} + p - 5\right)$

In general $p = y$ $\quad V = \pi\left(e^y + 4e^{\frac{y}{2}} + y - 5\right)$ $\quad \frac{dV}{dy} = \pi\left(e^y + 2e^{\frac{y}{2}} + 1\right)$

given $\frac{dy}{dt} = 0.2$ cm min^{-1} $\quad \frac{dV}{dt} = \frac{dV}{dy} \times \frac{dy}{dt}$

$\frac{dV}{dt} = \pi\left(e^y + 2e^{\frac{y}{2}} + 1\right) \times 0.2$ $\quad$ when $y = p = 4$

$\frac{dV}{dt} = 0.2\pi\left(e^4 + 2e^2 + 1\right) = 44$ cm^3 min^{-1} (2sf)