

GCE

Edexcel GCE

Core Mathematics C2 (6664)

June 2006

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Mark Scheme
(Results)

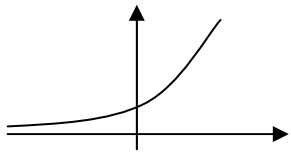
June 2006
6664 Core Mathematics C2
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Question number	Scheme	Marks
1.	$(2+x)^6 = 64 \dots$ $+ (6 \times 2^5 \times x) + \left(\frac{6 \times 5}{2} \times 2^4 \times x^2 \right), \quad + 192x, \quad + 240x^2$	B1 M1, A1, A1 (4) 4
	<p>The terms can be 'listed' rather than added.</p> <p>M1: Requires correct structure: 'binomial coefficients' (perhaps from Pascal's triangle), increasing powers of one term, decreasing powers of the other term (this may be 1 if factor 2 has been taken out). Allow 'slips'.</p> <p>$\binom{6}{1}$ and $\binom{6}{2}$ or equivalent are acceptable, or even $\binom{6}{1}$ and $\binom{6}{2}$.</p> <p><u>Decreasing powers of x:</u> Can score only the M mark.</p> <p>64(1 +), even if all terms in the bracket are correct, scores max. B1M1A0A0.</p>	

Question number	Scheme	Marks
2.	$\int (3x^2 + 5 + 4x^{-2}) dx = \frac{3x^3}{3} + 5x + \frac{4x^{-1}}{-1} \quad (= x^3 + 5x - 4x^{-1})$ $[x^3 + 5x - 4x^{-1}]_1^2 = (8 + 10 - 2) - (1 + 5 - 4), = 14$	M1 A1 A1 M1, A1 (5) 5
	<p><u>Integration:</u></p> <p>Accept any correct version, simplified or not.</p> <p>All 3 terms correct: M1 A1 A1, Two terms correct: M1 A1 A0, One power correct: M1 A0 A0.</p> <p>The <u>given</u> function must be integrated to score M1, and not e.g. $3x^4 + 5x^2 + 4$.</p> <p><u>Limits:</u></p> <p>M1: Substituting 2 and 1 into a 'changed function' and subtracting, either way round.</p>	

Question number	Scheme	Marks
3.	(i) 2 (ii) $2\log 3 = \log 3^2$ (or $2\log p = \log p^2$) $\log_a p + \log_a 11 = \log_a 11p, = \log_a 99$ (Allow e.g. $\log_a (3^2 \times 11)$)	B1 (1) B1 M1, A1 (3) 4
	(ii) Ignore 'missing base' or wrong base. The correct answer with no working scores full marks. $\log_a 9 \times \log_a 11 = \log_a 99$, or similar mistakes, score M0 A0.	

Question number	Scheme	Marks
4.	<p>(a) $f(-2) = 2(-2)^3 + 3(-2)^2 - 29(-2) - 60$ M: Attempt $f(2)$ or $f(-2)$ $= -16 + 12 + 58 - 60 = -6$ A1 (2)</p> <p>(b) $f(-3) = 2(-3)^3 + 3(-3)^2 - 29(-3) - 60$ M: Attempt $f(3)$ or $f(-3)$ $(= -54 + 27 + 87 - 60) = 0 \therefore (x + 3)$ is a factor A1 (2)</p> <p>(c) $(x + 3)(2x^2 - 3x - 20)$ M1 A1 $= (x + 3)(2x + 5)(x - 4)$ M1 A1 (4)</p>	8
	<p>(a) <u>Alternative (long division):</u> Divide by $(x + 2)$ to get $(2x^2 + ax + b)$, $a \neq 0, b \neq 0$. [M1] $(2x^2 - x - 27)$, remainder = -6 [A1]</p> <p>(b) A1 requires zero and a simple conclusion (even just a tick, or Q.E.D.).</p> <p>(c) First M requires division by $(x + 3)$ to get $(2x^2 + ax + b)$, $a \neq 0, b \neq 0$. Second M for the attempt to factorise their quadratic. Usual rule: $(2x^2 + ax + b) = (2x + c)(x + d)$, where $cd = b$.</p> <p><u>Alternative (first 2 marks):</u> $(x + 3)(2x^2 + ax + b) = 2x^3 + (6 + a)x^2 + (3a + b)x + 3b = 0$, then compare coefficients to find <u>values</u> of a and b. [M1] $a = -3, b = -20$ [A1]</p> <p><u>Alternative:</u> Factor theorem: Finding that $f\left(-\frac{5}{2}\right) = 0 \therefore$ factor is, $(2x + 5)$ [M1, A1] Finding that $f(4) = 0 \therefore$ factor is, $(x - 4)$ [M1, A1]</p> <p>“Combining” all 3 factors is <u>not</u> required. If just one of these is found, score the <u>first 2 marks</u> M1 A1 M0 A0.</p> <p><u>Losing a factor of 2:</u> $(x + 3)\left(x + \frac{5}{2}\right)(x - 4)$ scores M1 A1 M1 A0.</p> <p><u>Answer only, one sign wrong:</u> e.g. $(x + 3)(2x - 5)(x - 4)$ scores M1 A1 M1 A0.</p>	

Question number	Scheme	Marks
5.	<p>(a) </p> <p style="text-align: right;">Shape</p> <p>(0, 1), or just 1 on the y-axis, or seen in table for (b)</p> <p>(b) Missing values: 1.933, 2.408 (Accept awrt)</p> <p>(c) $\frac{1}{2} \times 0.2, \{(1+3) + 2(1.246 + 1.552 + 1.933 + 2.408)\}$ $= 1.8278$ (awrt 1.83)</p>	<p>B1</p> <p>B1 (2)</p> <p>B1, B1 (2)</p> <p>B1, M1 A1ft</p> <p>A1 (4)</p> <p style="text-align: right;">8</p>
	<p>Beware the order of marks!</p> <p>(a) Must be a curve (not a straight line). Curve must extend to the left of the y-axis, and must be increasing. Curve can 'touch' the x-axis, but must not go below it. Otherwise, be generous in cases of doubt.</p> <p>The B1 for (0, 1) is independent of the sketch.</p> <p>(c) Bracketing mistake: i.e. $\frac{1}{2} \times 0.2(1+3) + 2(1.246 + 1.552 + 1.933 + 2.408)$</p> <p>scores B1 M1 A0 A0 <u>unless</u> the final answer implies that the calculation has been done correctly (then full marks can be given).</p>	

Question number	Scheme	Marks
6.	(a) $\tan \theta = 5$ (b) $\tan \theta = k$ $(\theta = \tan^{-1} k)$ $\theta = 78.7, \quad 258.7$ (Accept awrt)	B1 (1) M1 A1, A1ft (3) 4
	<p>(a) Must be seen explicitly, e.g. $\tan \theta = \tan^{-1} 5 = 78.7$ or equiv. is B0, unless $\tan \theta = 5$ is also seen.</p> <p>(b) The M mark may be implied by working in (a).</p> <p>A1ft for $180 + \alpha$. ($\alpha \neq k$).</p> <p>Answers in radians would lose both the A marks.</p> <p>Extra answers between 0 and 360: Deduct the final mark.</p> <p><u>Alternative:</u> Using $\cos^2 \theta = 1 - \sin^2 \theta$ (or equiv.) and proceeding to $\sin \theta = k$ (or equiv.): M1 then A marks as in main scheme.</p>	

Question number	Scheme	Marks
7.	<p>(a) Gradient of PQ is $-\frac{1}{3}$</p> $y - 2 = -\frac{1}{3}(x - 2) \quad (3y + x = 8)$ <p>(b) $y = 1: \quad 3 + x = 8 \quad x = 5 \quad (*)$</p> <p>(c) $(5 - 2)^2 + (1 - 2)^2$ M: Attempt PQ^2 or PQ</p> $(x - 5)^2 + (y - 1)^2 = 10$ M: $(x \pm a)^2 + (y \pm b)^2 = k$	<p>B1</p> <p>M1 A1 (3)</p> <p>B1 (1)</p> <p>M1 A1</p> <p>M1 A1 (4)</p> <p style="text-align: right;">8</p>
	<p>(a) M1: eqn. of a straight line through (2, 2) with any gradient except 3, 0 or ∞. <u>Alternative:</u> Using (2, 2) in $y = mx + c$ to find a value of c scores M1, but an equation (general or specific) must be seen. If the given value $x = 5$ is used to find the gradient of PQ, maximum marks are (a) B0 M1 A1 (b) B0.</p> <p>(c) For the first M1, condone <u>one</u> slip, numerical or sign, <u>inside</u> a bracket. The first M1 can be scored if <u>their</u> x-coord. is used instead of 5. For the second M1, allow any equation in this form, with non-zero a, b and k.</p>	

Question number	Scheme	Marks
8.	<p>(a) $r\theta = 2.12 \times 0.65$ 1.38 (m)</p> <p>(b) $\frac{1}{2}r^2\theta = \frac{1}{2} \times 2.12^2 \times 0.65$ 1.46 (m²)</p> <p>(c) $\frac{\pi}{2} - 0.65$ 0.92 (radians) (α)</p> <p>(d) ΔACD: $\frac{1}{2}(2.12)(1.86)\sin \alpha$ (With the value of α from part (c)) Area = “1.46” + “1.57”, 3.03 (m²)</p>	<p>M1 A1 (2)</p> <p>M1 A1 (2)</p> <p>M1 A1 (2)</p> <p>M1</p> <p>M1 A1 (3)</p> <p>9</p>
	<p>(a) M1: Use of $r\theta$ with $r = 2.12$ or 1.86, and $\theta = 0.65$, or equiv. method for the angle changed to degrees (allow awrt 37°).</p> <p>(b) M1: Use of $\frac{1}{2}r^2\theta$ with $r = 2.12$ or 1.86, and $\theta = 0.65$, or equiv. method for the angle changed to degrees (allow awrt 37°).</p> <p>(c) M1: Subtracting 0.65 from $\frac{\pi}{2}$, or subtracting awrt 37 from 90 (degrees), (perhaps implied by awrt 53).</p> <p><u>Angle changed to degrees wrongly and used throughout (a), (b) and (c):</u> Penalise ‘method’ only once, so could score M0A0, M1A0, M1A0.</p> <p>(d) First M1: Other area methods must be fully correct. Second M1: Adding answer to (b) to their ΔACD.</p> <p><u>Failure to round to 2 d.p:</u> Penalise only once, on the first occurrence, then accept awrt.</p> <p><u>If 0.65 is taken as degrees throughout:</u> Only award marks in part (d).</p>	

Question number	Scheme	Marks
9.	<p>(a) $ar = 4$, $\frac{a}{1-r} = 25$ (These can be seen elsewhere)</p> <p>$a = 25(1-r)$ $25r(1-r) = 4$ M: Eliminate a</p> <p>$25r^2 - 25r + 4 = 0$ (*)</p> <p>(b) $(5r-1)(5r-4) = 0$ $r = \dots$, $\frac{1}{5}$ or $\frac{4}{5}$</p> <p>(c) $r = \dots \Rightarrow a = \dots$, 20 or 5</p> <p>(d) $S_n = \frac{a(1-r^n)}{1-r}$, but $\frac{a}{1-r} = 25$, so $S_n = 25(1-r^n)$ (*)</p> <p>(e) $25(1-0.8^n) > 24$ and proceed to $n = \dots$ (or $>$, or $<$) with no unsound algebra.</p> <p>$\left(n > \frac{\log 0.04}{\log 0.8} (= 14.425\dots) \right)$ $n = 15$</p>	<p>B1, B1</p> <p>M1</p> <p>A1cso (4)</p> <p>M1, A1 (2)</p> <p>M1, A1 (2)</p> <p>B1 (1)</p> <p>M1</p> <p>A1 (2)</p> <p>11</p>
	<p>(a) The M mark is not dependent, but both expressions must contain both a and r.</p> <p>(b) <u>Special case</u>: One correct r value given, with no method (or perhaps trial and error): B1 B0.</p> <p>(c) M1: Substitute one r value back to find a value of a.</p> <p>(d) Sufficient here to verify with just one pair of values of a and r.</p> <p>(e) Accept “=” rather than inequalities throughout, and also allow the <u>wrong</u> inequality to be used at any stage. M1 requires use of <u>their</u> larger value of r. A correct answer with no working scores both marks. For “trial and error” methods, to score M1, a value of n between 12 and 18 (inclusive) must be tried.</p>	

Question number	Scheme	Marks
10.	<p>(a) $\frac{dy}{dx} = 3x^2 - 16x + 20$ $3x^2 - 16x + 20 = 0 \quad (3x - 10)(x - 2) = 0 \quad x = \dots, \quad \frac{10}{3} \text{ and } 2$</p> <p>(b) $\frac{d^2y}{dx^2} = 6x - 16 \quad \text{At } x = 2, \frac{d^2y}{dx^2} = \dots$ $-4 \quad (\text{or } < 0, \text{ or both}), \text{ therefore maximum}$</p> <p>(c) $\int (x^3 - 8x^2 + 20x) dx = \frac{x^4}{4} - \frac{8x^3}{3} + \frac{20x^2}{2} (+C)$</p> <p>(d) $4 - \frac{64}{3} + 40 \quad \left(= \frac{68}{3} \right)$ $A: x = 2: \quad y = 8 - 32 + 40 = 16 \quad (\text{May be scored elsewhere})$ $\text{Area of } \Delta = \frac{1}{2} \left(\frac{10}{3} - 2 \right) \times 16 \quad \left(\frac{1}{2} (x_B - x_A) \times y_A \right) \quad \left(= \frac{32}{3} \right)$ $\text{Shaded area} = \frac{68}{3} + \frac{32}{3} = \frac{100}{3} \quad \left(= 33 \frac{1}{3} \right)$</p>	<p>M1 A1 dM1, A1 (4) M1 A1ft (2) M1 A1 A1 (3) M1 B1 M1 M1 A1 (5) 14</p>
	<p>(a) The second M is dependent on the first, and requires an attempt to solve a 3 term quadratic.</p> <p>(b) M1: Attempt second differentiation and substitution of one of the x values. A1ft: Requires correct second derivative and negative value of the second derivative, but ft from their x value.</p> <p>(c) All 3 terms correct: M1 A1 A1, Two terms correct: M1 A1 A0, One power correct: M1 A0 A0.</p> <p>(d) Limits M1: Substituting their lower x value into a 'changed' expression.</p> <p>Area of triangle M1: Fully correct method. Alternative for the triangle (finding an equation for the straight line then integrating) requires a fully correct method to score the M mark.</p> <p>Final M1: Fully correct method (beware valid alternatives!)</p>	

GENERAL PRINCIPLES FOR C2 MARKING

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm p)^2 \pm q \pm c$, $p \neq 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but will be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may gain no credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Misreads

(See the next sheet for a simple example).

A misread must be consistent for the whole question to be interpreted as such.

These are not common. In clear cases, please deduct the first 2 A (or B) marks which would have been lost by following the scheme. (Note that 2 marks is the maximum misread penalty, but that misreads which alter the nature or difficulty of the question cannot be treated so generously and it will usually be necessary here to follow the scheme as written).

Sometimes following the scheme as written is more generous to the candidate than applying the misread rule, so in this case use the scheme as written.

MISREADS

Question 1. $(2 + x)^6$ misread as $(2 + x)^8$

1.	$(2 + x)^8 = 256 \dots$	B0
	$+ (8 \times 2^7 \times x) + \left(\frac{8 \times 7}{2} \times 2^6 \times x^2 \right),$	$+ 1024x, + 1792x^2$ M1, A0, A1
