Mark Scheme 4724
June 2006

1		$\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$	**B1**		s.o.i. e.g. $2x\frac{dy}{dx} + y$
		$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$	**B1**		
		Substitute (1,2) into their differentiated equation and attempt to solve for $\frac{dy}{dx}$. [Allow subst of (2,1)]	**M1** dep at least 1 x **B1**		Or attempt to solve their diff equation for $\frac{dy}{dx}$ and then substitute (1,2)
		$\frac{dy}{dx} = -2$	**A1**	**4**	
2	**(i)**	$1 + (-2)(-3x) + \frac{(-2)(-3)}{1.2}(-3x)^2$ (+ … ignore)	**M1**		State or imply; accept $-3x^2$ & $-9x^2$
		$= 1 + 6x$	**B1**		Correct first 2 terms
		… $+27x^2$	**A1**	**3**	Correct third term
	(ii)	$(1+2x)^2(1-3x)^{-2}$	**M1**		For changing into suitable form, seen/implied
		Attempt to expand $(1+2x)^2$ & select (at least) 2 relevant products and add	**M1**		Selection may be after multiplying out
		55 (Accept $55x^2$)	**A2√**	**4**	If (i) is $a + bx + cx^2$, f.t. $4(a+b) + c$
		<u>SR 1</u> For expansion of $(1+2x)^2$ with 1 error, A1√			
		<u>SR 2</u> For expansion of $(1+2x)^2$ & > 1 error, A0			
		<u>Alternative Method</u>			
		For correct method idea of long division	**M1**		
		1 ….. $+10x$ ….. $+55x^2$	**A1,A1,A1(4)**		
3	**(i)**	$\frac{A}{x} + \frac{B}{3-x}$ & c-u rule or $A(3-x) + Bx \equiv 3 - 2x$	**M1**		Correct format + suitable method
		$\frac{1}{x}$	**A1**		seen in (i) or (ii)
		$-\frac{1}{3-x}$	**A1**	**3**	ditto; $\frac{1}{x} - \frac{1}{3-x}$ scores 3 immediately
	(ii)	$\int \frac{1}{x}(dx) = \ln x$ or $\ln\|x\|$	**B1**		
		$\int \frac{1}{3-x}(dx) = -\ln(3-x)$ or $-\ln\|3-x\|$	**B1**		Check sign carefully; do not allow $\ln(x-3)$
		Correct method idea of substitution of limits	**M1**		Dep on an attempt at integrating
		ln 2 (+ ln 1 − ln 1) − ln 2 = 0	**A1**	**4**	Clearly seen; WWW **AG**
		<u>Alternative Method</u>			
		If ignoring PFs, $\ln x(3-x)$ immediately	**B2**		$\ln x(x-3) \to 0$
		As before	**M1,A1**	**(4)**	
	(iii)	Suitable statement or clear implication e.g. Equal amounts (of area) above and below (axis) or graph crosses axis or there's a root (Be lenient)	**B1**	**1**	

4	**(i)** Working out **b – a** or **a – b** or **c – a** or **a – c**	**M1**	)		Irrespective of label
	= ± (– 3**i** – **j** – **k**) or ± (– 2**i** + **j** – 2**k**)	**A1**	)		If not scored ,these 1st 3 marks can be
	Method for finding magnitude of <u>any</u> vector	**M1**	)		awarded in part (ii)
	Method for finding scalar product of <u>any</u> 2 vectors	**M1**			
	Using $\cos\theta = \frac{a.b}{\lvert a\rvert\lvert b\rvert}$ AEF for <u>any</u> 2 vectors	**M1**			
	<u>[Alternative cosine rule method</u> $\lvert\overrightarrow{BC}\rvert = \sqrt{6}$	**B1**			
	Cosine rule used	**M1**			'Recognisable' form
	$45.3°, 0.79(0), \frac{\pi}{3.97}$ (45.289378, 0.7904487)	**A1**		**6**	Do not accept supplement (134.7 etc)
	(ii) Use of $\frac{1}{2}\lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{AC}\rvert\sin\theta$	**M1**			Accept $\lvert\frac{1}{2}\overrightarrow{AB} \times \overrightarrow{AC}\rvert$
	3.54 (3.5355) or $\frac{5\sqrt{2}}{2}$	**A1**		**2**	Accept from correct supp (134.7 etc)
5	**(i)** $\frac{dA}{dt}$ or kA^2 seen	**M1**			
	$\frac{dA}{dt} = kA^2$	**A1**		**2**	
	(ii) Separate variables + attempt to integrate	***M1**			Accept if based on $\frac{dA}{dt} = kA^2$ or A^2
	$-\frac{1}{A} = kt + c$ or $-\frac{1}{kA} = t + c$ or $-\frac{1}{A} = t + c$	**A1**			
	Subst one of (0,0),(1,1000) or (2,2000) into eqn.	**dep*M1**			Equation must contain k and/or c
	Subst another of (0,),(1,1000) or (2,2000) into eqn	**dep*M1**			This equation must contain k <u>and</u> c
	Substitute $A = 3000$ into eqn with k and c subst	**dep*M1**			
	$t = \frac{7}{3}$ ISW	**A1**		**6**	Accept 2.33, 2h 20 m
6	**(i)** Attempt to connect du and dx e.g. $\frac{du}{dx} = e^x$	**M1**			But not $du = dx$
	Use of $e^{2x} = (e^x)^2$ or $(u-1)^2$ s.o.i.	**A1**			
	Simplification to $\int \frac{u-1}{u}(du)$ WWW	**A1**		**3**	**AG**
	(ii) Change $\frac{u-1}{u}$ to $1 - \frac{1}{u}$ or use parts	**M1**			If parts, may be twice if $\int \ln x\, dx$ is involved
	$\int \frac{1}{u}\, du = \ln u$	**A1**			Seen anywhere in this part
	<u>Either</u> attempt to change limits <u>or</u> resubstitute	**M1** (indep)			Expect new limits e+1 & 2
	Show as $e + 1 - \ln(e+1) - \{2 \text{ or } (1+1)\} + \ln 2$	**A1**			
	WWW show final result as $e - 1 - \ln\left(\frac{e+1}{2}\right)$	**A1**		**5**	**AG**

7	**(i)**	Produce at least 2 of the 3 relevant eqns in λ and μ	**M1**		e.g. $1 + 3\lambda = -8 + \mu$, $-2 + \lambda = 2 - 2\mu$
		Solve the 2 eqns in λ & μ as far as $\lambda = \ldots$ or $\mu = \ldots$	**M1**		
		1st solution: $\lambda = -2$ or $\mu = 3$	**A1**		
		2nd solution: $\mu = 3$ or $\lambda = -2$ f.t.	**A1√**		
		Substitute their λ and μ into 3rd eqn and find 'a'	**M1**		
		Obtain $a = 2$ <u>& clearly state that *a* cannot be 2</u>	**A1**	**6**	
	(ii)	Subst their λ or μ (& poss a) into either line eqn	**M1**		
		Point of intersection is $-5\mathbf{i} - 4\mathbf{j}$	**A1**	**2**	Accept any format <u>No f.t. here</u>
		N.B. In this question, award marks irrespective of labelling of parts			
8	**(i)**	<u>Integration method</u>			
		Attempt to change $\cos^2 6x$ into $\mathrm{f}(\cos 12x)$	**M1**		
		$\cos^2 6x = \frac{1}{2}(1 + \cos 12x)$	**A1**		with $\cos^2 6x$ as the subject of the formula
		$\int = \frac{1}{2}x + \frac{1}{24}\sin 12x + \mathrm{c}$	**A1**		**AG** Accept $\frac{1}{2}\left(x + \frac{1}{12}\sin 12x\right)$
		<u>Differentiation method</u>			
		Differentiate RHS producing $\frac{1}{2} + \frac{1}{2}\cos 12x$ ---(E)	**B1**		
		Attempt to change $\cos 12x$ into $\mathrm{f}(\cos 6x)$	**M1**		Accept $+/-2\cos^2 6x +/- 1$
		Simplify (E) WWW to $\cos^2 6x$ + satis finish	**A1**	**3**	

	(ii)	Parts with $u = x, \mathrm{d}v = \cos^2 6x$	***M1**		
		$x\left(\frac{1}{2}x + \frac{1}{24}\sin 12x\right) - \int\left(\frac{1}{2}x + \frac{1}{24}\sin 12x\right)\mathrm{d}x$	**A1**		Correct expression only
		$\int \sin 12x\,\mathrm{d}x = -\frac{1}{12}\cos 12x$	**B1**		Clear indication somewhere in this part
		Correct use of limits to <u>whole</u> integral	**dep*M1**		Accept ( ) (-0)
		$\frac{\pi^2}{288} - \frac{\pi^2}{576} - \frac{1}{288} - \frac{1}{288}$	**A1**		AE unsimp exp. Accept 12x24,sin π here
		$\frac{\pi^2}{576} - \frac{1}{144}$	**+A1**	**6**	Tolerate e.g. $\frac{2}{288}$ here
		S.R. If final marks are A0 + A0, allow SR A1 for			0.01/0.010/0.0101/0.0102/0.0101902

9	(i)	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	M1		Used, not just quoted
		$\frac{dx}{dt} = -4\sin t$ or $\frac{dy}{dt} = 3\cos t$	*B1		
		$\frac{dy}{dx} = -\frac{3\cos t}{4\sin t}$ or $\frac{3\cos t}{-4\sin t}$ ISW	dep*A1	3	Also $\frac{-3\cos t}{4\sin t}$ provided B0 not awarded
		SR: M1 for Cartesian eqn attempt + B1 for $\frac{d}{dx}\left(y^2\right) = 2y\frac{dy}{dx}$ **+ A1** as before(must be in terms of *t*)			
	(ii)	$y - 3\sin p = \left(\text{their } \frac{dy}{dx}\right)(x - 4\cos p)$	M1		Accept *p* or *t* here
		or $y = \left(\text{their } \frac{dy}{dx}\right)x + c$ & subst cords to find c			Ditto
		$4y\sin p - 12\sin^2 p = -3x\cos p + 12\cos^2 p$	A1		Correct equation cleared of fractions
		or c = $\frac{12\sin^2 p + 12\cos^2 p}{4\sin p}$			
		$3x\cos p + 4y\sin p = 12$ WWW	A1	3	**AG** Only *p* here. Mixture earlier $\rightarrow$ A0
	(iii)	Subst *x* = 0 and *y* = 0 separately in tangent eqn	M1		to find *R* & *S*
		Produce $\frac{3}{\sin p}$ and $\frac{4}{\cos p}$	A1		Accept $\frac{12}{4\sin p}$ and/or $\frac{12}{3\cos p}$
		Use $\Delta = \frac{1}{2}\left(\frac{3}{\sin p}.\frac{4}{\cos p}\right) = \frac{12}{\sin 2p}$ WWW	A1	3	**AG**
	(iv)	Least area = 12	B1		
		$p = \frac{1}{4}\pi$ as final or only answer	B2	3	These B marks are independent.
		S.R. $45^\circ \rightarrow$ B1 ;			S.R. [-12 and e.g. $-\pi/4 \rightarrow$ B1]