

3.

Figure 1

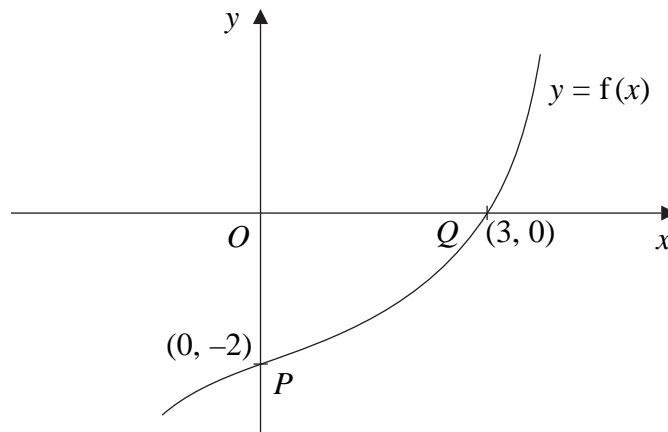


Figure 1 shows part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$, where f is an increasing function of x . The curve passes through the points $P(0, -2)$ and $Q(3, 0)$ as shown.

In separate diagrams, sketch the curve with equation

(a) $y = |f(x)|$, (3)

(b) $y = f^{-1}(x)$, (3)

(c) $y = \frac{1}{2} f(3x)$. (3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.



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Question 3 continued



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4. A heated metal ball is dropped into a liquid. As the ball cools, its temperature, $T^\circ\text{C}$, t minutes after it enters the liquid, is given by

$$T = 400 e^{-0.05t} + 25, \quad t \geq 0.$$

- (a) Find the temperature of the ball as it enters the liquid. (1)

- (b) Find the value of t for which $T = 300$, giving your answer to 3 significant figures. (4)

- (c) Find the rate at which the temperature of the ball is decreasing at the instant when $t = 50$. Give your answer in $^\circ\text{C}$ per minute to 3 significant figures. (3)

- (d) From the equation for temperature T in terms of t , given above, explain why the temperature of the ball can never fall to 20°C . (1)



5.

Figure 2

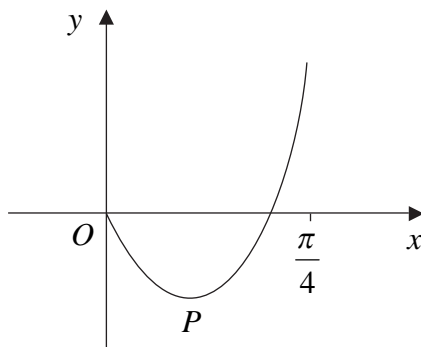


Figure 2 shows part of the curve with equation

$$y = (2x - 1) \tan 2x, \quad 0 \leq x < \frac{\pi}{4}.$$

The curve has a minimum at the point P . The x -coordinate of P is k .

(a) Show that k satisfies the equation

$$4k + \sin 4k - 2 = 0.$$

(6)

The iterative formula

$$x_{n+1} = \frac{1}{4}(2 - \sin 4x_n), \quad x_0 = 0.3,$$

is used to find an approximate value for k .

(b) Calculate the values of x_1 , x_2 , x_3 and x_4 , giving your answers to 4 decimal places.

(3)

(c) Show that $k = 0.277$, correct to 3 significant figures.

(2)



7. For the constant k , where $k > 1$, the functions f and g are defined by

$$\begin{aligned} f: x &\mapsto \ln(x+k), & x > -k, \\ g: x &\mapsto |2x-k|, & x \in \mathbb{R}. \end{aligned}$$

(a) On separate axes, sketch the graph of f and the graph of g .

On each sketch state, in terms of k , the coordinates of points where the graph meets the coordinate axes.

(5)

(b) Write down the range of f .

(1)

(c) Find $fg\left(\frac{k}{4}\right)$ in terms of k , giving your answer in its simplest form.

(2)

The curve C has equation $y = f(x)$. The tangent to C at the point with x -coordinate 3 is parallel to the line with equation $9y = 2x + 1$.

(d) Find the value of k .

(4)



