Mark Scheme (Results)
Summer 2007

GCE Mathematics

Core Mathematics C3 (6665)

Question Number	Scheme	Marks
1. (a)	$\ln 3x = \ln 6$ or $\ln x = \ln \left(\dfrac{6}{3} \right)$ [implied by 0.69…] or $\ln \left(\dfrac{3x}{6} \right) = 0$	M1
	$x = 2$ (only this answer)	A1 (cso) (2)
(b)	$(e^x)^2 - 4e^x + 3 = 0$ (any 3 term form)	M1
	$(e^x - 3)(e^x - 1) = 0$	
	$e^x = 3$ or $e^x = 1$ Solving quadratic	M1 dep
	$x = \ln 3$, $x = 0$ (or ln 1)	M1 A1 (4)
		(6 marks)

Notes: (a) Answer $x = 2$ with no working or no incorrect working seen: M1A1

Beware $x = 2$ from $\ln x = \dfrac{\ln 6}{\ln 3} = \ln 2$ M0A0

$\ln x = \ln 6 - \ln 3 \Rightarrow x = e^{(\ln 6 - \ln 3)}$ allow M1, $x = 2$ (no wrong working) A1

(b) 1st M1 for attempting to multiply through by e^x: Allow y, X, even x, for e^x

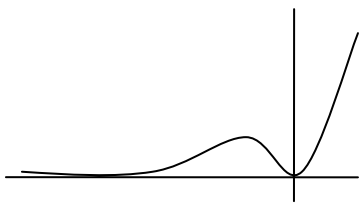Be generous for M1   e.g $e^{2x} + 3 = 4$, $e^{x^2} + 3 = 4e^x$,

$$3 y^2 + 1 = 12y \text{ (from } 3 e^{-x} = \frac{1}{3e^x} \text{), } e^x + 3 = 4e^x$$

2nd M1 is for solving quadratic (may be by formula or completing the square)
   as far as getting two values for e^x or y or X etc
3rd M1 is for converting their answer(s) of the form $e^x = k$ to $x = \ln k$ (must be exact)
   A1 is for ln3 **and** ln1 or 0 (Both required and no further solutions)

2.	(a)	$2x^2 + 3x - 2 = (2x-1)(x+2)$ at any stage	B1
		$f(x) = \dfrac{(2x+3)(2x-1)-(9+2x)}{(2x-1)(x+2)}$ f.t. on error in denominator factors	M1, A1√
		(need not be single fraction)	
		Simplifying numerator to quadratic form $\left[\ = \dfrac{4x^2+4x-3-9-2x}{(2x-1)(x+2)}\ \right]$	M1
		Correct **numerator** $= \dfrac{4x^2+2x-12}{[(2x-1)(x+2)]}$	A1
		Factorising numerator, **with** a denominator $= \dfrac{2(2x-3)(x+2)}{(2x-1)(x+2)}$ o.e.	M1
		$\left[\ = \dfrac{2(2x-3)}{2x-1}\ \right]$ $= \dfrac{4x-6}{2x-1}$ (✱)	A1 cso (7)
Alt.(a)		$2x^2 + 3x - 2 = (2x-1)(x+2)$ at any stage B1	
		$f(x) = \dfrac{(2x+3)(2x^2+3x-2)-(9+2x)(x+2)}{(x+2)(2x^2+3x-2)}$ M1A1 f.t.	
		$= \dfrac{4x^3+10x^2-8x-24}{(x+2)(2x^2+3x-2)}$	
		$= \dfrac{2(x+2)(2x^2+x-6)}{(x+2)(2x^2+3x-2)}$ or $\dfrac{2(2x-3)(x^2+4x+4)}{(x+2)(2x^2+3x+2)}$ o.e.	
		Any one linear factor $\times$ quadratic factor in **numerator** M1, A1	
		$= \dfrac{2(x+2)(x+2)(2x-3)}{(x+2)(2x^2+3x-2)}$ o.e. M1	
		$= \dfrac{2(2x-3)}{2x-1}$ $\dfrac{4x-6}{2x-1}$ (✱) A1	
	(b)	Complete method for $f'(x)$; e.g $f'(x) = \dfrac{(2x-1)\times 4-(4x-6)\times 2}{(2x-1)^2}$ o.e	M1 A1
		$= \dfrac{8}{(2x-1)^2}$ or $8(2x-1)^{-2}$	A1 (3)
		Not treating f^{-1} (for f') as misread	**(10 marks)**

Notes: (a)   1st M1 in either version is for correct method

         1st A1   Allow $\dfrac{2x+3(2x-1)-(9+2x)}{(2x-1)(x+2)}$ or $\dfrac{(2x+3)(2x-1)-9+2x}{(2x-1)(x+2)}$ or $\dfrac{2x+3(2x-1)-9+2x}{(2x-1)(x+2)}$ (fractions)

         2nd M1 in (main a) is for forming 3 term quadratic in **numerator**

         3rd M1 is for factorising their quadratic (usual rules) ; factor of 2 need not be extracted

         (✱) A1 is given answer so is cso

    Alt (a)   3rd M1 is for factorising resulting quadratic

         Notice that B1 likely to be scored very late but on ePen scored first

      (b)   SC: For M allow $\pm$ given expression or one error in product rule

         Alt: Attempt at $f(x) = 2 - 4(2x-1)^{-1}$ and diff. M1; $k(2x-1)^{-2}$ A1; A1 as above

         Accept $8(4x^2 - 4x + 1)^{-2}$. Differentiating original function – mark as scheme.

Question Number	Scheme	Marks
3. (a)	$\dfrac{dy}{dx} = x^2 e^x + 2x e^x$	M1,A1,A1 (3)
(b)	If $\dfrac{dy}{dx} = 0$,　$e^x(x^2 + 2x) = 0$ 　　　　　setting $(a) = 0$	M1
	$[e^x \neq 0]$ 　　　　　$x(x+2) = 0$	
	$(x = 0)$　　or　　$x = -2$	A1
	$x = 0, y = 0$　**and**　$x = -2, y = 4e^{-2}$ $(= 0.54\ldots)$	A1 √ 　　(3)
(c)	$\dfrac{d^2 y}{dx^2} = x^2 e^x + 2x e^x + 2x e^x + 2e^x$ 　　$\left[= (x^2 + 4x + 2)e^x \right]$	M1, A1 　(2)
(d)	$x = 0,\ \dfrac{d^2 y}{dx^2} > 0$ (=2)　　　　$x = -2,\ \dfrac{d^2 y}{dx^2} < 0$ $[= -2e^{-2}\ (= -0.270\ldots)]$	M1
	M1: Evaluate, or state sign of, candidate's (c) for at least one of candidate's x value(s) from (b)	
	$\therefore$ minimum 　　　　　　　　　　　$\therefore$ maximum	A1 (cso)　(2)
Alt.(d)	For M1: Evaluate, or state sign of, $\dfrac{dy}{dx}$ at two appropriate values – on either side of at least one of their answers from (b)　　or Evaluate y at two appropriate values – on either side of at least one of their answers from (b)　　or Sketch curve	
		(10 marks)

Notes: (a) Generous M for attempt at $f(x)g'(x) + f'(x)g(x)$
　　　1st A1 for one correct, 2nd A1 for the other correct.
　　　Note that $x^2 e^x$ on its own scores no marks
　(b) 1st A1 $(x = 0)$ may be omitted, but for
　　2nd A1 both sets of coordinates needed ; f.t only on candidate's $x = -2$
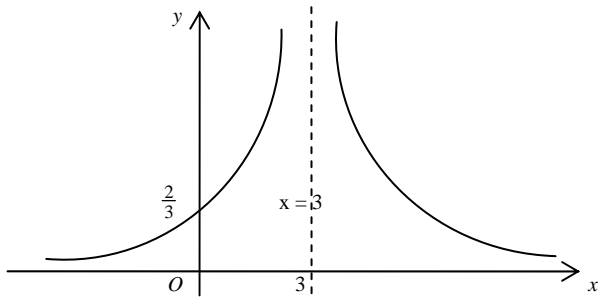
　(c) M1 requires complete method for candidate's (a), result may be unsimplified for A1

　(d) A1 is cso; $x = 0$, min, and $x = -2$, max and no incorrect working seen.,
　　or (in alternative) sign of $\frac{dy}{dx}$ either side correct, or values of y appropriate to t.p.
　　Need only consider the quadratic, as may assume $e^x > 0$.
　　If all marks gained in (a) and (c), and correct x values, give M1A1 for correct statements with no working

Question Number	Scheme	Marks
4. (a)	$x^2(3-x) - 1 = 0$ o.e. (e.g. $x^2(-x+3) = 1$)	M1
	$x = \sqrt{\dfrac{1}{3-x}}$ (✱)	A1 (cso) (2)
	Note(✱), answer is given: need to see appropriate working and A1 is cso	
	[Reverse process: Squaring and non-fractional equation M1, form f(x) A1]	
(b)	$x_2 = 0.6455$, $x_3 = 0.6517$, $x_4 = 0.6526$	B1; B1 (2)
	1st B1 is for one correct, 2nd B1 for other two correct	
	If all three are to greater accuracy, award B0 B1	
(c)	Choose values in interval (0.6525, 0.6535) or tighter and evaluate both	M1
	f(0.6525) = –0.0005 (372… f(0.6535) = 0.002 (101…	
	At least one correct "up to bracket", i.e. -0.0005 or 0.002	A1
	Change of sign, $\therefore x = 0.653$ is a root (correct) to 3 d.p.	A1 (3)
	Requires both correct "up to bracket" and conclusion as above	**(7 marks)**
Alt (i)	Continued iterations at least as far as x_6 M1	
	$x_5 = 0.6527$, $x_6 = 0.6527$, $x_{7 =}$ … two correct to at least 4 s.f. A1	
	Conclusion : Two values correct to 4 d.p., so 0.653 is root to 3 d.p. A1	
Alt (ii)	If use g(0.6525) = 0.6527..>0.6525 and g(0.6535) = 0.6528..<0.6535 M1A1	
	Conclusion : Both results correct, so 0.653 is root to 3 d.p. A1	
5. (a)	Finding g(4) = k and f(k) = …. or fg(x) = $\ln\left(\dfrac{4}{x-3} - 1\right)$	M1
	[f(2) = $\ln(2\times2 - 1)$ fg(4) = $\ln(4 - 1)$] = $\ln 3$	A1 (2)
(b)	$y = \ln(2x - 1) \implies e^y = 2x - 1$ or $e^x = 2y - 1$	M1, A1
	$f^{-1}(x) = \frac{1}{2}(e^x + 1)$ Allow $y = \frac{1}{2}(e^x + 1)$	A1
	Domain $x \in \Re$ [Allow $\Re$, all reals, $(-\infty, \infty)$] independent	B1 (4)
(c)		
	Shape, and x-axis should appear to be asymptote	B1
	Equation $x = 3$ needed, may see in diagram (ignore others)	B1 ind.
	Intercept $(0, \frac{2}{3})$ no other; accept y = ⅔ (0.67) or on graph	B1 ind (3)
(d)	$\dfrac{2}{x-3} = 3 \implies x = 3\frac{2}{3}$ or exact equiv.	B1
	$\dfrac{2}{x-3} = -3$, $\implies x = 2\frac{1}{3}$ or exact equiv.	M1, A1 (3)
	Note: 2 = 3(x + 3) or 2 = 3(–x – 3) o.e. is M0A0	
Alt:	Squaring to quadratic ($9x^2 - 54x + 77 = 0$) **and** solving M1; B1A1	**(12 marks)**

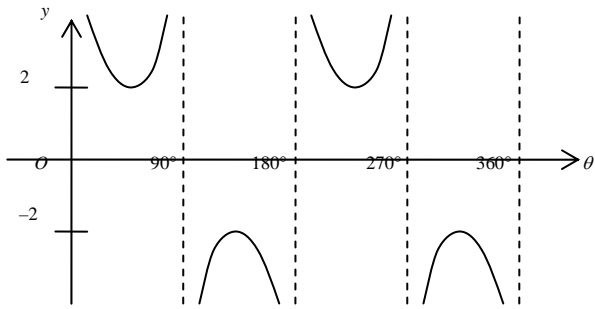| 6.	(a)	Complete method for R: e.g. $R\cos\alpha = 3$, $R\sin\alpha = 2$, $R = \sqrt{(3^2 + 2^2)}$	M1
		$\qquad\qquad R = \sqrt{13}$ or 3.61 (or more accurate)	A1
		Complete method for $\tan\alpha = \dfrac{2}{3}$ [Allow $\tan\alpha = \dfrac{3}{2}$]	M1
		$\qquad\qquad \alpha = 0.588$ (Allow $33.7°$)	A1 (4)
	(b)	Greatest value $= \left(\sqrt{13}\right)^4 = 169$	M1, A1 (2)
	(c)	$\sin(x + 0.588) = \dfrac{1}{\sqrt{13}}$ $(= 0.27735\ldots)$ $\sin(x + \text{their } \alpha) = \frac{1}{\text{their } R}$	M1
		$(x + 0.588)$ $= 0.281(03\ldots$ or $16.1°$	A1
		$(x + 0.588)$ $= \pi - 0.28103\ldots$ Must be $\pi -$ their 0.281 or $180° -$ their $16.1°$	M1
		or $(x + 0.588)$ $= 2\pi + 0.28103\ldots$ Must be $2\pi +$ their 0.281 or $360° +$ their $16.1°$	M1
		$x = 2.273$ or $x = 5.976$ (awrt) Both (radians only)	A1 (5)
		If 0.281 or 16.1° not seen, correct answers imply this A mark	**(11 marks)**

Notes: (a)   1$^{\text{st}}$ M1 on Epen for correct method for R, even if found second
              2$^{\text{nd}}$ M1 for correct method for $\tan\alpha$
              No working at all: M1A1 for √13, M1A1 for 0.588 or 33.7°.
              N.B. Rcos α = 2, Rsin α = 3 used, can still score M1A1 for R, but loses the A mark for α.
                   cosα = 3, sin α = 2: apply the same marking.

     (b)   M1 for realising $\sin(x + \alpha) = \pm 1$, so finding R^4.

     (c)   Working in mixed degrees/rads : first two marks available
          Working consistently in degrees: Possible to score first 4 marks
         [Degree answers, just for reference, Only are 130.2° and 342.4°]
        Third M1 can be gained for candidate's 0.281 – candidate's 0.588 + 2π or equiv. in degrees
        One of the answers correct in radians or degrees implies the corresponding M mark.

Alt: (c)   (i)   Squaring to form quadratic in sin x or cos x M1
                $[13\cos^2 x - 4\cos x - 8 = 0, \quad 13\sin^2 x - 6\sin x - 3 = 0]$
                Correct values for cos x = 0.953… , –0.646; or   sin x = 0.767, 2.27   awrt   A1
                For any one value of cos x or sin$x$, correct method for two values of x M1
                $x = 2.273$ or $x = 5.976$ (awrt) Both   seen anywhere   A1
                Checking other values (0.307, 4.011 or 0.869, 3.449) and discarding   M1

         (ii)   Squaring and forming equation of form $a\cos 2x + b\sin 2x = c$
                $9\sin^2 x + 4\cos^2 x + 12\sin 2x = 1 \Rightarrow 12\sin 2x + 5\cos 2x = 11$
                Setting up to solve using R formula e.g. $\sqrt{13}\cos(2x - 1.176) = 11$ M1
                          $(2x - 1.176) = \cos^{-1}\left(\dfrac{11}{\sqrt{13}}\right) = 0.562(0\ldots$ (α) A1
                          $(2x - 1.176) = 2\pi - \alpha, \; 2\pi + \alpha, \ldots\ldots$ M1
                $x = 2.273$ or $x = 5.976$ (awrt) Both   seen anywhere   A1
                Checking other values and discarding   M1

Question Number	Scheme	Marks
7. (a)	$\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}=\dfrac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}$	M1
	M1 Use of common denominator to obtain single fraction	
	$=\dfrac{1}{\cos\theta\sin\theta}$	M1
	M1 Use of appropriate trig identity (in this case $\sin^2\theta+\cos^2\theta=1$)	
	$=\dfrac{1}{\frac12\sin2\theta}$ $\qquad$ Use of $\sin2\theta=2\sin\theta\cos\theta$	M1
	$=2\mathrm{cosec}2\theta$ (✱)	A1 cso (4)
Alt.(a)	$\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}=\tan\theta+\dfrac{1}{\tan\theta}=\dfrac{\tan^2\theta+1}{\tan\theta}$ $\qquad$ M1	
	$=\dfrac{\sec^2\theta}{\tan\theta}$ $\qquad$ M1	
	$=\dfrac{1}{\cos\theta\sin\theta}=\dfrac{1}{\frac12\sin2\theta}$ $\qquad$ M1	
	$=2\mathrm{cosec}2\theta$ (✱) (cso) $\qquad$ A1	
	If show two expressions are equal, need conclusion such as QED, tick, true.	
(b)	Shape (May be translated but need to see 4"sections")	B1
	T.P.s at $y=\pm2$, asymptotic at correct x-values (dotted lines not required)	B1 dep. (2)
(c)	$2\mathrm{cosec}2\theta=3$	
	$\sin2\theta=\dfrac{2}{3}$ $\qquad$ Allow $\dfrac{2}{\sin2\theta}=3$ $\qquad$ [M1 for equation in $\sin2\theta$]	M1, A1
	$(2\theta)=[\,41.810\ldots°,\,138.189\ldots°\,;\qquad401.810\ldots°,\,498.189\ldots°]$ 1st M1 for $\alpha,\,180-\alpha$; 2nd M1 adding 360° to at least one of values	M1; M1
	$\theta=20.9°,\,69.1°,\,200.9°,\,249.1°$ (1 d.p.) $\qquad\qquad$ awrt	
Note	1st A1 for any two correct, 2nd A1 for other two Extra solutions in range lose final A1 only **SC: Final 4 marks: $\theta=20.9°$, after M0M0 is B1; record as M0M0A1A0**	A1,A1 (6)
Alt.(c)	$\tan\theta+\dfrac{1}{\tan\theta}=3$ and form quadratic , $\tan^2\theta-3\tan\theta+1=0$ $\qquad$ M1, A1	
	(M1 for attempt to multiply through by tanθ, A1 for correct equation above)	
	Solving quadratic $\quad[\tan\theta=\dfrac{3\pm\sqrt5}{2}\quad=2.618\ldots$ or $=0.3819\ldots]$ $\qquad$ M1	
	$\theta=69.1°,\,249.1°$ $\qquad\qquad\theta=20.9°,\,200.9°$ (1 d.p.) M1, A1, A1 (M1 is for one use of $180°+\alpha°$, A1A1 as for main scheme)	**(12 marks)**

Question Number	Scheme	Marks
8. (*a*)	$D = 10$, $t = 5$, $\quad x = 10\mathrm{e}^{-\frac{1}{8}\times 5}$ $\qquad\qquad = 5.353 \qquad\qquad\qquad\qquad$ awrt	M1 A1 $\quad$ (2)
(*b*)	$D = 10 + 10\mathrm{e}^{-\frac{5}{8}}$, $t = 1$, $\quad x = 15.3526\ldots\times\mathrm{e}^{-\frac{1}{8}}$ $\qquad\qquad\qquad x = 13.549 \qquad$ (✱)	M1 A1 $\;$ cso $\;$ (2)
Alt.(*b*)	$x = 10\mathrm{e}^{-\frac{1}{8}\times 6} + 10\mathrm{e}^{-\frac{1}{8}\times 1}$ $\;$ M1 $\qquad\quad x = 13.549$ $\;$ (✱) $\qquad$ A1 cso	
(*c*)	$15.3526\ldots\mathrm{e}^{-\frac{1}{8}T} = 3$ $\mathrm{e}^{-\frac{1}{8}T} = \dfrac{3}{15.3526\ldots} = 0.1954\ldots$ $-\dfrac{1}{8}T = \ln 0.1954\ldots$ $\qquad T = 13.06\ldots$ or 13.1 or 13	M1 M1 A1 $\quad$ (3)
		(7 marks)

Notes: (b) (main scheme) M1 is for $(10 + 10\mathrm{e}^{-\frac{5}{8}})\mathrm{e}^{-\frac{1}{8}}$, or $\{10 + \text{their(a)}\}\mathrm{e}^{-(1/8)}$

$\qquad$ **N.B.** The answer is given. There are many correct answers seen which deserve M0A0

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ or $\quad$ M1A0

$\quad$ (If adding two values, these should be 4.724 and 8.825)

$\quad$ (c) $\;$ 1$^{\text{st}}$ M is for $(10 + 10\mathrm{e}^{-\frac{5}{8}})\; e^{-\frac{T}{8}} = 3$

$\qquad$ 2$^{\text{nd}}$ M is for converting $e^{-\frac{T}{8}} = k$ (k > 0) to $-\dfrac{T}{8} = \ln k$. This is independent of 1$^{\text{st}}$ M.

$\qquad$ Trial and improvement: M1 as scheme,
$\qquad\qquad\qquad\qquad\qquad\qquad$ M1 correct process for their equation (two equal to 3 s.f.)
$\qquad\qquad\qquad\qquad\qquad\qquad$ A1 as scheme