

**Mark Scheme 4724
June 2007**

1	(i) Correct format $\frac{A}{x+2} + \frac{B}{x-3}$ $A = 1$ and $B = 2$ (ii) $-A(x+2)^{-2} - B(x-3)^{-2}$ f.t. Convincing statement that each denom > 0 State whole exp < 0 AG	M1 A1 2 $\sqrt{A1}$ B1 B1 3	s.o.i. in answer for both accept ≥ 0 . Do not accept $x^2 > 0$. <u>Dep on previous 4 marks.</u>	5
2	Use parts with $u = x^2, dv = e^x$ Obtain $x^2 e^x - \int 2xe^x (dx)$ Attempt parts again with $u = (-)(2)x, dv = e^x$ Final = $(x^2 - 2x + 2)e^x$ AEF incl brackets Use limits correctly throughout $e^{(1)} - 2$ ISW Exact answer only	*M1 A1 M1 A1 dep*M1 A1 6	obtaining a result $f(x) + /- \int g(x)(dx)$ s.o.i. eg $e + (-2x + 2)e^x$ Tolerate (their value for $x = 1$) (-0) Allow 0.718 \rightarrow M1	6
3	Volume = $(k) \int_0^{\pi} \sin^2 x (dx)$ Suitable method for integrating $\sin^2 x$ $\int \sin^2 x (dx) = \frac{1}{2} \int 1 - \cos 2x (dx)$ $\int \cos 2x (dx) = \frac{1}{2} \sin 2x$ Use limits correctly Volume = $\frac{1}{2} \pi^2$ WWW Exact answer	B1 *M1 A1 A1 dep*M1 A1 6	where $k = \pi, 2\pi$ or 1; limits necessary eg $\int + /- 1 + /- \cos 2x (dx)$ or single integ by parts & connect to $\int \sin^2 x (dx)$ or $-\sin x \cos x + \int \cos^2 x (dx)$ or $-\sin x \cos x + \int 1 - \sin^2 x (dx)$ Beware: wrong working leading to $\frac{1}{2} \pi^2$	6
4	(i) $(1 + \frac{x}{2})^{-2}$ $= 1 + (-2)(\frac{x}{2}) + \frac{-2 \cdot -3}{2} (\frac{x}{2})^2 + \frac{-2 \cdot -3 \cdot -4}{3!} (\frac{x}{2})^3$ $= 1 - x$ $+ \frac{3}{4} x^2 - \frac{1}{2} x^3$ $(2+x)^{-2} = \frac{1}{4} (\text{their exp of } (1+ax)^{-2})$ mult out $ x < 2$ or $-2 < x < 2$ (but not $ \frac{1}{2}x < 1$) (ii) If (i) is $a + bx + cx^2 + dx^3$ evaluate $b + d$ $-\frac{3}{8} (x^3)$	M1 B1 A1 $\sqrt{B1}$ B1 5 M1 $\sqrt{A1}$ 2	Clear indication of method of ≥ 3 terms First two terms, not dependent on M1 For both third and fourth terms Correct: $\frac{1}{4} - \frac{1}{4}x + \frac{3}{16}x^2 - \frac{1}{8}x^3$ Follow-through from $b + d$	7

<p>5(i) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ $= \frac{-4 \sin 2t}{-\sin t}$ $= 8 \cos t$ ≤ 8 AG</p> <p>(ii) Use $\cos 2t = 2 \cos^2 t + / - 1$ or $1 - 2 \cos^2 t$ Use correct version $\cos 2t = 2 \cos^2 t - 1$ Produce WWW $y = 4x^2 + 1$ AG</p> <p>(iii) U-shaped parabola above x-axis, sym abt y-axis Portion between $(-1, 5)$ and $(1, 5)$ N.B. If (ii) answered or quoted before (i) attempted, allow in part</p>	<p>M1 A1 A1 A1 M1 A1 A1 B1 B1</p>	<p>Accept $\frac{4 \sin 2t}{\sin t}$ WWW 4 with brief explanation eg $\cos t \leq 1$ If starting with $y = 4x^2 + 1$, then Subst $x = \cos t, y = 3 + 2 \cos 2t$ M1 3 Either substitute a formula for $\cos 2t$ M1 Obtain $0=0$ or $4\cos^2 t + 1 = 4\cos^2 t + 1$ A1 Or Manip to give formula for $\cos 2t$ M1 Obtain corr formula & say it's correct A1 Any labelling must be correct 2 either $x = \pm 1$ or $y = 5$ must be marked (i) B2 for $\frac{dy}{dx} = 8x$ +B1,B1 if earned. 9</p>
<p>6 (i) $\frac{d}{dx}(y^2) = 2y \frac{dy}{dx}$ Using $d(uv) = u dv + v du$ for the $(3)xy$ term $\frac{d}{dx}(x^2 + 3xy + 4y^2) = 2x + 3x \frac{dy}{dx} + 3y + 8y \frac{dy}{dx}$ Solve for $\frac{dy}{dx}$ & subst $(x, y) = (2, 3)$ $\frac{dy}{dx} = -\frac{13}{30}$ Grad normal = $\frac{30}{13}$ follow-through Find equ any line thro $(2, 3)$ with any num grad $30x - 13y - 21 = 0$ AEF</p>	<p>B1 M1 A1 M1 A1 √B1 M1 A1</p>	<p>or v.v. Subst now or at normal eqn stage; (M1 dep on either/both B1 M1 earned) Implied if grad normal = $\frac{30}{13}$ This f.t. mark awarded only if numerical 8 No fractions in final answer 8</p>
<p>7 (i) Leading term in quotient = $2x$ Suff evidence of division or identity process Quotient = $2x + 3$ Remainder = x</p> <p>(ii) their quotient + $\frac{\text{their remainder}}{x^2 + 4}$</p> <p>(iii) Working with their expression in part (ii) their $Ax + B$ integrated as $\frac{1}{2}Ax^2 + Bx$ their $\frac{Cx}{x^2 + 4}$ integrated as $k \ln(x^2 + 4)$ $k = \frac{1}{2}C$ Limits used correctly throughout $14 + \frac{1}{2} \ln \frac{13}{5}$</p>	<p>B1 M1 A1 A1 √B1 √B1 M1 √A1 M1 A1</p>	<p>Stated or in relevant position in division 4 Accept $\frac{x}{x^2 + 4}$ as remainder 1 $2x + 3 + \frac{x}{x^2 + 4}$ Ignore any integration of $\frac{D}{x^2 + 4}$ 5 logs need not be combined. 10</p>

8	<p>(i) Sep variables eg $\int \frac{1}{6-h} (dh) = \int \frac{1}{20} (dt)$ LHS = $-\ln(6-h)$ RHS = $\frac{1}{20}t + c$ Subst $t=0, h=1$ into equation containing 'c' Correct value of their c = $-(20)\ln 5$ WWW Produce $t = 20 \ln \frac{5}{6-h}$ WWW AG</p> <p>(ii) When $h=2, t = 20 \ln \frac{5}{4} = 4.46(2871)$</p> <p>(iii) Solve $10 = 20 \ln \frac{5}{6-h}$ to $\frac{5}{6-h} = e^{0.5}$ $h = 2.97(2.9673467\dots)$ [In (ii),(iii) accept non-decimal (exact) answers but – 1 once.] Accept truncated values in (ii),(iii).</p> <p>(iv) Any indication of (approximately) 6 (m)</p>	<p>*M1 A1 A1 dep*M1 A1 A1 A1 B1 M1 A1 B1</p> <p>6 1 2 1</p>	<p>s.o.i. Or $\frac{dt}{dh} = \frac{20}{6-h} \rightarrow M1$ & then $t = -20 \ln(6-h) + c \rightarrow A1+A1$ or $(20)\ln 5$ if on LHS Must see $\ln 5 - \ln(6-h)$ Accept 4.5, $4\frac{1}{2}$ or $\frac{6-h}{5} = e^{-0.5}$ or suitable $\frac{1}{2}$-way stage $6 - 5e^{-0.5}$ or $6 - e^{1.109}$</p>
10			
9	<p>(i) Use $-6\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}$ and $\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ only Correct method for scalar product Correct method for magnitude 68 or 68.5 (68.47546); 1.2(0) (1.1951222) rad [N.B. 61 (60.562) will probably have been generated by $5\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ and $3\mathbf{i} - 8\mathbf{j}$]</p> <p>(ii) Indication that relevant vectors are parallel $c = -4$</p> <p>(iii) Produce 2/3 equations containing t, u (& c) Solve the 2 equations not containing 'c' $t = 2, u = 1$ Subst their (t, u) into equation containing c $c = -3$ <u>Alternative method for final 4 marks</u> Solve two equations, one with 'c', for t and u in terms of c, and substitute into third equation $c = -3$</p>	<p>M1 M1 M1 A1 M1 A1 M1 A1 M1 A1 (M2) (A2)</p> <p>4 2 5</p>	<p>of <u>any</u> two vectors ($-6 + 24 - 4 = 14$) of <u>any</u> vector ($\sqrt{36 + 64 + 4} = \sqrt{104}$ or $\sqrt{1 + 9 + 4} = \sqrt{14}$) $-6\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}$ & $3\mathbf{i} + c\mathbf{j} + \mathbf{k}$ with some indic of method of attack eg $-6\mathbf{i} + 8\mathbf{j} - 2\mathbf{k} = \lambda(3\mathbf{i} + c\mathbf{j} + \mathbf{k})$ $c = -4$ WW \rightarrow B2 eg $3 + t = 2 + 3u, -8 + 3t = 1 + cu$ and $2t = 3 + u$</p>
11			