Mark Scheme 4736
June 2007

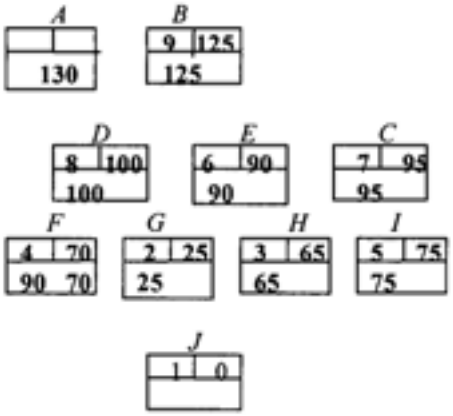
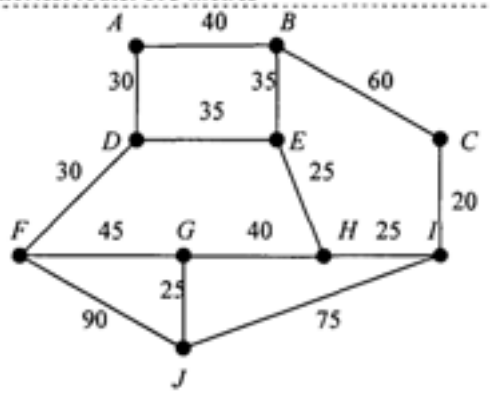
SOLUTIONS 4736 D1 June 2007 FINAL

1	(i)	Example: $N-P-Q-T-S-R-N$ or: $P-Q-S-P$	B1	1	Any valid cycle (closed and does not repeat vertices, need not be a Hamiltonian cycle)
	(ii)	It passes through Y twice	B1	1	Or, it includes a cycle (accept 'loop')
	(iii)	5	B1	1	
	(iv)	A: neither B: semi-Eulerian	B1 B1	2	If graphs are not specified, assume A is first
	(v)	A: 2 B: 1	B1 B1	2	If graphs are not specified, assume A is first A: 1, B: 0 $\Rightarrow$ B1 only
	(vi)	There are 4 odd nodes (N, P, S and Z) To connect these we must add 2 arcs	M1 A1	2 9	Seen or implied For 2

2	(i)	$d+f+g=120$	B1	1	For this equality. Condone an inequality
	(ii)	"(Area of) grass is not more than 4 times (area of) decking"	B1	1	Identifying the constraint in words (not just 'grass is less than or equal to 4 times decking' though)
	(iii)	$d \leq f$	B1	1	Do not accept $d < f$
	(iv)	$g \geq 40$ min $d = 10$ min $f = 20$	B1 B1 B1	3	Do not accept $g > 40$ $d \geq 10$ $f \geq 20$
	(v)	$5g + 10d + 20f$ or $g + 2d + 4f$	B1	1	Or any positive multiple of this
	(vi)	Minimise $g + 2d + 4f$ Subject to $d+f+g=120$ $g-4d+s=0$ $d-f+t=0$ $g \geq 40$, and $d \geq 10, f \geq 20, s \geq 0, t \geq 0$	M1 B1 A1	3 10	For a reasonable attempt at setting up the minimisation problem using their expressions For dealing with this slack variable correctly (variables on LHS and constant on RHS) For a completely correct formulation (accept d and $f \geq 0$, or their min values for d, f)

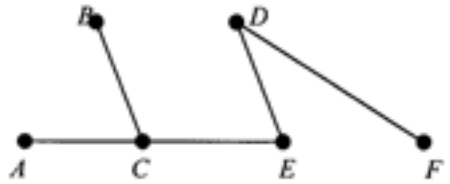
3	(i)	8 6 9 7 5 Comps Swaps After 1st pass: 6 8 9 7 5 1 1 After 2nd pass: 6 8 9 7 5 1 0 After 3rd pass: 6 7 8 9 5 3 2 After 4th pass: 5 6 7 8 9 4 4 Comparisons must be 1, 2, 3 or 4 with total ≤ 10 Swaps must be 0, 1, 2, 3 or 4 and no more than corresponding number of comparisons	 M1 M1 M1 A1 B1 B1	6	Bubble sort or decreasing order loses first 4 marks 1st pass correct 2nd pass correct, follow through from 1st pass 3rd pass correct, follow through from 2nd pass 4th pass correct Counting comparisons for at least three passes Counting swaps for at least three passes
	(ii)	Step 1 A = 8 6 9 7 5 Step 2 A = 6 9 7 5 X = 8 Step 3 A = 9 7 5 B = 6 Step 4 A = 7 5 C = 9 Step 4 A = 5 B = 6 7 Step 4 A is empty B = 6 7 5 Step 6 N = 3 Step 7 A = 6 7 5 8 9 Step 8 Display 6 7 5 8 9	 M1 M1 M1 M1 A1	5 11	 For identifying that 6 → B or the sublist {6} For identifying that 9 → C or the sublist {9} For identifying that 7 → B For identifying that 5 → B For the final A list or the display correct

4	(i)	<table><tr><td>P</td><td>x</td><td>y</td><td>s</td><td>t</td><td>u</td><td></td></tr><tr><td>1</td><td>-3</td><td>5</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>5</td><td>1</td><td>0</td><td>0</td><td>12</td></tr><tr><td>0</td><td>1</td><td>-5</td><td>0</td><td>1</td><td>0</td><td>10</td></tr><tr><td>0</td><td>3</td><td>10</td><td>0</td><td>0</td><td>1</td><td>45</td></tr></table>	B1 B1 B1 3	For correct use of three slack variable columns For $\pm$ (-3 5) in objective row For 1 5 12, 1 -5 10 and 3 10 45 in constraint rows
	(ii)	Pivot on second 1 in x column x column has a negative entry in objective row $12 \div 1 = 12$, $10 \div 1 = 10$, $45 \div 3 = 15$ Least non-negative ratio is 10 so pivot on the second 1	B1 B1 2	For correct pivot choice (cao) For 'negative in top row for x', or equivalent, and a correct explanation of choice of row 'least ratio $10 \div 1$' (ft their pivot column)
	(iii)	<table><tr><td>P</td><td>x</td><td>y</td><td>z</td><td>s</td><td>t</td><td></td></tr><tr><td>1</td><td>0</td><td>-10</td><td>0</td><td>3</td><td>0</td><td>30</td></tr><tr><td>0</td><td>0</td><td>10</td><td>1</td><td>-1</td><td>0</td><td>2</td></tr><tr><td>0</td><td>1</td><td>-5</td><td>0</td><td>1</td><td>0</td><td>10</td></tr><tr><td>0</td><td>0</td><td>25</td><td>0</td><td>-3</td><td>1</td><td>15</td></tr></table> $x = 10$, $y = 0$ $P = 30$	ft their tableau if possible for method marks M1 M1 M1 A1 B1 B1 6	 For correct method evident for objective row For a correct method evident for pivot row For a correct method evident for other rows For correct tableau CAO For correct values from their tableau For correct value from their tableau
	(iv)	$11 + 5(0.2) = 12$ or $s = 0$ $11 - 5(0.2) = 10$ or $t = 0$ $3(11) + 10(0.2) = 35$ or $u = 10$ so all the constraints are satisfied $P = 3(11) - 5(0.2) = 32$ which is bigger than 30 from (iii)	B1 B1 2 13	For showing (not just stating) that constraints are satisfied For calculating 32, or equivalent (eg $3x$ has increased by 3 but $-5y$ has only decreased by 1)

5	(i)				ANSWERED ON INSERT
				M1	For correct initial temporary labels at *F*, *G*, *I*
				M1	For correctly updating *F* and label at *H*
				A1	For all temporary labels correct (including *A*) (allow extra 100 at *C*, 105 at *D*, 75 at *H* only)
				B1	For order of becoming permanent correct
				B1	For all permanent labels correct (*A* need not have a permanent label)
		Shortest path from *J* to *B*: *J G H E B*		B1	For correct route (condone omission of *J* or *B*)
		Length of path: 125 metres		B1 7	For 125
	(ii)	Odd nodes: *B C E J*		B1	For identifying or using *B C E J* or implied
		$BC = 60$ $BE = 35$ $BJ = 125$ $EJ = 90$ $CJ = 95$ $CE = 70$ 150 130 195		M1	For any three of these weights correct, or implied or ft from their (i)
		Repeat *BE* and *CJ* (or *BE*, *JI*, *IC*)		A1	For identifying the pairing *BE*, *CJ* to repeat or 130 (not ft)
		130 + 765		M1	For 765 + their 130 (a valid pairs total)
		Shortest route: 895 metres		A1 5	For 895 (cao)
	(iii)				
				B1	For graph structure correct
				M1	For a reasonable attempt at arc weights (at least 9 correct, including the three given)
				A1	For all arc weights correct
		Travelling salesperson problem		4 B1 16	For identifying TSP by name

6	(i)			ANSWERED ON INSERT

	1	5	2	4	3	6
	A	*B*	*C*	*D*	*E*	*F*
A	-	6	3	-	-	-
B	6	-	**5**	6	-	14
C	**3**	5	-	8	4	10
D	-	6	8	-	**3**	8
E	-	-	**4**	3	-	-
F	-	14	10	**8**	-	-

	Order: *A C E D B F*	B1 For correct order, listed or marked on arrows or table, or arcs listed *AC CE ED CB DF*
	Minimum spanning tree: 	B1 For tree (correct or follow through from table, provided solution forms a spanning tree)
	Total weight: 23 miles	B1 For 23 (or follow through from table or diagram, provided solution forms a spanning tree) **6**
(ii)	MST for reduced network = 18 Two shortest arcs from *B* = 5 + 6 = 11 Lower bound = 29 miles	M1 For their 18 seen or implied M1 For 11 seen or implied A1 **3** For 29 (cao)
(iii)	*F – D – E – C – A – B – F* 8 + 3 + 4 + 3 + 6 + 14 = 38 miles	M1 For *F–D–E–C–A–B* A1 For correct tour M1 **4** For a substantially correct attempt at sum A1 **13** For 38 (cao)