RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT **4752/01**

MATHEMATICS (MEI)

Concepts for Advanced Mathematics (C2)

THURSDAY 7 JUNE 2007

Morning
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **6** printed pages and **2** blank pages.

[Turn over

Section A (36 marks)

1 **(i)** State the exact value of $\tan 300°$. [1]

(ii) Express $300°$ in radians, giving your answer in the form $k\pi$, where k is a fraction in its lowest terms. [2]

2 Given that $y = 6x^{\frac{3}{2}}$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

Show, without using a calculator, that when $x = 36$ the value of $\frac{d^2y}{dx^2}$ is $\frac{3}{4}$. [5]

3

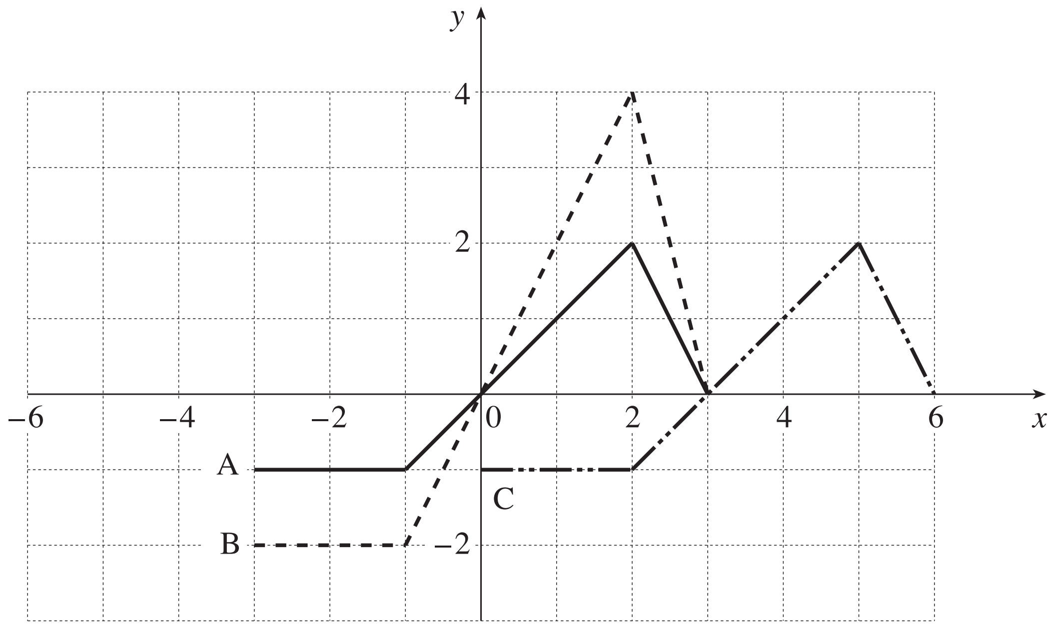

Fig. 3

Fig. 3 shows sketches of three graphs, A, B and C. The equation of graph A is $y = f(x)$.

State the equation of

(i) graph B, [2]

(ii) graph C. [2]

4 **(i)** Find the second and third terms of the sequence defined by the following.

$$t_{n+1} = 2t_n + 5$$
$$t_1 = 3$$ [2]

(ii) Find $\sum_{k=1}^{3} k(k+1)$. [2]

5 A sector of a circle of radius 5 cm has area 9 cm^2.

Find, in radians, the angle of the sector.

Find also the perimeter of the sector. [5]

6 **(i)** Write down the values of $\log_a 1$ and $\log_a a$, where $a > 1$. [2]

(ii) Show that $\log_a x^{10} - 2\log_a\left(\frac{x^3}{4}\right) = 4\log_a(2x)$. [3]

7 **(i)** Sketch the graph of $y = 3^x$. [2]

(ii) Use logarithms to solve the equation $3^x = 20$. Give your answer correct to 2 decimal places. [3]

8 **(i)** Show that the equation $2\cos^2\theta + 7\sin\theta = 5$ may be written in the form

$$2\sin^2\theta - 7\sin\theta + 3 = 0.$$ [1]

(ii) By factorising this quadratic equation, solve the equation for values of θ between 0° and 180°. [4]

Section B (36 marks)

9 The equation of a cubic curve is $y = 2x^3 - 9x^2 + 12x - 2$.

(i) Find $\frac{dy}{dx}$ and show that the tangent to the curve when $x = 3$ passes through the point $(-1, -41)$. [5]

(ii) Use calculus to find the coordinates of the turning points of the curve. You need not distinguish between the maximum and minimum. [4]

(iii) Sketch the curve, given that the only real root of $2x^3 - 9x^2 + 12x - 2 = 0$ is $x = 0.2$ correct to 1 decimal place. [3]

10 Fig. 10 shows the speed of a car, in metres per second, during one minute, measured at 10-second intervals.

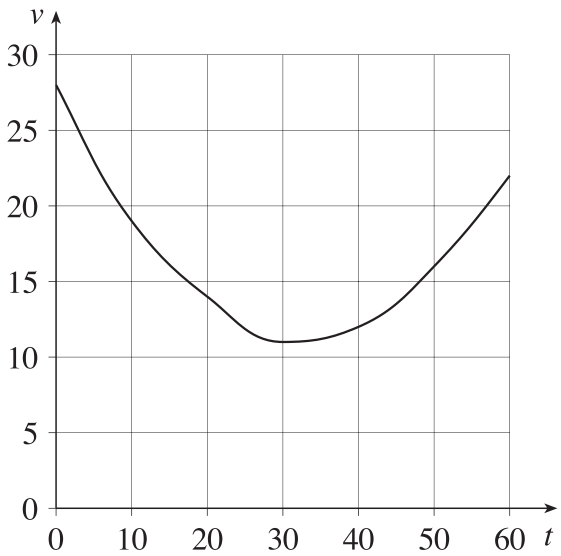

Fig. 10

The measured speeds are shown below.

Time (t seconds)	0	10	20	30	40	50	60
Speed ($v\,\mathrm{m\,s^{-1}}$)	28	19	14	11	12	16	22

(i) Use the trapezium rule with 6 strips to find an estimate of the area of the region bounded by the curve, the line $t = 60$ and the axes. [This area represents the distance travelled by the car.] [4]

(ii) Explain why your calculation in part **(i)** gives an overestimate for this area. Use appropriate rectangles to calculate an underestimate for this area. [3]

The speed of the car may be modelled by $v = 28 - t + 0.015t^2$.

(iii) Show that the difference between the value given by the model when $t = 10$ and the measured value is less than 3% of the measured value. [2]

(iv) According to this model, the distance travelled by the car is

$$\int_0^{60} (28 - t + 0.015t^2)\,dt.$$

Find this distance. [3]

11 **(a)** André is playing a game where he makes piles of counters. He puts 3 counters in the first pile. Each successive pile he makes has 2 more counters in it than the previous one.

(i) How many counters are there in his sixth pile? [1]

(ii) André makes ten piles of counters. How many counters has he used altogether? [2]

(b) In another game, played with an ordinary fair die and counters, Betty needs to throw a six to start.

The probability P_n of Betty starting on her nth throw is given by

$$P_n = \frac{1}{6} \times \left(\frac{5}{6}\right)^{n-1}.$$

(i) Calculate P_4. Give your answer as a fraction. [2]

(ii) The values $P_1, P_2, P_3, \ldots$ form an infinite geometric progression. State the first term and the common ratio of this progression.

Hence show that $P_1 + P_2 + P_3 + \ldots = 1$. [3]

(iii) Given that $P_n < 0.001$, show that n satisfies the inequality

$$n > \frac{\log_{10} 0.006}{\log_{10}\left(\frac{5}{6}\right)} + 1.$$

Hence find the least value of n for which $P_n < 0.001$. [4]