4723 Core Mathematics 3

1	Either:	Obtain $x = 0$	**B1**	ignoring errors in working
		Form linear equation with signs of $4x$ and $3x$ different	**M1**	ignoring other sign errors
		State $4x - 5 = -3x + 5$	**A1**	or equiv without brackets
		Obtain $\frac{10}{7}$ and no other non-zero value(s)	**A1**	or exact equiv
			4	
	Or:	Obtain $16x^2 - 40x + 25 = 9x^2 - 30x + 25$	**B1**	or equiv
		Attempt solution of quadratic equation	**M1**	at least as far as factorisation or use of formula
		Obtain $\frac{10}{7}$ and no other non-zero value(s)	**A1**	or exact equiv
		Obtain 0	**B1**	ignoring errors in working
			4	
2	**(i)**	Show graph indicating attempt at reflection in $y = x$	**M1**	with correct curvature and crossing negative y-axis and positive x-axis
		Show correct graph with x-coord 2 and y-coord -3 indicated	**A1**	
			2	
	(ii)	Show graph indicating attempt at reflection in x-axis	**M1**	with correct curvature and crossing each negative axis
		Show correct graph with x-coord -3 indicated	**A1**	
		… and y-coord -4 indicated	**A1**	
		[SC: Incorrect curve earning M0 but both correct intercepts indicated	B1]	
			3	
3		Attempt use of product rule	**M1**	… + … form
		Obtain $2x \ln x + x^2 . \frac{1}{x}$	**A1**	or equiv
		Substitute e to obtain 3e for gradient	**A1**	or exact (unsimplified) equiv
		Attempt eqn of straight line with numerical gradient	**M1**	allowing approx values
		Obtain $y - e^2 = 3e(x - e)$	**A1√**	or equiv; following their gradient provided obtained by diffn attempt; allow approx values
		Obtain $y = 3ex - 2e^2$	**A1**	in terms of e now and in requested form
			6	
4	**(i)**	Differentiate to obtain form $kx(2x^2 + 9)^n$	**M1**	any constant k; any $n < \frac{5}{2}$
		Obtain correct $10x(2x^2 + 9)^{\frac{3}{2}}$	**A1**	or (unsimplified) equiv
		Equate to 100 and confirm $x = 10(2x^2 + 9)^{-\frac{3}{2}}$	**A1**	**AG**; necessary detail required
			3	
	(ii)	Attempt relevant calculations with 0.3 and 0.4	**M1**	
		Obtain at least one correct value	**A1**	x f(x) x – f(x) f′(x) 0.3 0.3595 –0.0595 83.4 0.4 0.3515 0.0485 113.8
		Obtain two correct values and conclude appropriately	**A1**	noting sign change or showing 0.3 < f(0.3) and 0.4 > f(0.4) or showing gradients either side of 100
			3	

	(iii)	Obtain correct first iterate	**B1**	
		Carry out correct process	**M1**	finding at least 3 iterates in all
		Obtain 0.3553	**A1**	answer required to exactly 4 dp
			3	
		[0.3 → 0.35953 → 0.35497 → 0.35534 → 0.35531; 0.35 → 0.35575 → 0.35528 → 0.35532 (→ 0.35531); 0.4 → 0.35146 → 0.35563 → 0.35529 → 0.35532]		
5	**(a)**	Obtain expression of form $\frac{a\tan\alpha}{b+c\tan^2\alpha}$	**M1**	any non-zero constants a, b, c
		State correct $\frac{2\tan\alpha}{1-\tan^2\alpha}$	**A1**	or equiv
		Attempt to produce polynomial equation in $\tan\alpha$	**M1**	using sound process
		Obtain at least one correct value of $\tan\alpha$	**A1**	$\tan\alpha = \pm\sqrt{\frac{4}{5}}$
		Obtain 41.8	**A1**	allow 42 or greater accuracy; allow 0.73
		Obtain 138.2 and no other values between 0 and 180	**A1**	allow 138 or greater accuracy
		[SC: Answers only 41.8 or … B1; 138.2 or … and no others B1]	**6**	
	(b)(i)	State $\frac{7}{6}$	**B1**	
			1	
	(ii)	Attempt use of identity linking $\cot^2\beta$ and $\text{cosec}^2\beta$	**M1**	or equiv retaining exactness; condone sign errors
		Obtain $\frac{13}{36}$	**A1**	or exact equiv
			2	
6		Integrate k_1e^{nx} to obtain k_2e^{nx}	**M1**	any constants involving π or not; any n
		Obtain correct indefinite integral of their k_1e^{nx}	**A1**	
		Substitute limits to obtain $\frac{1}{6}\pi(e^3-1)$ or $\frac{1}{6}(e^3-1)$	**A1**	or exact equiv perhaps involving e^0
		Integrate $k(2x-1)^n$ to obtain $k'(2x-1)^{n+1}$	**M1**	any constants involving π or not; any n
		Obtain correct indefinite integral of their $k(2x-1)^n$	**A1**	
		Substitute limits to obtain $\frac{1}{18}\pi$ or $\frac{1}{18}$	**A1**	or exact equiv
		Apply formula $\int\pi y^2\text{d}x$ at least once	**B1**	for $y=e^{3x}$ and/or $y=(2x-1)^4$
		Subtract, correct way round, attempts at volumes y^2	**M1**	allow with π missing but must involve
		Obtain $\frac{1}{6}\pi e^3 - \frac{2}{9}\pi$	**A1**	or similarly simplified exact equiv
			9	
7	**(i)**	State $A = 42$	**B1**	
		State $k = \frac{1}{9}$	**B1**	or 0.11 or greater accuracy
		Attempt correct process for finding m	**M1**	involving logarithms or equiv
		Obtain $\frac{1}{9}\ln 2$ or 0.077	**A1**	or 0.08 or greater accuracy
			4	
	(ii)	Attempt solution for t using either formula	**M1**	using correct process (log'ms or T&I or …)
		Obtain 11.3	**A1**	or greater accuracy; allow 11.3 ± 0.1
			2	
	(iii)	Differentiate to obtain form Be^{mt}	**M1**	where B is different from A
		Obtain $3.235e^{0.077t}$	**A1**√	or equiv; following their A and m
		Obtain 47.9	**A1**	allow 48 or greater accuracy
			3	

8	**(i)**	Show at least correct $\cos\theta\cos 60 + \sin\theta\sin 60$ or $\cos\theta\cos 60 - \sin\theta\sin 60$	**B1**	
		Attempt expansion of both with exact numerical values attempted	**M1**	and with $\cos 60 \neq \sin 60$
		Obtain $\frac{1}{2}\sqrt{3}\sin\theta + \frac{5}{2}\cos\theta$	**A1**	or exact equiv
			3	
	(ii)	Attempt correct process for finding R	**M1**	whether exact or approx
		Attempt recognisable process for finding α	**M1**	allowing sin / cos muddles
		Obtain $\sqrt{7}\sin(\theta + 70.9)$	**A1**	allow 2.65 for R; allow 70.9 ± 0.1 for α
			3	
	(iii)	Attempt correct process to find any value of θ + their α	**M1**	
		Obtain any correct value for $\theta + 70.9$	**A1**	–158, –22, 202, 338, …
		Attempt correct process to find θ + their α in 3rd quadrant	**M1**	or several values including this
		Obtain 131	**A1**	or greater accuracy and no other
		[SC for solutions with no working shown: Correct answer only B4; 131 with other answers B2]		
			4	
9	**(i)**	Attempt use of quotient rule	***M1**	or equiv; allow u / v muddles
		Obtain $\dfrac{75 - 15x^2}{(x^2 + 5)^2}$	**A1**	or (unsimplified) equiv; this **M1A1** available at any stage of question
		Equate attempt at first derivative to zero and rearrange to solvable form	**M1**	dep ***M**
		Obtain $x = \sqrt{5}$ or 2.24	**A1**	or greater accuracy
		Recognise range as values less than y-coord of st pt	**M1**	allowing < here
		Obtain $0 \leq y \leq \frac{3}{2}\sqrt{5}$	**A1**	any notation; with ≤ now; any exact equiv
			6	
	(ii)	State $\sqrt{5}$	**B1**√	following their x-coord of st pt; condone answer $x \geq \sqrt{5}$ but not inequality with k
			1	
	(iii)	Equate attempt at first derivative to –1 and attempt simplification	***M1**	and dependent on first **M** in part **(i)**
		Obtain $x^4 - 5x^2 + 100 = 0$	**A1**	or equiv involving 3 non-zero terms
		Attempt evaluation of discriminant or equiv	**M1**	dep ***M**
		Obtain –375 or equiv and conclude appropriately	**A1**	
			4	