

- 1.** The point P lies on the curve with equation

$$y = 4e^{2x+1}.$$

The y -coordinate of P is 8.

- (a) Find, in terms of $\ln 2$, the x -coordinate of P .

(2)

- (b) Find the equation of the tangent to the curve at the point P in the form $y = ax + b$, where a and b are exact constants to be found.

(4)



2.

$$f(x) = 5\cos x + 12\sin x$$

Given that $f(x) = R \cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$,

- (a) find the value of R and the value of α to 3 decimal places.

(4)

- (b) Hence solve the equation

$$5\cos x + 12\sin x = 6$$

for $0 \leq x < 2\pi$.

(5)

- (c) (i) Write down the maximum value of $5\cos x + 12\sin x$.

(1)

- (ii) Find the smallest positive value of x for which this maximum value occurs.

(2)

[illegible]

3.

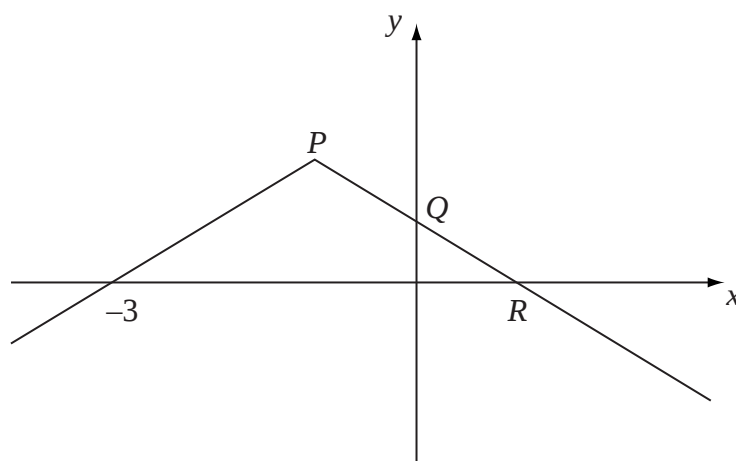


Figure 1

Figure 1 shows the graph of $y = f(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point P .

The graph cuts the y -axis at the point Q and the x -axis at the points $(-3, 0)$ and R .

Sketch, on separate diagrams, the graphs of

$$(a) \quad y = |f(x)|, \quad (2)$$

$$(b) \quad y = f(-x).$$

Given that $f(x) = 2 - |x + 1|$,

(c) find the coordinates of the points P , Q and R ,

(d) solve $f(x) = \frac{1}{2}x$. (5)



Question 3 continued

[illegible]

[illegible]

6. (a) Differentiate with respect to x ,

$$(i) \quad e^{3x}(\sin x + 2 \cos x), \quad (3)$$

$$\text{(ii) } x^3 \ln(5x+2).$$

Given that $y = \frac{3x^2 + 6x - 7}{(x+1)^2}$, $x \neq -1$,

(b) show that $\frac{dy}{dx} = \frac{20}{(x+1)^3}$. (5)

(c) Hence find $\frac{d^2y}{dx^2}$ and the real values of x for which $\frac{d^2y}{dx^2} = -\frac{15}{4}$. (3)



[illegible]

7.

$$f(x) = 3x^3 - 2x - 6$$

- (a) Show that $f(x) = 0$ has a root, α , between $x = 1.4$ and $x = 1.45$

(2)

- (b) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}, \quad x \neq 0.$$

(3)

- (c) Starting with $x_0=1.43$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$$

to calculate the values of x_1 , x_2 and x_3 , giving your answers to 4 decimal places.

(3)

- (d) By choosing a suitable interval, show that $\alpha = 1.435$ is correct to 3 decimal places.

(3)

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END

