C3 June 2009

(3) 1/ i) $\sec x$ ii) $\cot x$ iii) $\sin^{-1} x$.

(5) 2/ $V = \pi \int_0^{3/2} (2x-3)^4 \, dx = \pi \left[\frac{(2x-3)^5}{5 \times 2} \right]_0^{3/2} = \pi \left[\frac{(2\times\frac{3}{2}-3)^5}{10} - \frac{(2\times 0-3)^5}{10} \right]$

$= \frac{243\pi}{10}$.

3/ i) $\sec^2\alpha - \sec^2\beta = 16$.

$(1+\tan^2\alpha) - (1+\tan^2\beta) = 16$.

$\tan^2\alpha - \tan^2\beta = 16$.

$(m+2)^2 - m^2 = 16$.

(3) $4m = 12 \Rightarrow m = 3$.

(3) ii) $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{5+3}{1-5\times 3} = -\frac{4}{7}$

4/ i) $\int_a^{3a} e^{3x} + e^x \, dx = 100$

$\left[\frac{1}{3}e^{3x} + e^x \right]_a^{3a} = 100$

$\left(\frac{1}{3}e^{9a} + e^{3a} \right) - \left(\frac{1}{3}e^{3a} + e^a \right) = 100$.

$\frac{1}{3}e^{9a} + \frac{2}{3}e^{3a} - e^a = 100$

$e^{9a} = 300 + 3e^a - 2e^{3a}$

(5) $9a = \ln(300 + 3e^a - 2e^{3a})$

$a = \frac{1}{9}\ln(300 + 3e^a - 2e^{3a})$

ii) $a_{n+1} = \frac{1}{9}\ln(300 + 3e^{a_n} - 2e^{3a_n})$

(4) $a_1 = 0.6$ $a_2 = 0.63127$ $a_3 = 0.63088$ $a_4 = 0.63081$ $a_5 = 0.63089$

$\therefore a = 0.6309$ to 4dp.

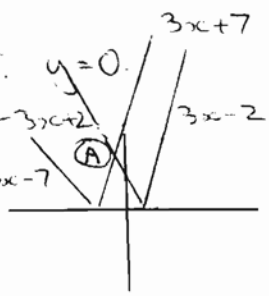
(3) 5/ i) $fg(x) = 0 \Rightarrow 3(3x+7) - 2 = 0$ $9x + 19 = 0$ $x = \frac{-19}{9}$. $y = 0$.

(3) ii) $g(x) = g^{-1}(x) = x$ $3x + 7 = x \Rightarrow x = -\frac{7}{2}$ $y = -\frac{7}{2}$.

iii) $|3x-2| = |3x+7|$

$3x - 2 = 3x + 7$ (A) $2 - 3x = 3x + 7$

(4) no solution. $-5 = 6x \Rightarrow x = -\frac{5}{6}$ $y = \frac{9}{2}$.



6/ i) $x = (37 + 10y - 2y^2)^{1/2}$

(2) $\frac{dx}{dy} = \frac{1}{2}(37 + 10y - 2y^2)^{-\frac{1}{2}} \times (10 - 4y)$.

u) (7,3) $\frac{dx}{dy} = \frac{10-4\times3}{2(37+10\times3-2\times3^2)^{1/2}} = \frac{-1}{7}$ or $\frac{dy}{dx} = -7$

(5) $x-7 = -\frac{1}{7}(y-3)$ $\quad y-3 = -7(x-7)$

$7x-49 = -y+3$ $\quad y = -7x+52$

$y = 52-7x$.

7) i) $8\sin\theta - 6\cos\theta \equiv R\sin(\theta-\alpha)$

$R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$ $\quad R\sin\alpha = 6$ $\quad R\cos\alpha = 8$

(3) $R = \sqrt{8^2+6^2} = 10$ $\quad \alpha = \tan^{-1}\left(\frac{6}{8}\right) = 36.9$ $\quad 10\sin(\theta-36.9)$.

ii) a) $8\sin\theta - 6\cos\theta = 9$

$10\sin(\theta-36.9) = 9$.

$\sin(\theta-36.9) = 0.9$

(4) $\theta-36.9 = \sin^{-1}(0.9) = 64.2, 115.8$

$\theta = 101^\circ, 153^\circ$.

b) $32\sin x - 24\cos x - (16\sin y - 12\cos y)$

$= 4(8\sin x - 6\cos x) - 2(8\sin y - 6\cos y)$

$= 40\sin(x-36.9) - 20\sin(y-36.9)$

(3) max $\sin(x-36.9) = 1$ $\quad$ min $\sin(y-36.9) = -1$.

$\therefore$ max $40+20 = 60$.

(3) 8) i) Vertical stretch s.f 2, Horizontal Translation of +6.

ii) $\ln x = 2\ln(x-6)$

$x = (x-6)^2 \Rightarrow x^2-13x+36 = 0$.

(4) $(x-9)(x-4) = 0$

$x = 9$ or 4 (not valid as $\ln(4-6) = \ln(-2)$). $\therefore x = 9$ only.

(3) iii) $A = \frac{1}{3}\times1\times\{2\ln1 + 2\ln3 + 4\times2\ln2\} = 2.58$.

9) a) $y = \frac{kx^2-1}{kx^2+1}$ $\quad \frac{dy}{dx} = \frac{(kx^2+1)\times2kx - (kx^2-1)\times2kx}{(kx^2+1)^2} = \frac{4kx}{(kx^2+1)^2} = 0$ when

(5) $x = 0$ only $\therefore$ only 1 stationary point.

b) $y = e^{mx}(x^2+mx)$ $\quad \frac{dy}{dx} = me^{mx}(x^2+mx) + e^{mx}(2x+m)$

$= e^{mx}(mx^2 + (m^2+2)x + m) = 0$

if $mx^2 + (m^2+2)x + m = 0$.

(7) $b^2-4ac = (m^2+2)^2 - 4\times m\times m$

$= m^4+4m^2+4-4m^2 = m^4+4 > 0$ for all m

$\therefore$ 2 roots $\Rightarrow$ 2 stationary points.