## ADVANCED SUBSIDIARY GCE MATHEMATICS (MEI)

Candidates answer on the Answer Booklet
OCR Supplied Materials:

- 8 page Answer Booklet
- Insert for Question 10 (inserted)
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Friday 22 May 2009
Morning
Duration: 1 hour 30 minutes


## INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer all the questions.
- Do not write in the bar codes.
- $\quad$ There is an insert for use in Question 10.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.


## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- $\quad$ The total number of marks for this paper is 72.
- This document consists of 4 pages. Any blank pages are indicated.


## Section A (36 marks)

1 Use an isosceles right-angled triangle to show that $\cos 45^{\circ}=\frac{1}{\sqrt{2}}$.

2 Find $\int_{1}^{2}\left(12 x^{5}+5\right) d x$.
[4]
(i) Find $\sum_{k=3}^{8}\left(k^{2}-1\right)$.
(ii) State whether the sequence with $k$ th term $k^{2}-1$ is convergent or divergent, giving a reason for your answer.

4 A sector of a circle of radius 18.0 cm has arc length 43.2 cm .
(i) Find in radians the angle of the sector.
(ii) Find this angle in degrees, giving your answer to the nearest degree.

5 (i) On the same axes, sketch the graphs of $y=\cos x$ and $y=\cos 2 x$ for values of $x$ from 0 to $2 \pi$.
(ii) Describe the transformation which maps the graph of $y=\cos x$ onto the graph of $y=3 \cos x$.

6 Use calculus to find the $x$-coordinates of the turning points of the curve $y=x^{3}-6 x^{2}-15 x$.
Hence find the set of values of $x$ for which $x^{3}-6 x^{2}-15 x$ is an increasing function.

7 Show that the equation $4 \cos ^{2} \theta=4-\sin \theta$ may be written in the form

$$
\begin{equation*}
4 \sin ^{2} \theta-\sin \theta=0 \tag{5}
\end{equation*}
$$

Hence solve the equation $4 \cos ^{2} \theta=4-\sin \theta$ for $0^{\circ} \leqslant \theta \leqslant 180^{\circ}$.

8 The gradient of a curve is $3 \sqrt{x}-5$. The curve passes through the point $(4,6)$. Find the equation of the curve.

9 Simplify
(i) $10-3 \log _{a} a$,
(ii) $\frac{\log _{10} a^{5}+\log _{10} \sqrt{a}}{\log _{10} a}$.

## Section B (36 marks)

## 10 Answer part (i) of this question on the insert provided.

Ash trees grow quickly for the first years of their life, then more slowly. This table shows the height of a tree at various ages.

| Age $(t$ years $)$ | 4 | 7 | 10 | 15 | 20 | 40 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Height $(h \mathrm{~m})$ | 4 | 9 | 12 | 17 | 19 | 26 |

The height, $h \mathrm{~m}$, of an ash tree when it is $t$ years old may be modelled by an equation of the form

$$
h=a \log _{10} t+b
$$

(i) On the insert, complete the table and plot $h$ against $\log _{10} t$, drawing by eye a line of best fit. [3]
(ii) Use your graph to find an equation for $h$ in terms of $\log _{10} t$ for this model.
(iii) Find the height of the tree at age 100 years, as predicted by this model.
(iv) Find the age of the tree when it reaches a height of 29 m , according to this model.
(v) Comment on the suitability of the model when the tree is very young.

11 (i) In a 'Make Ten' quiz game, contestants get $£ 10$ for answering the first question correctly, then a further $£ 20$ for the second question, then a further $£ 30$ for the third, and so on, until they get a question wrong and are out of the game.
(A) Haroon answers six questions correctly. Show that he receives a total of $£ 210$.
(B) State, in a simple form, a formula for the total amount received by a contestant who answers $n$ questions correctly.

Hence find the value of $n$ for a contestant who receives $£ 10350$ from this game.
(ii) In a 'Double Your Money' quiz game, contestants get $£ 5$ for answering the first question correctly, then a further $£ 10$ for the second question, then a further $£ 20$ for the third, and so on doubling the amount for each question until they get a question wrong and are out of the game.
(A) Gary received $£ 75$ from the game. How many questions did he get right?
(B) Bethan answered 9 questions correctly. How much did she receive from the game?
(C) State a formula for the total amount received by a contestant who answers $n$ questions correctly.

Hence find the value of $n$ for a contestant in this game who receives $£ 2621435$.

## [Question 12 is printed overleaf.]

12 (i) Calculate the gradient of the chord joining the points on the curve $y=x^{2}-7$ for which $x=3$ and $x=3.1$.
(ii) Given that $\mathrm{f}(x)=x^{2}-7$, find and simplify $\frac{\mathrm{f}(3+h)-\mathrm{f}(3)}{h}$.
(iii) Use your result in part (ii) to find the gradient of $y=x^{2}-7$ at the point where $x=3$, showing your reasoning.
(iv) Find the equation of the tangent to the curve $y=x^{2}-7$ at the point where $x=3$.
(v) This tangent crosses the $x$-axis at the point P. The curve crosses the positive $x$-axis at the point Q . Find the distance PQ , giving your answer correct to 3 decimal places.

## Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations, is given to all schools that receive assessment material and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.
If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.
For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1PB.
OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

