

**ADVANCED GCE** 

MATHEMATICS Core Mathematics 3 4723

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required: None

Monday 1 June 2009 Morning

Duration: 1 hour 30 minutes



## INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer all the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is 72.
- This document consists of 4 pages. Any blank pages are indicated.



2

Each diagram above shows part of a curve, the equation of which is one of the following:

 $y = \sin^{-1} x$ ,  $y = \cos^{-1} x$ ,  $y = \tan^{-1} x$ ,  $y = \sec x$ ,  $y = \csc x$ ,  $y = \cot x$ .

State which equation corresponds to

(i) Fig. 1,	[1]
(ii) Fig. 2,	[1]

(iii) Fig. 3.

2

1



The diagram shows the curve with equation  $y = (2x - 3)^2$ . The shaded region is bounded by the curve and the lines x = 0 and y = 0. Find the exact volume obtained when the shaded region is rotated completely about the x-axis. [5]

3 The angles  $\alpha$  and  $\beta$  are such that

$$\tan \alpha = m + 2$$
 and  $\tan \beta = m$ ,

where *m* is a constant.

- (i) Given that  $\sec^2 \alpha \sec^2 \beta = 16$ , find the value of *m*. [3]
- (ii) Hence find the exact value of  $tan(\alpha + \beta)$ .

[3]

[1]

(i) Show that 
$$a = \frac{1}{9}\ln(300 + 3e^a - 2e^{3a})$$
. [5]

3

- (ii) Use an iterative process, based on the equation in part (i), to find the value of *a* correct to 4 decimal places. Use a starting value of 0.6 and show the result of each step of the process. [4]
- 5 The functions f and g are defined for all real values of x by

$$f(x) = 3x - 2$$
 and  $g(x) = 3x + 7$ .

Find the exact coordinates of the point at which

- (i) the graph of y = fg(x) meets the x-axis,
- (ii) the graph of y = g(x) meets the graph of  $y = g^{-1}(x)$ , [3]
- (iii) the graph of y = |f(x)| meets the graph of y = |g(x)|.

6



The diagram shows the curve with equation  $x = (37 + 10y - 2y^2)^{\frac{1}{2}}$ .

- (i) Find an expression for  $\frac{dx}{dy}$  in terms of y. [2]
- (ii) Hence find the equation of the tangent to the curve at the point (7, 3), giving your answer in the form y = mx + c. [5]
- 7 (i) Express  $8\sin\theta 6\cos\theta$  in the form  $R\sin(\theta \alpha)$ , where R > 0 and  $0^{\circ} < \alpha < 90^{\circ}$ . [3]
  - (ii) Hence
    - (a) solve, for  $0^{\circ} < \theta < 360^{\circ}$ , the equation  $8 \sin \theta 6 \cos \theta = 9$ , [4]
    - (b) find the greatest possible value of

 $32\sin x - 24\cos x - (16\sin y - 12\cos y)$ 

as the angles *x* and *y* vary.

[3]

[3]

[4]



4

The diagram shows the curves  $y = \ln x$  and  $y = 2\ln(x-6)$ . The curves meet at the point *P* which has *x*-coordinate *a*. The shaded region is bounded by the curve  $y = 2\ln(x-6)$  and the lines x = a and y = 0.

- (i) Give details of the pair of transformations which transforms the curve  $y = \ln x$  to the curve  $y = 2\ln(x-6)$ . [3]
- (ii) Solve an equation to find the value of *a*.
- (iii) Use Simpson's rule with two strips to find an approximation to the area of the shaded region.

[3]

[4]

9 (a) Show that, for all non-zero values of the constant k, the curve

$$y = \frac{kx^2 - 1}{kx^2 + 1}$$

has exactly one stationary point.

(b) Show that, for all non-zero values of the constant *m*, the curve

$$y = e^{mx}(x^2 + mx)$$

has exactly two stationary points.

[7]

[5]



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