OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

Core Mathematics 4

4724

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Friday 5 June 2009
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Find the quotient and the remainder when $3x^4 - x^3 - 3x^2 - 14x - 8$ is divided by $x^2 + x + 2$. **[4]**

2 Use the substitution $x = \tan\theta$ to find the exact value of

$$\int_1^{\sqrt{3}} \frac{1 - x^2}{1 + x^2}\,dx. \qquad [7]$$

3 **(i)** Expand $(a + x)^{-2}$ in ascending powers of x up to and including the term in x^2. **[4]**

(ii) When $(1 - x)(a + x)^{-2}$ is expanded, the coefficient of x^2 is 0. Find the value of a. **[3]**

4 **(i)** Differentiate $e^x(\sin 2x - 2\cos 2x)$, simplifying your answer. **[4]**

(ii) Hence find the exact value of $\int_0^{\frac{1}{4}\pi} e^x \sin 2x\,dx$. **[3]**

5 A curve has parametric equations

$$x = 2t + t^2, \quad y = 2t^2 + t^3.$$

(i) Express $\frac{dy}{dx}$ in terms of t and find the gradient of the curve at the point $(3, -9)$. **[5]**

(ii) By considering $\frac{y}{x}$, find a cartesian equation of the curve, giving your answer in a form not involving fractions. **[4]**

6 The expression $\frac{4x}{(x-5)(x-3)^2}$ is denoted by $f(x)$.

(i) Express $f(x)$ in the form $\frac{A}{x-5} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$, where A, B and C are constants. **[4]**

(ii) Hence find the exact value of $\int_1^2 f(x)\,dx$. **[5]**

7 **(i)** The vector $\mathbf{u} = \frac{3}{13}\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ is perpendicular to the vector $4\mathbf{i} + \mathbf{k}$ and to the vector $4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$. Find the values of b and c, and show that $\mathbf{u}$ is a unit vector. **[6]**

(ii) Calculate, to the nearest degree, the angle between the vectors $4\mathbf{i} + \mathbf{k}$ and $4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$. **[3]**

8 **(i)** Given that $14x^2 - 7xy + y^2 = 2$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{28x - 7y}{7x - 2y}$. **[4]**

(ii) The points L and M on the curve $14x^2 - 7xy + y^2 = 2$ each have x-coordinate 1. The tangents to the curve at L and M meet at N. Find the coordinates of N. **[6]**

9 A tank contains water which is heated by an electric water heater working under the action of a thermostat. The temperature of the water, $\theta\,°\mathrm{C}$, may be modelled as follows. When the water heater is first switched on, $\theta = 40$. The heater causes the temperature to increase at a rate k_1 °C per second, where k_1 is a constant, until $\theta = 60$. The heater then switches off.

(i) Write down, in terms of k_1, how long it takes for the temperature to increase from 40 °C to 60 °C. **[1]**

The temperature of the water then immediately starts to decrease at a variable rate $k_2(\theta - 20)$ °C per second, where k_2 is a constant, until $\theta = 40$.

(ii) Write down a differential equation to represent the situation as the temperature is decreasing. **[1]**

(iii) Find the total length of time for the temperature to increase from 40 °C to 60 °C and then decrease to 40 °C. Give your answer in terms of k_1 and k_2. **[8]**