OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4761

Mechanics 1

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Thursday 11 June 2009
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

Section A (36 marks)

1 The velocity-time graph shown in Fig. 1 represents the straight line motion of a toy car. All the lines on the graph are straight.

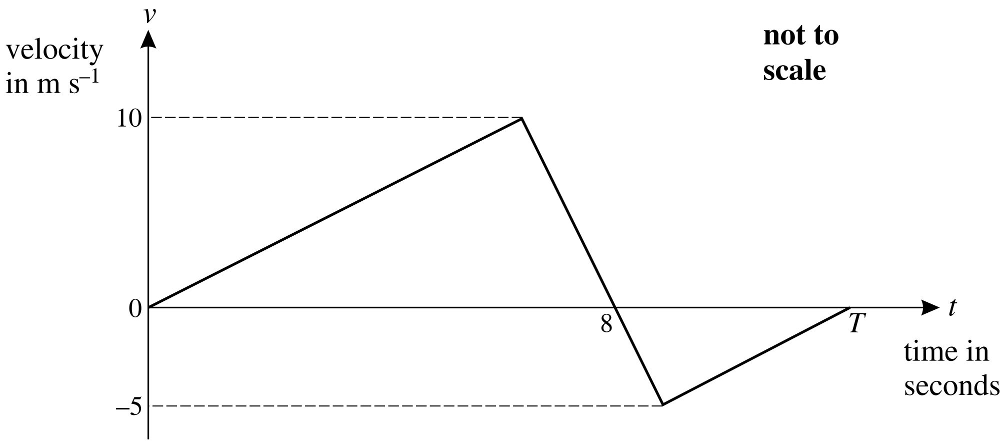

Fig. 1

The car starts at the point A at $t = 0$ and in the next 8 seconds moves to a point B.

(i) Find the distance from A to B. **[2]**

T seconds after leaving A, the car is at a point C which is a distance of 10 m from B.

(ii) Find the value of T. **[3]**

(iii) Find the displacement from A to C. **[1]**

2 A small box has weight $\mathbf{W}$ N and is held in equilibrium by two strings with tensions $\mathbf{T}_1$ N and $\mathbf{T}_2$ N. This situation is shown in Fig. 2 which also shows the standard unit vectors $\mathbf{i}$ and $\mathbf{j}$ that are horizontal and vertically upwards, respectively.

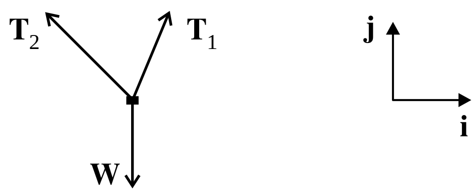

Fig. 2

The tension $\mathbf{T}_1$ is $10\mathbf{i} + 24\mathbf{j}$.

(i) Calculate the magnitude of $\mathbf{T}_1$ and the angle between $\mathbf{T}_1$ and the vertical. **[3]**

The magnitude of the weight is w N.

(ii) Write down the vector $\mathbf{W}$ in terms of w and $\mathbf{j}$. **[1]**

The tension $\mathbf{T}_2$ is $k\mathbf{i} + 10\mathbf{j}$, where k is a scalar.

(iii) Find the values of k and of w. **[3]**

3 Fig. 3 is a sketch of the velocity-time graph modelling the velocity of a sprinter at the start of a race.

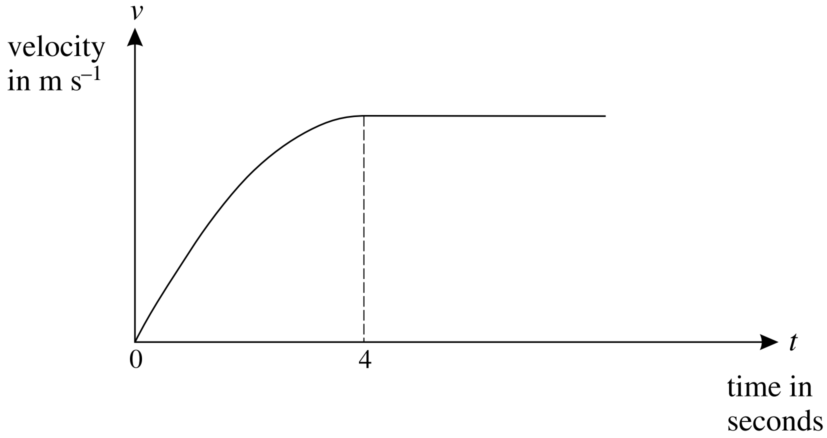

Fig. 3

(i) How can you tell from the sketch that the acceleration is not modelled as being constant for $0 \leqslant t \leqslant 4$? **[1]**

The velocity of the sprinter, $v\,\text{m s}^{-1}$, for the time interval $0 \leqslant t \leqslant 4$ is modelled by the expression

$$v = 3t - \tfrac{3}{8}t^2.$$

(ii) Find the acceleration that the model predicts for $t = 4$ and comment on what this suggests about the running of the sprinter. **[3]**

(iii) Calculate the distance run by the sprinter from $t = 1$ to $t = 4$. **[4]**

4 Fig. 4 shows a particle projected over horizontal ground from a point O at ground level. The particle initially has a speed of $32\,\mathrm{m\,s^{-1}}$ at an angle α to the horizontal. The particle is a horizontal distance of 44.8 m from O after 5 seconds. Air resistance should be neglected.

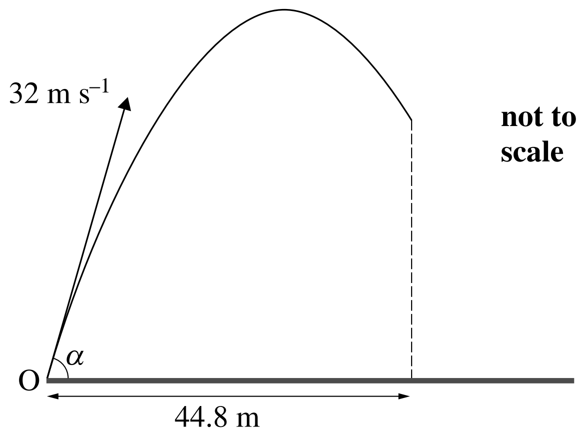

Fig. 4

(i) Write down an expression, in terms of α and t, for the horizontal distance of the particle from O at time t seconds after it is projected. **[1]**

(ii) Show that $\cos\alpha = 0.28$. **[2]**

(iii) Calculate the greatest height reached by the particle. **[4]**

5 The position vector of a toy boat of mass 1.5 kg is modelled as $\mathbf{r} = (2 + t)\mathbf{i} + (3t - t^2)\mathbf{j}$ where lengths are in metres, t is the time in seconds, $\mathbf{i}$ and $\mathbf{j}$ are horizontal, perpendicular unit vectors and the origin is O.

(i) Find the velocity of the boat when $t = 4$. **[3]**

(ii) Find the acceleration of the boat and the horizontal force acting on the boat. **[3]**

(iii) Find the cartesian equation of the path of the boat referred to x- and y-axes in the directions of $\mathbf{i}$ and $\mathbf{j}$, respectively, with origin O. You are not required to simplify your answer. **[2]**

Section B (36 marks)

6 An empty open box of mass 4 kg is on a plane that is inclined at 25° to the horizontal.

In one model the plane is taken to be smooth.

The box is held in equilibrium by a string with tension T N parallel to the plane, as shown in Fig. 6.1.

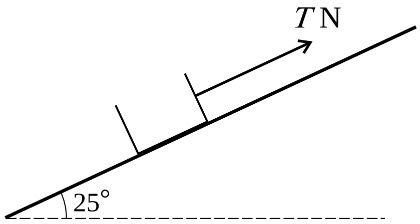

Fig. 6.1

(i) Calculate T. **[2]**

A rock of mass m kg is now put in the box. The system is in equilibrium when the tension in the string, still parallel to the plane, is 50 N.

(ii) Find m. **[3]**

In a refined model the plane is rough.

The empty box, of mass 4 kg, is in equilibrium when a frictional force of 20 N acts down the plane and the string has a tension of P N inclined at 15° to the plane, as shown in Fig. 6.2.

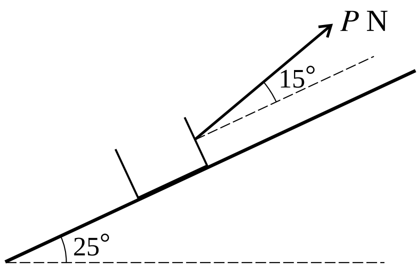

Fig. 6.2

(iii) Draw a diagram showing all the forces acting on the box. **[2]**

(iv) Calculate P. **[5]**

(v) Calculate the normal reaction of the plane on the box. **[4]**

7 A box of mass 8 kg slides on a horizontal table against a constant resistance of 11.2 N.

(i) What horizontal force is applied to the box if it is sliding with acceleration of magnitude $2\ \text{m s}^{-2}$? **[3]**

Fig. 7 shows the box of mass 8 kg on a long, rough, horizontal table. A sphere of mass 6 kg is attached to the box by means of a light inextensible string that passes over a smooth pulley. The section of the string between the pulley and the box is parallel to the table. The constant frictional force of 11.2 N opposes the motion of the box. A force of 105 N parallel to the table acts on the box in the direction shown, and the acceleration of the system is in that direction.

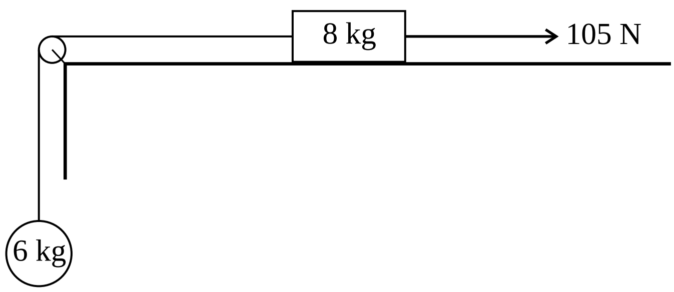

Fig. 7

(ii) What information in the question indicates that while the string is taut the box and sphere have the same acceleration? **[1]**

(iii) Draw two separate diagrams, one showing all the horizontal forces acting on the box and the other showing all the forces acting on the sphere. **[2]**

(iv) Show that the magnitude of the acceleration of the system is $2.5\ \text{m s}^{-2}$ and find the tension in the string. **[7]**

The system is stationary when the sphere is at point P. When the sphere is 1.8 m above P the string breaks, leaving the sphere moving upwards at a speed of $3\ \text{m s}^{-1}$.

(v) (*A*) Write down the value of the acceleration of the sphere after the string breaks. **[1]**

(*B*) The sphere passes through P again at time T seconds after the string breaks. Show that T is the positive root of the equation $4.9T^2 - 3T - 1.8 = 0$. **[2]**

(*C*) Using part (*B*), or otherwise, calculate the total time that elapses after the sphere moves from P before the sphere again passes through P. **[4]**