

# **Mathematics (MEI)**

Advanced Subsidiary GCE 4751

Introduction to Advanced Mathematics (C1)

## **Mark Scheme for June 2010**

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## SECTION A

<b>1</b>	$y = 3x + c$ or $y - y_1 = 3(x - x_1)$  $y - 5 = \text{their } m(x - 4)$ o.e.  $y = 3x - 7$ or simplified equiv.	<b>M1</b> allow M1 for 3 clearly stated/ used as gradient of required line  <b>M1</b> or (4, 5) subst in their $y = mx + c$ ; allow M1 for $y - 5 = m(x - 4)$ o.e.  <b>A1</b> condone $y = 3x + c$ and $c = -7$ or <b>B3</b> www
<b>2</b>	(i) $250a^6b^7$  (ii) 16 cao  (iii) 64	<b>2</b> <b>B1</b> for two elements correct; condone multiplication signs left in SC1 for eg $250 + a^6 + b^7$  <b>1</b>  <b>2</b> condone $\pm 64$ <b>M1</b> for $[\pm]4^3$ or for $\sqrt{4096}$ or for only $-64$
<b>3</b>	$ac = \sqrt{y} - 5$ o.e. $ac + 5 = \sqrt{y}$ o.e. $[y =](ac + 5)^2$ o.e. isw	<b>M1</b> <b>M1</b> for each of 3 correct or ft correct steps s.o.i. leading to y as subject  <b>M1</b>  <b>M1</b> <u>or</u> some/all steps may be combined; allow <b>B3</b> for $[y =](ac + 5)^2$ o.e. isw or <b>B2</b> if one error
<b>4 (i)</b>	$2 - 2x > 6x + 5$  $-3 > 8x$ o.e. or ft  $x < -3/8$ o.e. or ft isw	<b>M1</b> or $1 - x > 3x + 2.5$  <b>M1</b> for collecting terms of their inequality correctly on opposite sides eg $-8x > 3$ <b>M1</b> allow <b>B3</b> for correct inequality found after working with equation allow <b>SC2</b> for $-3/8$ o.e. found with equation or wrong inequality
<b>4 (ii)</b>	$-4 < x < 1/2$ o.e.	<b>2</b> accept as two inequalities <b>M1</b> for one 'end' correct or for $-4$ and $1/2$
<b>5 (i)</b>	$7\sqrt{3}$	<b>2</b> <b>M1</b> for $\sqrt{48} = 4\sqrt{3}$ or $\sqrt{27} = 3\sqrt{3}$

5 (ii)	$\frac{10+15\sqrt{2}}{7}$ www isw	3	<b>B1</b> for 7 [B0 for 7 wrongly obtained]  and <b>B2</b> for $10+15\sqrt{2}$ or <b>B1</b> for one term of numerator correct;  if <b>B0</b> , then <b>M1</b> for attempt to multiply num and denom by $3+\sqrt{2}$
6	$5 + 2k$ soi  $k = 12$  attempt at $f(3)$  $27 + 36 + m = 59$ o.e.  $m = -4$ cao	<b>M1</b>  <b>A1</b>  <b>M1</b>  <b>A1</b>  <b>A1</b>	allow <b>M1</b> for expansion with $5x^3 + 2kx^3$ and no other $x^3$ terms or <b>M1</b> for $(29 - 5) / 2$ soi  must substitute 3 for $x$ in cubic not product or long division as far as obtaining $x^2 + 3x$ in quotient or from division $m - (-63) = 59$ o.e. or for $27 + 3k + m = 59$ or ft their $k$
7	$1 + 2x + \frac{3}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{16}x^4$ oe (must be simplified) isw	4	<b>B3</b> for 4 terms correct, or <b>B2</b> for 3 terms correct or for all correct but unsimplified (may be at an earlier stage, but factorial or ${}^nC_r$ notation must be expanded/worked out) or <b>B1</b> for 1, 4, 6, 4, 1 soi or for $1 + \dots + \frac{1}{16}x^4$ [must have at least one other term]
8	$5(x + 2)^2 - 14$	4	<b>B1</b> for $a = 5$ , and <b>B1</b> for $b = 2$ and <b>B2</b> for $c = -14$ or <b>M1</b> for $c = 6 -$ their $ab^2$ or <b>M1</b> for [their $a$ ](6/their $a -$ their $b^2$ ) [no ft for $a = 1$ ]
9	mention of $-5$ as a square root of 25 or $(-5)^2 = 25$  $-5 - 5 \neq 0$ o.e. or $x + 5 = 0$	<b>M1</b>  <b>M1</b>	condone $-5^2 = 25$  or, dep on first <b>M1</b> being obtained, allow <b>M1</b> for showing that 5 is the only soln of $x - 5 = 0$  allow <b>M2</b> for $x^2 - 25 = 0$ $(x + 5)(x - 5) [= 0]$ so $x - 5 = 0$ or $x + 5 = 0$

## SECTION B

<b>10 (i)</b>	$(2x - 3)(x + 1)$  $x = 3/2$ and $-1$ obtained	<b>M2</b>  <b>B1</b>	<b>M1</b> for factors with one sign error or giving two terms correct allow <b>M1</b> for $2(x - 1.5)(x + 1)$ with no better factors seen  or ft their factors
<b>10 (ii)</b>	graph of quadratic the correct way up and crossing both axes  crossing $x$ -axis only at $3/2$ and $-1$ or ft from their roots in (i), or their factors if roots not given  crossing $y$ -axis at $-3$	<b>B1</b>  <b>B1</b>  <b>B1</b>	for $x = 3/2$ condone 1 and 2 marked on axis and crossing roughly halfway between; intns must be shown labelled or worked out nearby
<b>10 (iii)</b>	use of $b^2 - 4ac$ with numbers subst (condone one error in substitution) (may be in quadratic formula)  $25 - 40 < 0$ or $-15$ obtained	<b>M1</b>  <b>A1</b>	may be in formula or $(x - 2.5)^2 = 6.25 - 10$ or $(x - 2.5)^2 + 3.75 = 0$ oe (condone one error)  or $\sqrt{-15}$ seen in formula or $(x - 2.5)^2 = -3.75$ oe or $x = 2.5 \pm \sqrt{-3.75}$ oe
<b>10 (iv)</b>	$2x^2 - x - 3 = x^2 - 5x + 10$ o.e.  $x^2 + 4x - 13 [= 0]$  use of quad. formula on resulting eqn (do not allow for original quadratics used)  $-2 \pm \sqrt{17}$ cao	<b>M1</b>  <b>M1</b>  <b>M1</b>  <b>A1</b>	attempt at eliminating $y$ by subst or subtraction  or $(x + 2)^2 = 17$ ; for rearranging to form $ax^2 + bx + c [= 0]$ or to completing square form condone one error for each of 2 <sup>nd</sup> and 3 <sup>rd</sup> <b>M1s</b>  or $x + 2 = \pm\sqrt{17}$ o.e. 2 <sup>nd</sup> and 3 <sup>rd</sup> <b>M1s</b> may be earned for good attempt at completing square as far as roots obtained

11 (i)	$\text{grad AB} = \frac{1-3}{5-(-1)} [= -1/3]$ $y - 3 = \text{their grad } (x - (-1)) \text{ or}$ $y - 1 = \text{their grad } (x - 5)$ $y = -1/3x + 8/3 \text{ or } 3y = -x + 8 \text{ o.e. isw}$	<b>M1</b>                      <b>A1</b>	or use of $y = \text{their gradient } x + c$ with coords of A or B or <b>M2</b> for $\frac{y-3}{1-3} = \frac{x-(-1)}{5-(-1)}$ o.e.  o.e. eg $x + 3y - 8 = 0$ or $6y = 16 - 2x$ allow <b>B3</b> for correct eqn www
11 (ii)	when $y = 0, x = 8$ ; when $x = 0, y = 8/3$ or ft their (i)  [Area =] $\frac{1}{2} \times 8/3 \times 8$ o.e. cao isw	<b>M1</b>                      <b>M1</b>	allow $y = 8/3$ used without explanation if already seen in eqn in (i)  NB answer $32/3$ given; allow $4 \times 8/3$ if first M1 earned; or <b>M1</b> for $\int_0^8 \left[ \frac{1}{3}(8-x) \right] dx = \left[ \frac{1}{3} \left( 8x - \frac{1}{2}x^2 \right) \right]_0^8$ and M1 dep for $\frac{1}{3}(64 - 32[-0])$
11 (iii)	grad perp = $-1/\text{grad AB}$ stated, or used after their grad AB stated in this part  midpoint [of AB] = (2, 2)  $y - 2 = \text{their grad perp } (x - 2)$ or ft their midpoint  <u>alt method working back from ans:</u>  grad perp = $-1/\text{grad AB}$ and showing/stating same as given line  finding intn of their $y = -1/3x - 8/3$ and $y = 3x - 4$ is (2, 2)  showing midpt of AB is (2, 2)	<b>M1</b>                      <b>M1</b>                      <b>M1</b>                      <b>M1</b>	or showing $3 \times -1/3 = -1$ if (i) is wrong, allow the first M1 here ft, provided the answer is correct ft  must state 'midpoint' or show working  for <b>M3</b> this must be correct, starting from grad AB = $-1/3$ , and also needs correct completion to given ans $y = 3x - 4$  <b>or</b> mark one method or the other, to benefit of candidate, not a mixture  eg stating $-1/3 \times 3 = -1$  or showing that (2, 2) is on $y = 3x - 4$ , having found (2, 2) first  [for both methods: for <b>M3</b> must be fully correct]

11 (iv)	<p>subst <math>x = 3</math> into <math>y = 3x - 4</math> and obtaining centre = (3, 5)</p> <p><math>r^2 = (5 - 3)^2 + (1 - 5)^2</math> o.e.</p> <p><math>r = \sqrt{20}</math> o.e. cao</p> <p>eqn is <math>(x - 3)^2 + (y - 5)^2 = 20</math> or ft their <math>r</math> and <math>y</math>-coord of centre</p>	<p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>B1</b></p>	<p>or using <math>(-1 - 3)^2 + (3 - b)^2 = (5 - 3)^2 + (1 - b)^2</math> and finding (3, 5)</p> <p>or <math>(-1 - 3)^2 + (3 - 5)^2</math> or ft their centre using A or B</p> <p>condone <math>(x - 3)^2 + (y - b)^2 = r^2</math> o.e. or <math>(x - 3)^2 + (y - \text{their } 5)^2 = r^2</math> o.e. (may be seen earlier)</p>
12 (i)	<p>trials of at calculating <math>f(x)</math> for at least one factor of 30</p> <p>details of calculation for <math>f(2)</math> or <math>f(-3)</math> or <math>f(-5)</math></p> <p>attempt at division by <math>(x - 2)</math> as far as <math>x^3 - 2x^2</math> in working</p> <p>correctly obtaining <math>x^2 + 8x + 15</math></p> <p>factorising a correct quadratic factor</p> <p><math>(x - 2)(x + 3)(x + 5)</math></p>	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p><b>M0</b> for division or inspection used</p> <p>or equiv for <math>(x + 3)</math> or <math>(x + 5)</math>; or inspection with at least two terms of quadratic factor correct</p> <p>or B2 for another factor found by factor theorem</p> <p>for factors giving two terms of quadratic correct; M0 for formula without factors found</p> <p>condone omission of first factor found; ignore '= 0' seen</p> <p>allow last four marks for <math>(x - 2)(x + 3)(x + 5)</math> obtained; for all 6 marks must see factor theorem use first</p>
12 (ii)	<p>sketch of cubic right way up, with two turning points</p> <p>values of intns on <math>x</math> axis shown, correct <math>(-5, -3, \text{ and } 2)</math> or ft from their factors/ roots in (i)</p> <p><math>y</math>-axis intersection at <math>-30</math></p>	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p>	<p>0 if stops at <math>x</math>-axis</p> <p>on graph or nearby in this part</p> <p>mark intent for intersections with both axes</p> <p>or <math>x = 0, y = -30</math> seen in this part if consistent with graph drawn</p>

<b>12 (iii)</b>	<p><math>(x - 1)</math> substituted for <math>x</math> in either form of eqn for <math>y = f(x)</math></p> <p><math>(x - 1)^3</math> expanded correctly (need not be simplified) or two of their factors multiplied correctly</p> <p>correct completion to given answer [condone omission of 'y =']</p>	<p><b>M1</b></p> <p><b>M1 dep</b></p> <p><b>M1</b></p>	<p>correct or ft their (i) or (ii) for factorised form; condone one error; allow for new roots stated as <math>-4, -2</math> and <math>3</math> or ft</p> <p>or <b>M1</b> for correct or correct ft multiplying out of all 3 brackets at once, condoning one error [<math>x^3 - 3x^2 + 4x^2 + 2x^2 + 8x - 6x - 12x - 24</math>]</p> <p>unless all 3 brackets already expanded, must show at least one further interim step allow <b>SC1</b> for <math>(x + 1)</math> subst <u>and</u> correct exp of <math>(x + 1)^3</math> or two of their factors ft</p> <p><u>or</u>, for those using given answer: <b>M1</b> for roots stated or used as <math>-4, -2</math> and <math>3</math> or ft <b>A1</b> for showing all 3 roots satisfy given eqn <b>B1</b> for comment re coefft of <math>x^3</math> or product of roots to show that eqn of translated graph is not a multiple of RHS of given eqn</p>
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**Section B Total: 36**