

# **Mathematics (MEI)**

Advanced Subsidiary GCE 4761

Mechanics 1

## **Mark Scheme for June 2010**

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Q 1		mark	notes
(i)	$v^2 = 0^2 + 2 \times 9.8 \times 0.75$  $v = \pm 3.8340\dots$ so 3.83 m s <sup>-1</sup> (3. s. f.)	M1 A1 A1 3	Use of $v^2 = u^2 + 2as$ with $u = 0$ and $a = \pm g$ . Accept muddled units and sign errors. Allow wrong or wrongly converted units not sign errors cao [SC2 for 38.3... seen WWW and SC3 for 3.83... seen WWW]
		3	

Q 2		mark	notes
(i)	Resolving  $\leftarrow 250 \sin 70 = 234.92\dots$ so 235 N (3 s. f.)  $\uparrow 250 \cos 70 = 85.5050\dots$ so 85.5 N (3 s. f.)	M1 A1 A1 3	Resolving in at least 1 of horiz or vert. Accept $\sin \leftrightarrow \cos$ . No extra terms.  Either both expressions correct (neglect direction) or one correct in correct direction  cao Both evaluated and directions correct
(ii)	$250 \div 2 = 125$ N	B1 1	Accept 125g only if tension taken to be 250g in (i)
		4	

Q 3		mark	notes
(i)	$\begin{pmatrix} -1 \\ 14 \\ -8 \end{pmatrix} + \begin{pmatrix} 3 \\ -9 \\ 10 \end{pmatrix} + \mathbf{F} = 4 \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$  $\mathbf{F} = \begin{pmatrix} -6 \\ 3 \\ 14 \end{pmatrix}$	M1 M1 A1 A1 4	N2L. Allow sign errors in applying N2L. Do not condone $\mathbf{F} = m\mathbf{g}\mathbf{a}$ . Allow one given force omitted.  Attempt to add $\begin{pmatrix} -1 \\ 14 \\ -8 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -9 \\ 10 \end{pmatrix}$  Two components correct cao
(ii)	$\mathbf{v} = \begin{pmatrix} -3 \\ 3 \\ 6 \end{pmatrix} + 3 \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} -6 \\ 9 \\ 18 \end{pmatrix} \text{ so } \begin{pmatrix} -6 \\ 9 \\ 18 \end{pmatrix} \text{ m s}^{-1}.$  speed is $\sqrt{(-6)^2 + 9^2 + 18^2} = 21 \text{ m s}^{-1}.$	M1 A1 M1 F1 4	$\mathbf{v} = \mathbf{u} + t\mathbf{a}$ with given $\mathbf{u}$ and $\mathbf{a}$ . Could go via $\mathbf{s}$ . If integration used, require arbitrary constant (need not be evaluated) cao isw Allow $-6^2$ even if interpreted as $-36$ . Only FT <b>their</b> $\mathbf{v}$ . FT their $\mathbf{v}$ only. [Award M1 F1 for 21 seen WWW]
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Q 4		mark	notes
(i)	Diagram for P or Q Other diagram	B1 B1 2	Must be properly labelled with arrows Must be properly labelled with arrows consistent with 1 <sup>st</sup> diagram Accept single diagram if clear.
(ii)	Let tension in rope be $T$ N and accn $\uparrow a \text{ m s}^{-2}$  For box P: N2L $\uparrow$ $1030 - 75g - T = 75a$ For box Q: N2L $\uparrow$ $T - 25g = 25a$	M1  A1  A1 3	N2L applied correctly to either part. Allow $F = mga$ and sign errors. Do not condone missing or extra forces.  Direction of $a$ consistent with equation for P. [Condone taking + ve downwards in either equation. +ve direction must be consistent in both equations to receive both A1s]
(iii)	tension is 257.5 N	M1 A1 2	Solving for $T$ <b>their</b> simultaneous equations with 2 variables. cao CWO
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Q 5		mark	notes
(i)	$270 - \arctan\left(\frac{6}{4}\right)$  $= 213.69\dots$ so $214^\circ$	M1 A1 2	Award for $\arctan p$ seen where $p = \pm \frac{6}{4}$ or $\frac{4}{6}$ , or equivalent cao
(ii)	Need $(-4 + 3k)\mathbf{i} + (-6 - 2k)\mathbf{j} = \lambda(7\mathbf{i} - 9\mathbf{j})$ *  <b>either</b> so $\frac{-4 + 3k}{-6 - 2k} = \frac{7}{-9}$ . or equivalent  $k = 6$ <b>or</b> $-4 + 3k = 7\lambda$ $-6 - 2k = -9\lambda$ $k = 6$  <b>trial and error method</b>	M1  M1 A1 A1 M1 A1 A1 4	Attempt to get LHS in the direction of $(7\mathbf{i} - 9\mathbf{j})$ . Could be done by finding (tangents of) angles. Accept the use of $\lambda = 1$ .  Attempt to solve <b>their</b> *. Allow $= \frac{7}{9}, \frac{9}{7}, -\frac{9}{7}$ Expression correct Award full marks for $k = 6$ found WWW Attempt to solve <b>their</b> *. Must have both equations. Correct equations Award full marks for $k = 6$ found WWW  M1 any attempt to find the value of $k$ and 'test' M1 Systematic attempt in (the equivalent of) <b>their</b> * Award full marks for $k = 6$ found WWW
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Q6		mark	notes
(i)	Vertically $y = 8t - 4.9t^2$  Horizontally $x = 12t$	M1  A1  B1 3	Use of $s = ut + 0.5at^2$ with $g = \pm 9.8, \pm 10$ . Accept $u = 0$ or $14.4\dots$ or $14.4 \sin \theta$ or $u \sin \theta$ but not $12$ . Allow use of $+3.6$ . Accept derivation of $-4.9$ not clear. cao.
(ii)	<b>either</b> Require $y = -3.6$ so $-3.6 = 8t - 4.9t^2$ Use of formula or $4.9(t-2)(t+\frac{18}{49}) = 0$  Roots are 2 and $-\frac{18}{49}$ ( $= -0.367346\dots$ )  Horizontal distance is $12 \times 2 = 24$  so 24 m  <b>or</b> Require $y = -3.6$ so $-3.6 = 8t - 4.9t^2$ Eliminate $t$ between $x = 12t$ and $-3.6 = 8t - 4.9t^2$  so $0 = 3.6 + \frac{8x}{12} - \frac{4.9x^2}{144}$  Use of formula or factorise  +ve root is 24 so 24m  <b>or</b> Methods that divide the motion into sections Projection to highest point (A) Highest point to level of jetty (B) Level of jetty to sea (C) Combination of A, B and C may be used  (A) 0.8163.. s; 9.7959.. m: (B) 0.816...s; 9.7959.. m (C): 0.3673... s; 4.4081... m	M1  M1  A1  M1  F1  M1  M1  A1  M1  F1  M1 M1 A1 A1 A1 5	Equating <b>their</b> $y$ to $\pm 3.6$ or equiv. Any form.  A method for solving a 3 term quadratic to give at least 1 root. Allow <b>their</b> $y$ and re-arrangement errors.  WWW. Accept no reference to 2 <sup>nd</sup> root [Award SC3 for $t = 2$ seen WWW]  FT <b>their</b> $x$ and $t$ .  FT only <b>their</b> $t$ (as long as it is +ve and is not obtained with sign error(s) e.g. -ve sign just dropped)  Equating <b>their</b> $y$ to $\pm 3.6$ or equiv. Any form.  Expressions in any form. Elimination must be complete  Accept in any form. May be implied.  A method for solving a 3 term quadratic to give at least 1 root. Allow <b>their</b> $y$ and re-arrangement errors.  FT from <b>their</b> quadratic after re-arrangement. Must be +ve.  Attempt to find times or distances for sections that give the total horizontal distance travelled Correct method for one section to find time or distance Any time or distance for a section correct  2 <sup>nd</sup> time or distance correct ( The two sections must not be A and B) cao
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Q 7		mark	notes
(i)			
(A)	4 m	B1	
(B)	$12 - (-4) = 16$ m	M1 A1	Looking for distance. Need evidence of taking account of +ve and -ve displacements.
(C)	$1 < t < 3.5$	B1 B1	The values 1 and 3.5 Strict inequality
(D)	$t = 1, t = 3.5$	B1 6	Do not award if extra values given.
(ii)	$v = -8t + 8$ $a = -8$	M1 A1 F1 3	Differentiating
(iii)	$-8t + 8 = 4$ so $t = 0.5$ so 0.5 s $-8t + 8 = -4$ so $t = 1.5$ so 1.5 s	B1 B1 2	FT <b>their</b> $v$ . FT <b>their</b> $v$ .
(iv)	<b>method 1</b> Need velocity at $t = 3$ $v(3) = -8 \times 3 + 8 = -16$ <b>either</b> $v = \int 32 \, dt = 32t + C$ $v = -16$ when $t = 3$ gives $v = 32t - 112$ $y = \int (32t - 112) \, dt = 16t^2 - 112t + D$ $y = 0$ when $t = 3$ gives $y = 16t^2 - 112t + 192$ <b>or</b> $y = -16 \times (t - 3) + \frac{1}{2} \times 32 \times (t - 3)^2$  (so $y = 16t^2 - 112t + 192$ )  <b>method 2</b> Since accn is constant, the displacement $y$ is a quadratic function. Since we have $y = 0$ at $t = 3$ and $t = 4$ $y = k(t - 3)(t - 4)$  When $t = 3.5$ , $y = -4$ so $-4 = k \times \frac{1}{2} \times -\frac{1}{2}$ so $k = 16$ (and $y = 16t^2 - 112t + 192$ )	B1  M1 A1 M1  A1  M1 A1         M1 A1 B1  M1 A1 5	FT <b>their</b> $v$ from (ii)  Accept $32t + C$ or $32t$ . SC1 if $\int_3^4 32 \, dt$ attempted. Use of <b>their</b> -16 from an attempt at $v$ when $t=3$ FT <b>their</b> $v$ of the form $pt + q$ with $p \neq 0$ and $q \neq 0$ . Accept if at least 1 term correct. Accept no $D$ .  cao.  Use of $s = ut + \frac{1}{2}at^2$ Use of <b>their</b> -16 (not 0) from an attempt at $v$ when $t=3$ and 32. Condone use of just $t$ Use of $t \pm 3$ cao   Use of a quadratic function (condone no $k$ ) Correct use of roots $k$ present  Or consider velocity at $t = 3$ cao. Accept $k$ without $y$ simplified.
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Q8		mark	notes
(i)	N2L <b>i</b> direction $150 = 250a$ $a = 0.6$ so $0.6 \text{ m s}^{-2}$	M1 A1 2	Use of N2L. Allow $F = mga$ . Accept no reference to direction
(ii)	150 N – <b>i</b> direction	B1 B1 2	Allow correct description or arrow  [Accept ‘– 150 in <b>i</b> direction’ for B1 B1]
(iii)	For force only in direction perp to <b>i</b> $300 \sin 40 = 450 \sin \theta$  $\theta = 25.37300\dots$ so $25.4^\circ$ (3 s. f.)  In <b>i</b> direction $300 \cos 40 + 150 + 450 \cos \theta$  786.4017... so 786 <b>i</b> N (3 s. f.)	M1  B1 A1  M1 A1  A1 6	Resolution of both terms attempted. Allow $\sin \leftrightarrow \cos$ if in both terms. Allow 250 or 250g present.  300sin40 or 450sin $\theta$ Accept $\pm$ . Accept answer rounding to 25.5. Allow SC1 if seen in this part.  Proper resolution attempted of 450 <b>and</b> 300. Allow $\sin \leftrightarrow \cos$ if in both terms. Accept use of <b>their</b> $\theta$ or just $\theta$ . Either resolution correct. Accept <b>their</b> $\theta$ or just $\theta$ . Accept $\sin/\cos$ consistent with use for cpt perpendicular to <b>i</b> . Accept no reference to direction cao. Allow SC1 WW
(iv)	Using $s = ut + 0.5at^2$ $1 = 0.5a \times 2^2$ $a = 0.5$  Using N2L in <b>i</b> direction $786.4017\dots - F = 250 \times 0.5$  661.4017... so 661 N (3 s. f.)	M1 A1  M1 A1 E1 5	Appropriate (sequence of) <i>suvat</i>  [WW M0 A0]  Use of $F = ma$ with <b>their</b> 786.4 and <b>their</b> $a$ . No extra forces. Allow sign errors. All correct using <b>their</b> 786.4 and $a$ Use of N2L clearly shown. (Accept 0.5 used WW)
(v)	Using N2L in <b>i</b> direction <b>either</b> $125 - 200 = 250a_1$ <b>or</b> (starting again) $786.4017\dots - (200 + 661.4017\dots) = 250 a_1$  so $a_1 = -0.3$ Using $v^2 = u^2 + 2 a_1 s$  $v^2 = 1.8^2 + 2 \times (-0.3) \times 1.65$ $v = 1.5$ so $1.5 \text{ m s}^{-1}$	M1  F1 M1  F1 A1 5	Use of $F = ma$ with <b>their</b> values.  Allow 1 force missing  FT only <b>their</b> 786... and <b>their</b> 661 Appropriate (sequence of) <i>suvat</i> with $u \neq 0$ . Must be ‘new’ $a$ obtained by using N2L. Only FT use of $\pm$ <b>their</b> $a_1$ cao
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