

GCE

Mathematics (MEI)

Advanced Subsidiary GCE **4766**

Statistics 1

Mark Scheme for June 2010

Q1 (i)	Positive skewness	B1	1																								
(ii)	<p>Inter-quartile range = $10.3 - 8.0 = 2.3$</p> <p>Lower limit $8.0 - 1.5 \times 2.3 = 4.55$</p> <p>Upper limit $10.3 + 1.5 \times 2.3 = 13.75$</p> <p>Lowest value is 7 so no outliers at lower end</p> <p>Highest value is 17.6 so at least one outlier at upper end.</p>	<p>B1</p> <p>M1 for $8.0 - 1.5 \times 2.3$</p> <p>M1 for $10.3 + 1.5 \times 2.3$</p> <p>A1</p> <p>A1</p>	5																								
(iii)	<p>Any suitable answers</p> <p>Eg minimum wage means no very low values</p> <p>Highest wage earner may be a supervisor or manager or specialist worker or more highly trained worker</p>	<p>E1 one comment relating to low earners</p> <p>E1 one comment relating to high earners</p>	2																								
		TOTAL	8																								
Q2 (i)	<p>$4k + 6k + 6k + 4k = 1$</p> <p>$20k = 1$</p> <p>$k = 0.05$</p>	<p>M1</p> <p>A1 NB Answer given</p>	2																								
(ii)	<p>$E(X) = 1 \times 0.2 + 2 \times 0.3 + 3 \times 0.3 + 4 \times 0.2 = 2.5$ (or by inspection)</p> <p>$E(X^2) = 1 \times 0.2 + 4 \times 0.3 + 9 \times 0.3 + 16 \times 0.2 = 7.3$</p> <p>$\text{Var}(X) = 7.3 - 2.5^2 = 1.05$</p>	<p>M1 for $\sum rp$ (at least 3 terms correct)</p> <p>A1 CAO</p> <p>M1 for $\sum r^2 p$ (at least 3 terms correct)</p> <p>M1dep for – their $E(X)^2$</p> <p>A1 FT their $E(X)$</p> <p>provided $\text{Var}(X) > 0$</p>	5																								
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Q3 (i)	<table border="1"> <thead> <tr> <th>Lifetime (x hours)</th><th>Frequency</th><th>Width</th><th>FD</th></tr> </thead> <tbody> <tr> <td>$0 < x \leq 20$</td><td>24</td><td>20</td><td>1.2</td></tr> <tr> <td>$20 < x \leq 30$</td><td>13</td><td>10</td><td>1.3</td></tr> <tr> <td>$30 < x \leq 50$</td><td>14</td><td>20</td><td>0.7</td></tr> <tr> <td>$50 < x \leq 65$</td><td>21</td><td>15</td><td>1.4</td></tr> <tr> <td>$65 < x \leq 100$</td><td>18</td><td>35</td><td>0.51</td></tr> </tbody> </table> 	Lifetime (x hours)	Frequency	Width	FD	$0 < x \leq 20$	24	20	1.2	$20 < x \leq 30$	13	10	1.3	$30 < x \leq 50$	14	20	0.7	$50 < x \leq 65$	21	15	1.4	$65 < x \leq 100$	18	35	0.51	<p>M1 for fds</p> <p>A1 CAO</p> <p>Accept any suitable unit for fd such as eg freq per 10 hours.</p> <p>L1 linear scales on both axes and label on vert axis</p> <p>W1 width of bars</p> <p>H1 height of bars</p>	5
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(ii)	Median lies in third class interval ($30 < x \leq 50$) Median = 45.5th lifetime (which lies beyond 37 but not as far as 51)	B1 CAO E1 <i>dep</i> on B1	2
		TOTAL	7
Q4 (i)	$1 \times \frac{1}{5} = \frac{1}{5}$	M1 A1	2
(ii)	$1 \times \frac{4}{5} \times \frac{3}{5} \times \frac{2}{5} \times \frac{1}{5} = \frac{24}{625} = 0.0384$	M1 For $1 \times \frac{4}{5} \times \text{or just } \frac{4}{5} \times$ M1 <i>dep</i> for fully correct product A1	3
(iii)	$1 - 0.0384 = 0.9616$ or 601/625	B1	1
		TOTAL	6
Q5 (i)	Mean = $\frac{0 \times 37 + 1 \times 23 + 2 \times 11 + 3 \times 3 + 4 \times 0 + 5 \times 1}{75} = \frac{59}{75} = 0.787$ $S_{xx} =$ $0^2 \times 37 + 1^2 \times 23 + 2^2 \times 11 + 3^2 \times 3 + 4^2 \times 0 + 5^2 \times 1 - \frac{59^2}{75} = 72.59$ $s = \sqrt{\frac{72.59}{74}} = 0.99$	M1 A1 M1 for Σfx^2 s.o.i. M1 <i>dep</i> for good attempt at S_{xx} BUT NOTE M1M0 if their $S_{xx} < 0$ A1 CAO	5
(ii)	New mean = $0.787 \times £1.04 = £0.818$ or 81.8 pence New s = $0.99 \times £1.04 = £1.03$ or 103 pence	B1 fit their mean B1 fit their s B1 for correct units <i>dep</i> on at least 1 correct (ft)	3
		TOTAL	8
Section B			
Q6 (i)	$X \sim B(18, 0.1)$ (A) $P(2 \text{ faulty tiles}) = \binom{18}{2} \times 0.1^2 \times 0.9^{16} = 0.2835$ OR from tables $0.7338 - 0.4503 = 0.2835$ (B) $P(\text{More than 2 faulty tiles}) = 1 - 0.7338 = 0.2662$	M1 $0.1^2 \times 0.9^{16}$ M1 $\binom{18}{2} \times p^2 q^{16}$ A1 CAO OR: M2 for $0.7338 - 0.4503$ A1 CAO M1 $P(X \leq 2)$ M1 <i>dep</i> for $1 - P(X \leq 2)$ A1 CAO	3 3

	(C) $E(X) = np = 18 \times 0.1 = 1.8$	M1 for product 18×0.1 A1 CAO	2
(ii)	(A) Let p = probability that a randomly selected tile is faulty $H_0: p = 0.1$ $H_1: p > 0.1$	B1 for definition of p in context B1 for H_0 B1 for H_1	3
	(B) H_1 has this form as the manufacturer believes that the number of faulty tiles may <u>increase</u> .	E1	1
(iii)	Let $X \sim B(18, 0.1)$ $P(X \geq 4) = 1 - P(X \leq 3) = 1 - 0.9018 = 0.0982 > 5\%$ $P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.9718 = 0.0282 < 5\%$ So critical region is $\{5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\}$	B1 for 0.0982 B1 for 0.0282 M1 for at least one comparison with 5% A1 CAO for critical region <i>dep</i> on M1 and at least one B1	4
(iv)	4 does not lie in the critical region, (so there is insufficient evidence to reject the null hypothesis and we conclude that there is not enough evidence to suggest that the number of faulty tiles has increased.	M1 for comparison A1 for conclusion in context	2
		TOTAL	18
Q7 (i)		G1 first set of branches G1 <i>indep</i> second set of branches G1 <i>indep</i> third set of branches G1 labels	4

(ii)	<p>(A) $P(\text{all on time}) = 0.95^3 = 0.8574$</p> <p>(B) $P(\text{just one on time}) =$ $0.95 \times 0.05 \times 0.4 + 0.05 \times 0.6 \times 0.05 + 0.05 \times 0.4 \times 0.6$ $= 0.019 + 0.0015 + 0.012 = 0.0325$</p> <p>(C) $P(1200 \text{ is on time}) =$ $0.95 \times 0.95 \times 0.95 + 0.95 \times 0.05 \times 0.6 + 0.05 \times 0.6 \times 0.95 +$ $0.05 \times 0.4 \times 0.6 = 0.857375 + 0.0285 + 0.0285 + 0.012 = 0.926375$</p>	<p>M1 for 0.95^3 A1 CAO</p> <p>M1 first term M1 second term M1 third term A1 CAO</p> <p>M1 any two terms M1 third term M1 fourth term A1 CAO</p>	<p>2</p> <p>4</p> <p>4</p>
(iii)	<p>$P(1000 \text{ on time given } 1200 \text{ on time}) =$ $P(1000 \text{ on time and } 1200 \text{ on time}) / P(1200 \text{ on time}) =$ $\frac{0.95 \times 0.95 \times 0.95 + 0.95 \times 0.05 \times 0.6}{0.926375} = \frac{0.885875}{0.926375} = 0.9563$</p>	<p>M1 either term of numerator M1 full numerator M1 denominator A1 CAO</p>	<p>4</p>
		Total	18