RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced Subsidiary GCE

Unit **4721:** Core Mathematics 1

Mark Scheme for June 2011

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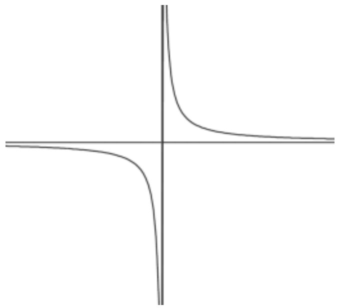
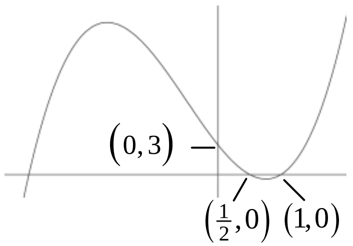
1	$3(x^2-6x)+4$	**B1**		$p=3$	If p, q, r found correctly, then **ISW** slips in format. $3(x - 3)^2 + 23$ **B1 B1 M0 A0** $3(x - 3) - 23$ **B1 B1 M1 A1 (BOD)** $3(x - 3x)^2 - 23$ **B1 B0 M1 A0** $3(x^2 - 3)^2 - 23$ **B1 B0 M1 A0** $3(x + 3)^2 - 23$ **B1 B0 M1 A1 (BOD)** $3x(x - 3)^2 - 23$ **B0 B1M1A1**
	$=3\left[(x-3)^2-9\right]+4$	**B1**		$(x-3)^2$ seen or $q=-3$	
	$=3(x-3)^2-23$	**M1**		$4-3q^2$ or $\frac{4}{3}-q^2$ (their q)	
		A1	4 **4**	$r=-23$	
2 (i)		**B1**		Reasonably correct curve for $y=\frac{1}{x}$ in 1st and 3rd quadrants only	N.B. Ignore ‘feathering’ now that answers are scanned. Reasonably correct shape, not touching axes more than twice.
		B1	2	Very good curves for $y=\frac{1}{x}$ in 1st and 3rd quadrants **SC** If 0, very good single curve in either 1st or 3rd quadrant and nothing in other three quadrants. **B1**	Correct shape, not touching axes, asymptotes clearly the axes. Allow slight movement away from asymptote at one end but not more. Not finite.
(ii)	Translation	**B1**		**Must** be translation/translated – not shift, move etc.	For “parallel to the y axis” allow “vertically”, “up”, “in the (positive) y direction”. **Do not accept** “in/on/across/up/along the y axis”
	4 units parallel to y axis	**B1**	2 **4**	Or $\begin{pmatrix}0\\4\end{pmatrix}$	
3 (i)	$\dfrac{16x^2\times 2x^3}{x}$	**B1**		32	
	$=32x^4$	**B1**	2	x^4	
(ii)	$\frac{1}{6}x$	**M1**		6 or $\frac{1}{36^{\frac{1}{2}}}$ or $\frac{1}{\sqrt{36}}$ seen	$\dfrac{1}{\frac{1}{\sqrt{36}}}$ is M0
		A1		$\frac{1}{6}$ in final answer	
		B1	3 **5**	x (Allow x^1) in final answer	$\pm\frac{1}{6}$ is **A0**

4	$2x^2 - 8x + 8 = 26 - 3x$	**M1**		Attempt to eliminate x or y	Must be a clear attempt to reduce to one variable. Condone poor algebra for first mark.
	$2x^2 - 5x - 18 (= 0)$	**A1**		Correct 3 term quadratic (not necessarily all in one side)	<u>If x eliminated:</u>
	$(2x-9)(x+2)(=0)$	**M1**		Correct method to solve quadratic	$y = 2(\frac{26-y}{3} - 2)^2$
	$x = \frac{9}{2}, x = -2$	**A1**		x values correct	Leading to $2y^2 - 89y + 800 = 0$
	$y = \frac{25}{2}, y = 32$	**A1**	5	y values correct	(2y – 25)(y – 32) =0 etc.
			5	**SR** If A0 A0, one correct pair of values, spotted or from correct factorisation **www B1**	
5 (i)	$10\sqrt{3} - 4\sqrt{3}$	**M1**		Attempt to express both surds in terms of $\sqrt{3}$	e.g. $\sqrt{3\text{x}100} - \sqrt{3\text{x}16}$
		B1		One term correct	
	$= 6\sqrt{3}$	**A1**	3	Fully correct (not $\pm 6\sqrt{3}$)	
(ii)	$\frac{\sqrt{5}(15+\sqrt{40})}{5}$	**M1**		Multiply numerator and denominator by $\sqrt{5}$ or - $\sqrt{5}$ **or** attempt to express both terms of numerator in terms of $\sqrt{5}$ (e.g. dividing both terms by $\sqrt{5}$)	Check both numerator and denominator have been multiplied
	$= \frac{15\sqrt{5} + 10\sqrt{2}}{5}$	**B1**		One of a, b correctly obtained	
	$= 3\sqrt{5} + 2\sqrt{2}$	**A1**	3 **6**	Both a = 3 and b=2 correctly obtained	

6	$k = x^{\frac{1}{4}}$	M1*		Use a substitution to obtain a quadratic or factorise into 2 brackets each containing $x^{\frac{1}{4}}$	**No marks** unless evidence of substitution (quadratic seen or square rooting or squaring of roots found). = 0 may be implied.
	$3k^2 - 8k + 4 = 0$				
	$(3k-2)(k-2) = 0$	DM1		Correct method to solve a quadratic	Allow $x = x^{\frac{1}{4}}$ as a substitution.
	$k = \frac{2}{3}$ or $k = 2$	A1			**No marks** if straight to quadratic formula to get x = "$\frac{2}{3}$" x = "2" and no further working
	$x = \left(\frac{2}{3}\right)^4$ or $x = 2^4$	M1		Attempt to calculate k^4	**No marks** if $k = x^{\frac{1}{4}}$ then $3k - 8k^2 + 4 = 0$
	$x = \frac{16}{81}$ or $x = 16$	A1	5 **5**		**SC** If **M0** Spotted solutions **www B1** each Justifies 2 solutions exactly **B3**
	<u>If candidates use $k = x^{\frac{1}{2}}$ and rearrange:</u>				
	$3k - 8\sqrt{k} + 4 = 0$ $8\sqrt{k} = 3k + 4$ $64k = 9k^2 + 24k + 16$	M1*		Substitute, rearrange and square both sides	
	$9k^2 - 40k + 16 = 0$ $(9k-4)(k-4) = 0$	DM1		Correct method to solve quadratic	
	$k = \frac{4}{9}$ or $k = 4$	A1			
	$x = \left(\frac{4}{9}\right)^2$ or $x = 4^2$	M1		Attempt to calculate k^2	
	$x = \frac{16}{81}$ or $x = 16$	A1			
7 (i)	$-14 \le 6x \le -5$	M1		2 equations or inequalities both dealing with all 3 terms resulting in $a \le 6x \le b$, $a \ne -9$, $b \ne 0$	**Do not ISW** after correct answer if contradictory inequality seen.
		A1		-14 and -5 seen **www**	
	$-\frac{7}{3} \le x \le -\frac{5}{6}$	A1	3	Accept as two separate inequalities provided not linked by "or" (must be <u>≤</u>)	Allow $-\frac{14}{6} \le x \le -\frac{5}{6}$
(ii)	$0 < x^2 - 4x - 12$	M1		Rearrange to collect all terms on one side	**Do not ISW** after correct answer if contradictory inequality seen.
	$(x-6)(x+2)$	M1		Correct method to find roots	
		A1		6, -2 seen	
	$x > 6$, $x < -2$	M1	5	Correct method to solve quadratic inequality i.e. $x >$ their higher root, $x <$ their lower root	
		A1	**8**	(not wrapped, strict inequalities, no 'and')	e.g. for last two marks, $-2 > x > 6$ scores **M1 A0**

8 (i)	$\frac{dy}{dx} = 6x + 6x^{-2}$	**M1** **A1**		Attempt to differentiate (one non-zero term correct) Completely correct	**NB** – $x = -1$ (and therefore possibly $y = 7$) can be found from equating the incorrect differential
	$6x + \frac{6}{x^2} = 0$	**M1**		Sets their $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	$\frac{dy}{dx} = 6x + 6$ to 0. This could score **M1A0 M1A0A1 ft**
	$x = -1$	**A1**		Correct value for x - **www**	
	$y = 7$	**A1 ft**	5	Correct value of y for *their* value of x	If more than one value of x found, allow **A1 ft** for one correct value of y
(ii)	$\frac{d^2y}{dx^2} = 6 - 12x^{-3}$	**M1**		Correct method e.g. substitutes their x from (i) into their $\frac{d^2y}{dx^2}$ (must involve x) and considers sign.	Allow comparing signs of their $\frac{dy}{dx}$ either side of their "– 1", comparing values of y to their "7"
	When $x = -1$, $\frac{d^2y}{dx^2} > 0$ so minimum pt	**A1 ft**	2 **7**	**ft** from their $\frac{dy}{dx}$ differentiated correctly and correct substitution of *their* value of x and consistent final conclusion **NB** If second derivate evaluated, it must be correct (18 for $x = -1$). If more than one value of x used, max **M1 A0**	**SC** $\frac{d^2y}{dx^2}$ = a constant correctly obtained from their $\frac{dy}{dx}$ and correct conclusion (ft) **B1**

9 (i)	Gradient of $AB = \frac{1-3}{7-1} = -\frac{1}{3}$ Gradient of $AC = \frac{-9-3}{-3-1} = 3$ Vertex A **OR:** Length of $AB = \sqrt{(7-1)^2+(1-3)^2} = \sqrt{40}$ $AC = \sqrt{(-3-1)^2+(-9-3)^2} = \sqrt{160}$ $BC = \sqrt{(-3-7)^2+(-9-1)^2} = \sqrt{200}$ Shows that $AB^2 + AC^2 = BC^2$ Vertex A	**M1*** **A1** **A1** **M1** **DB1** **M1*** **A1** **A1** **M1** **DB1**	5	Uses $\frac{y_2-y_1}{x_2-x_1}$ for any 2 points One correct gradient (may be for gradient of BC =1) Gradients for both AB and AC found correctly Attempts to show that $m_1 \times m_2 = -1$ oe, accept "negative reciprocal" Correct use of Pythagoras, square rooting not needed Any length or length squared correct All three correct Correct use of Pythagoras to show $AB^2 + AC^2 = BC^2$	Do not allow final mark if vertex A found from wrong working. (Dependent on 1st M 1 A1 A1) Accept BÂC etc for vertex A or "between AB and AC" Allow if marked on diagram. i.e must add squares of shorter two lengths
9 (ii)	Midpoint of BC is $\left(\frac{7+-3}{2}, \frac{1+-9}{2}\right)$ = (2, -4) Length of BC = $\sqrt{(-3-7)^2+(-9-1)^2} = \sqrt{200} = 10\sqrt{2}$ Radius = $5\sqrt{2}$ $(x-2)^2+(y+4)^2 = (5\sqrt{2})^2$ $(x-2)^2+(y+4)^2 = 50$ $x^2+y^2-4x+8y-30=0$	**M1*** **A1** **M1**** **DM1*** **DM1**** **A1** **A1**	7 **12**	Uses $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ o.e. for BC, AB or AC (3 out of 4 subs correct) Correct centre (**cao**) Correct method to find d or r or d^2 or r^2 o.e. for BC, AB or AC (must be consistent with their midpoint if found) $(x-a)^2 + (y-b)^2$ seen for their centre $(x-a)^2 + (y-b)^2$ = their r^2 Correct equation Correct equation in required form	<u>Substitution method 1</u> (into $x^2 + y^2 + ax + by + c = 0$) Substitutes all 3 points to get 3 equations in a,b,c **M1** At least 2 equations correct **A1** Correct method to find one variable **M1** One of a, b, c correct **A1** Correct method to find other values **M1** All values correct **A1** Correct equation in required form **A1** <u>Alternative markscheme for last 4 marks with f,g,c method</u>: $x^2 - 4x + y^2 + 8y$ for their centre **DM1*** $c = (\pm 2)^2 + 4^2 - 50$ **DM1**** $c = -30$ **A1** Correct equation in required form **A1** <u>Ends of diameter method (p, q) to (c, d):</u> Attempts to use $(x-p)(x-c) + (y-q)(y-d) = 0$ for BC,AC or AB **M2** $(x-7)(x+3) + (y-1)(y+9) = 0$ **A2** for both x brackets correct, **A2** for both y brackets correct $x^2 + y^2 - 4x + 8y - 30 = 0$ **A1** **SC** If **M2 A0 A0** then **B1** if both x brackets correct and **B1** if both y brackets correct for **AC** or **AB**

					Substitution method 2 into $(x-p)^2+(y-q)^2=$ their r^2 Correct method to find d or r or d^2 or r^2 ***M1** Substitutes all 3 points to get 3 equations in p,q **DM1** At least 2 equations correct **A1** Correct method to find one variable **M1** One of p, q correct **A1** Correct equation [$(x-2)^2+(y+4)^2=50$]**A1** Correct equation in required form [$x^2+y^2-4x+8y-30=0$]**A1**
10(i)		**B1** **B1** **B1**	 3	+ve cubic with 3 distinct roots (0, 3) labelled or indicated on y-axis (-3, 0), ($\frac{1}{2}$, 0) and (1, 0) labelled or indicated on x-axis and no other x- intercepts	For first **B1**, left end of curve must finish below x axis and right end must end above x axis. Allow slight wrong curvature at one end but not both ends. No cusp at either turning point. No straight lines drawn with a ruler. Condone (0, 3) as maximum point. To gain second and third **B** marks, there must be an attempt at a curve, not just points on axes. Final **B1** can be awarded for a negative cubic.
(ii)	$2x^2+5x-3$, x^2+2x-3, $2x^2-3x+1$ $(2x^2+5x-3)(x-1)$ $2x^3+3x^2-8x+3$ $\frac{dy}{dx}=6x^2+6x-8$ When $x=1$, gradient = 4	**B1** **M1** **A1** **M1** **A1** **A1**	 6	Obtain one quadratic factor (can be unsimplified) Attempt to multiply a quadratic by a linear factor Attempt to differentiate (one non-zero term correct) Fully correct expression **www** Confirms gradient = 4 at x = 1 ****AG**	Alternative for first 3 marks: Attempt to expand all 3 brackets with an appropriate number of terms (including an x^3 term) **M1** Expansion with at most 1 incorrect term **A1** Correct, answer (can be unsimplified) **A1** Allow if done in part(i) please check.
(iii)	Gradient of $l=4$ On curve, when $x=-2$, $y=15$ $y-15=4(x+2)$ $y=4x+23$	**B1** **B1** **M1** **A1**	 4	May be embedded in equation of line Correct y coordinate Correct equation of line using their values Correct answer **in correct form**	 **M** mark is for any equation of line with any non-zero numerical gradient through (-2, their evaluated y)
(iv)	Attempt to find gradient of curve when $x=-2$ $6(-2)^2+6(-2)-8=4$ So line is a tangent	**M1** **A1** **A1**	 3 **16**	Substitute $x=-2$ into their $\frac{dy}{dx}$ Obtain gradient of 4 **CWO** Correct conclusion	Alternatives **1)** Equates equation of l to equation of curve and attempts to divide resulting cubic by $(x+2)$ **M1** Obtains $(x+2)^2(2x-5)$ (=0) **A1** Concludes repeated root implies tangent at x = -2 **A1** **2)** Equates their gradient function to 4 and uses correct method to solve the resulting quadratic **M1** Obtains $(x+2)(x-1)=0$ oe **A1** Correctly concludes gradient = 4 when $x=-2$ **A1**

<u>Allocation of method mark for solving a quadratic</u>

e.g. $2x^2 - 5x - 18 = 0$

1) If the candidate attempts to solve by factorisation, their attempt when expanded must produce the **correct quadratic term** and **one other correct term** (with correct sign):

$(2x+2)(x-9)=0$	**M1**	$2x^2$ and -18 obtained from expansion
$(2x+3)(x-4)=0$	**M1**	$2x^2$ and $-5x$ obtained from expansion
$(2x-9)(x-2)=0$	**M0**	only $2x^2$ term correct

2) If the candidate attempts to solve by using the formula

a) If the formula is quoted incorrectly then **M0.**

b) If the formula is quoted correctly then one **sign** slip is permitted. Substituting the wrong numerical value for a or b or c scores **M0**

$\dfrac{-5 \pm \sqrt{(-5)^2 - 4 \times 2 \times -18}}{2 \times 2}$	earns **M1** (minus sign incorrect at start of formula)
$\dfrac{5 \pm \sqrt{(-5)^2 - 4 \times 2 \times 18}}{2 \times 2}$	earns **M1** (18 for *c* instead of −18)
$\dfrac{-5 \pm \sqrt{(-5)^2 - 4 \times 2 \times 18}}{2 \times 2}$	**M0** (2 sign errors: initial sign and *c* incorrect)
$\dfrac{5 \pm \sqrt{(-5)^2 - 4 \times 2 \times -18}}{2 \times -5}$	**M0** (2*b* on the denominator)

Notes – for equations such as $2x^2 - 5x - 18 = 0$, then $b^2 = 5^2$ would be condoned in the discriminant and would not be counted as a sign error. Repeating the sign error for *a* in both occurrences in the formula would be two sign errors and score **M0**.

c) If the formula is not quoted at all, substitution must be completely correct to earn the **M1**

3) If the candidate attempts to complete the square, they must get to the “square root stage” involving ±; we are looking for evidence that the candidate knows a quadratic has two solutions!

$$2x^2 - 5x - 18 = 0$$

$$2\left(x^2 - \frac{5}{2}x\right) - 18 = 0$$

$$2\left[\left(x - \frac{5}{4}\right)^2 - \frac{25}{16}\right] - 18 = 0$$

$$\left(x - \frac{5}{4}\right)^2 = \frac{169}{16}$$

$$x - \frac{5}{4} = \pm\sqrt{\frac{169}{16}}$$

This is where the **M1** is awarded – arithmetical errors may be condoned provided $x - \frac{5}{4}$ seen or implied

If a candidate makes repeated attempts (e.g. fails to factorise and then tries the formula), mark only what you consider to be their last full attempt.

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

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14 – 19 Qualifications (General)
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Facsimile: 01223 552627
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