GCE

Mathematics (MEI)

Mark Scheme for June 2011

SECTION A

1	$\frac{1}{2}x^4 + 3x$	**M1**	accept unsimplified	ignore + c
	F[5] – F[2]	**M1**	at least one term correctly integrated, may be implied by A1	condone omission of brackets
	[=327.5 − 14]			
	=313.5 o.e.	**A1**		313.5 unsupported scores 0
2	0.05, 2000, 1.25×10^{-6} or	**B2**	**B1** for two correct	
	$\frac{1}{20}$, 2000, $\frac{1}{800\,000}$ o.e.			
	divergent	**B1**	allow "alternate terms tend to zero and to infinity" o.e.	do *not* allow "oscillating", "getting bigger and smaller", "getting further apart"
3	(i) $m =$	**M1**		no marks for use of Chain Rule or any other attempt to differentiate
	$\dfrac{\sqrt{1+2\times 4.1} - \sqrt{1+2\times 4}}{4.1 - 4}$ s.o.i			
	grad $= \dfrac{\sqrt{9.2} - \sqrt{9}}{4.1 - 4}$ s.o.i	**M1**		**SC2** for 0.33…. appearing only embedded in equation of chord
	0.3315 cao	**A1**		
3	(ii) selection of value in (4, 4.1) and 4 or of two values in [3.9, 4.1] centred on 4	**M1**		allow selection of 4 and value in (3.9, 4)
	answer closer to 1/3 than 0.3315(...)	**A1**		
4	$6 = ab$ and $3.6 = ab^2$	**M1**	$\log 6 = \log a + \log b$ *and* $\log 3.6 = \log a + \log b^2$	
	$a = 10$, $b = 0.6$ c.a.o.	**A2**	**A1** each; if **M0** then **B3** for both, **B1** for one	

5	$[\dfrac{dy}{dx} =] 32x^3$ c.a.o.	**M1**		
	substitution of $x = \frac{1}{2}$ in their $\dfrac{dy}{dx}$	**M1**	$[= 4]$	must see kx^3
	grad normal $= \dfrac{-1}{their\,4}$	**M1**		their 4 must be obtained by calculus
	when $x = \frac{1}{2}$, $y = 4\frac{1}{2}$ o.e.	**B1**		
	$y - 4\frac{1}{2} = -\frac{1}{4}(x - \frac{1}{2})$ i.s.w	**A1**	$y = -\frac{1}{4}x + 4\frac{5}{8}$ o.e.	
6	$\dfrac{dy}{dx} = 6x^{\frac{1}{2}} - 2$ $y = kx^{\frac{3}{2}} - 2x + c$ o.e.	**M2**	**M1** for $k\,x^{\frac{3}{2}}$ and **M1** for $-2x + c$	$x^{\frac{1}{6}}$ is a mistake, not a misread
	$y = 4x^{\frac{3}{2}} - 2x + c$ o.e.	**A1**		"$y =$" need not be stated at this point, but must be seen at some point for full marks
	correct substitution of $x = 9$ and $y = 4$ in their equation of curve	**M1** dep	dependent on at least **M1** already awarded	must see "$+ c$"
	$y = 4x^{\frac{3}{2}} - 2x - 86$	**A1**	allow **A1** for $c = -86$ i.s.w. if simplified equation for y seen earlier	

7	$\dfrac{\sin\theta}{\cos\theta}=2\sin\theta$	**M1**	*may* be implied by $2\cos\theta - 1 = 0$ or better	or, if to advantage of candidate **B4** for all 5 correct
	$2\cos\theta - 1 = 0$ *and* $\sin\theta = 0$	**A1**		**B3** for 4 correct
	$[\theta =\,]$ 0, 180, 360,	**B1**		**B2** for 3 correct
	$[\theta =\,]$ 60, 300	**B1**		**B1** for 2 correct
	if 4 marks awarded, lose 1 mark for extra values in the range, ignore extra values outside the range			if extra value(s) in range, deduct one mark from total do not award if values embedded in trial and improvement approach
8	$\log p = \log s + \log t^n$	**M1**	or $\dfrac{p}{s} = t^n$	
	$\log p = \log s + n\log t$	**M1**	$n\log t = \log\left(\dfrac{p}{s}\right)$	or **A2** for $[n =\,]\log_t\left(\dfrac{p}{s}\right)$ [base t needed] following first M1
	$[n =]\dfrac{\log p - \log s}{\log t}$ or $\dfrac{\log\left(\dfrac{p}{s}\right)}{\log t}$ [base not required]	**A1**	as final answer (i.e. penalise further incorrect simplification)	
9	$\log 16^{\frac{1}{2}}$ or $[-]\log 5^2$ s.o.i.	**M1**		if $a = 10$ assumed, $x = 12$ c.a.o. scores **B3** www
	$\log(4\times 75)$ or $\log\dfrac{75}{25}$ s.o.i.	**M1**	$x = \dfrac{4\times 75}{25}$ implies **M1M1**	no follow through
	$x = 12$ www	**A1**		
10	$t_1 = -\sin\theta$	**B1**	www	
	$t_2 = \sin\theta$	**B1**	www	e.g. $\sin(\theta + 360) = \sin\theta + \sin 360 = \sin\theta$ **B0**

Section A Total: 36

SECTION B

11	(i) $200 - 2\pi r^2 = 2\pi rh$	**M1**	$100 = \pi r^2 + \pi rh$	**sc3** for complete argument working backwards: $V = 100r - \pi r^3$
	$h = \dfrac{200 - 2\pi r^2}{2\pi r}$ o.e.	**M1**	$100r = \pi r^3 + \pi r^2 h$	$\pi r^2 h = 100r - \pi r^3$ $\pi rh = 100 - \pi r^2$
	substitution of correct h into $V = \pi r^2 h$	**M1**	$100r = \pi r^3 + V$	$100 = \pi rh + \pi r^2$ $200 = A = 2\pi rh + 2\pi r^2$
	$V = 100r - \pi r^3$ convincingly obtained	**A1**	$V = 100r - \pi r^3$	**sc0** if argument is incomplete
			or	
			M1 for $h = \dfrac{V}{\pi r^2}$	
			M1 for $200 = 2\pi r^2 + 2\pi r \times \dfrac{V}{\pi r^2}$	
			M1 for $200 = 2\pi r^2 + 2\dfrac{V}{r}$	
			A1 for $V = 100r - \pi r^3$ convincingly obtained	
11	(ii) $\dfrac{dV}{dr} = 100 - 3\pi r^2$	**B2**	**B1** for each term	allow $9.42(....)\, r^2$ or better if decimalised
	$\dfrac{d^2V}{dr^2} = -6\pi r$	**B1**		$-18.8(...)\, r$ or better if decimalised

11	(iii) their $\dfrac{dV}{dr} = 0$ s.o.i.	**M1**	must contain r as the only variable	
	$r = 3.26$ c.a.o.	**A2**	**A1** for $r = (\pm)\sqrt{\dfrac{100}{3\pi}}$; may be implied by 3.25…	
	$V = 217$ c.a.o.	**A1**	deduct 1 mark only in this part if answers not given to 3 sf,	there must be evidence of use of calculus

12	(i)(A) 390	**B2**	**M1** for $500 - 11 \times 10$	
12	(i)(B) $S_{24} = \frac{24}{2}\left(2\times500+(24-1)\times-10\right)$ o.e. i.s.w. *or* $S_{24} = \frac{24}{2}\left(500+270\right)$ o.e. i.s.w. [=9240] (answer given)	**B2**	nothing simpler than $12(1000 + 23\times\text{-}10)$ or $\frac{24}{2}(1000-230)$ or $12(2\times500-230)$ if **B2** not awarded, then **M1** for use of a.p. formula for S_{24} with $n = 24$, $a = 500$ and $d = $ -10 or **M1** for $l = 270$ s.o.i.	condone omission of final bracket or "(23)-10" if recovered in later work if they write the sum out, all the terms must be listed for 2 marks $12 \times (1000 - 230)$ or 12×770 on its own do not score
12	(ii)(A) 368.33(...) or 368.34	**B2**	**M1** for 460×0.98^{11}	
12	(ii)(B) $J_{20} = 310$ $M_{20} = 313.36(...)$, 313.4, 313.3, 313.37 or 313 $J_{19} = 320$ $M_{19} = 319.76(...)$, 319.8 or 319.7	**B3**	**B3** for all 4 values correct or **B2** for 3 values correct or **B1** for 2 values correct	values which are clearly wrongly attributed do not score
12	(ii)(C) 8837 to 8837.06	**B2**	**M1** for $S_{24} = \frac{460\left(1-0.98^{24}\right)}{1-0.98}$ o.e.	
12	(ii)(D) $\frac{a(1-0.98^{24})}{(1-0.98)} = 9240$ o.e. 480.97 to 480.98	**M1** **A1**	f.t. their power of 24 from (ii)C	

13	(i) arc AC = 2.1 × 1.8	**M1**	$\dfrac{103}{360} \times 2\pi \times 2.1$	103° or better
	= 3.78 c.a.o.	**A1**		3.78 must be seen but may be embedded in area formula
	area = their 3.78 × 5.5 = 20.79 or 20.8 i.s.w.	**M1** **dep*** **A1**	dependent on first M1	
13	(ii) BD = 2.1 cos (π − 1.8) or 2.1cos1.3(4159…..) or 2.1sin0.2(292…) r.o.t to 1 d.p. or more	**M2**	**M1** for $\cos(\pi - 1.8) = \dfrac{BD}{2.1}$ o.e.	**M2** for BD = 2.1 cos 76.8675…° or 2.1sin13.1324…rounded to 2 or more sf or **M2** for CD = 2.045... r.o.t. to 3 s.f. or better and BD = √(2.1² − 2.045²)
	= 0.48	**A1**	allow any answer which rounds to 0.48	
13	(iii) sector area = 3.969	**M2**	**M1** for ½ × 2.1² × 1.8	or equivalent with degrees for first two Ms N.B. 5.5 × 3.969 = 21.8295 so allow M2 for 21.8295
	triangle area = 0.487 to 0.491	**M2**	**M1** for ½ × 2.1 × their 0.48 × sin (π − 1.8) *or* ½ × their 0.48 × 2.045.. r.o.t. to 3 s.f. or better	may be sin 1.8 instead of sin (π − 1.8) N.B. 5.5 × area = 2.6785 to 2.7005 so allow M2 for a value in this range
	24.5	**A1**	allow any answer which rounds to 24.5	

Section B Total: 36