RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4724:** Core Mathematics 4

Mark Scheme for June 2011

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

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1	Attempt to factorise **both** numerator & denominator	M1		completely or partially
	Num = e.g. $(x^2-1)(x^2-9)$ or $(x^2-2x-3)(x^2+2x-3)$	B1		or $(x-3)(x+3)(x-1)(x+1)$
	Denominator = e.g. $(x^2-2x-3)(x+5)(x+3)$	B1		or $(x-3)(x+1)(x+5)(x+3)$
	$\frac{x-1}{x+5}$ or $1-\frac{6}{x+5}$ WWW	A1	**4**	ISW but not if any further 'cancellation'
	Alternative start, attempting long division			
	Expand denom as quartic & attempt to divide $\frac{\text{numerator}}{\text{denominator}}$	M1		but not divide $\frac{\text{denominator}}{\text{numerator}}$
	Obtain quotient = 1 & remainder = $-6x^3-6x^2+54x+54$	B1		
	Final B1 A1 available as before			
				$\boxed{4}$
2	$2^2+(-3)^2+(\sqrt{12})^2$ soi e.g. 25 or 5	M1		Allow $2^2-3^2+\sqrt{12}^2$
	5	A1		May be implied by 5 or 1/5 in final answer
	$\frac{1}{5}\begin{pmatrix}2\\-3\\\sqrt{12}\end{pmatrix}$ or $\begin{pmatrix}\frac{2}{5}\\-\frac{3}{5}\\\frac{\sqrt{12}}{5}\end{pmatrix}$ AEF	√A1	**3**	FT their '5'. Accept $-\frac{1}{5}\begin{pmatrix}\\\\\end{pmatrix}$ or $\frac{1}{\pm 5}\begin{pmatrix}\\\\\end{pmatrix}$
				$\boxed{3}$
3	**(i)** The words quotient and remainder need not be explicit			
	Long division For leading term $3x$ in quotient	B1		
	Suff evidence of div process ($3x$, mult back, attempt sub)	M1		
	(Quotient) = $3x-1$	A1		
	(Remainder) = x **AG**	A1	**4**	No wrong working, partic on penult line
	Identity $3x^3-x^2+10x-3=Q(x^2+3)+R$	*M1		
	$Q=ax+b, R=cx+d$ & attempt at least 2 operations	dep*M1		If $a=3$, this $\Rightarrow$ 1 operation
	$a=3, b=-1$	A1		
	$c=1, d=0$	A1		No wrong working anywhere
	Inspection $3x^3-x^2+10x-3=(x^2+3)(3x-1)+x$	B2		or state quotient $=3x-1$
	Clear demonstration of LHS = RHS	B2		
(ii)	Change integrand to 'their (i) quotient' + $\frac{x}{x^2+3}$	M1		
	Correct FT integration of 'their (i) quotient'	√A1		
	$\int\frac{x}{x^2+3}\,dx=\frac{1}{2}\ln(x^2+3)$	A1		
	Exact value of integral = $\frac{1}{2}+\frac{1}{2}\ln 4-\frac{1}{2}\ln 3$ AEF ISW	A1	**4**	Answer as decimal value (only) $\rightarrow$ A0
				$\boxed{8}$

4	<u>Indefinite integral</u> Attempt to connect $\mathrm{d}x$ and $\mathrm{d}\theta$	M1		Incl $\frac{\mathrm{d}x}{\mathrm{d}\theta}=, \frac{\mathrm{d}\theta}{\mathrm{d}x}=$, $\mathrm{d}x = ...\mathrm{d}\theta$; not $\mathrm{d}x=\mathrm{d}\theta$
	Denominator $\left(1-9x^2\right)^{3/2}$ becomes $\cos^3\theta$	B1		
	Reduce original integral to $\frac{1}{3}\int \frac{1}{\cos^2\theta}\,\mathrm{d}\theta$	A1		May be implied, seen only as $\frac{1}{3}\int \sec^2\theta\,\mathrm{d}\theta$
	Change $\int \frac{1}{\cos^2\theta}\,\mathrm{d}\theta$ to $\tan\theta$	B1		Ignore $\frac{1}{3}$ at this stage
	Use <u>appropriate</u> limits for θ (allow degrees) or x	M1		Integration need not be accurate
	$\frac{\sqrt{3}}{9}$ AEF, exact answer required, ISW	A1	**6**	
				$\boxed{6}$
5 (i)	Attempt to set up 3 equations	M1		of type $4+3s=1, 6+2s=t, 4+s=-t$
	$(s,t)=(-1,4)$ or $(-1,-3)$ or $(-\frac{10}{3},-\frac{2}{3})$	*A1		or $s=-1$ & $-\frac{10}{3}$ <u>or</u> $t=$ two of $\left(4,-3,-\frac{2}{3}\right)$
	Show clear contradiction e.g. $3\neq -4,\ 4\neq -3,\ -6\neq 1$	dep*A1	**3**	Allow ✓ unsimpl contradictions. No ISW.
	<u>SC</u> If $s=\frac{-10}{3}$ found from 2nd & 3rd eqns and contradiction shown in 1st eqn, all 3 marks may be awarded.			
(ii)	Work with $\begin{pmatrix}3\\2\\1\end{pmatrix}$ and $\begin{pmatrix}0\\1\\-1\end{pmatrix}$	M1		
	Clear method for scalar product of any 2 vectors	M1		
	Clear method for modulus of any vector	M1		
	$79.1^{(\circ)}$ or better (79.1066..) 1.38 (rad) (1.38067..) ISW	A1	**4**	**(**From $\frac{1}{\sqrt{14}.\sqrt{2}}$)
(iii)	Use $\begin{pmatrix}4+3s\\6+2s\\4+s\end{pmatrix}.\begin{pmatrix}3\\2\\1\end{pmatrix}=0$	M1		
	Obtain $s=-2$	A1		from $12+9s+12+4s+4+s=0$
	A is $\begin{pmatrix}-2\\2\\2\end{pmatrix}$ or $-2\mathbf{i}+2\mathbf{j}+2\mathbf{k}$ final answer	<u>B</u>1	**3**	Accept $(-2,2,2)$
				$\boxed{10}$

6 $(1+ax)^{\frac{1}{2}} = 1+\frac{1}{2}ax$ …….. $+\dfrac{\frac{1}{2}\cdot\frac{-1}{2}}{2}(ax)^2$ B1, B1 N.B. third term = $-\frac{1}{8}a^2x^2$

Change $(4-x)^{-\frac{1}{2}}$ into $k\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$, where k is likely to be $\frac{1}{2}$ /2/4 / –2 , & work out expansion of $\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$

$\left(1-\frac{x}{4}\right)^{-\frac{1}{2}} = 1+\frac{1}{8}x$ …. $+\dfrac{\frac{-1}{2}\cdot\frac{-3}{2}}{2}\left(\dfrac{(-)x}{4}\right)^2$ B1, B1 N.B. third term = $\frac{3}{128}x^2$

OR Change $\{4-x\}^{\frac{1}{2}}$ into $l\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$, where l is likely to be $\frac{1}{2}$ / 2 / 4 /– 2,& work out expansion of $\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$

$\left(1-\frac{x}{4}\right)^{\frac{1}{2}} = 1-\frac{1}{8}x-\frac{1}{128}x^2$ B1 (for all 3 terms simplified)

$k=\frac{1}{2}$ (with possibility of M1 + A1 + A1 to follow) B1 $l=2$ (with no further marks available)

Multiply $(1+ax)^{\frac{1}{2}}$ by $(4-x)^{-\frac{1}{2}}$ or $\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$ M1 Ignore irrelevant products

The required three terms (with/without x^2) identified as

$-\frac{1}{16}a^2+\frac{1}{32}a+\frac{3}{256}$ or $\frac{-16a^2+8a+3}{256}$ AEF ISW A1+A1 **8** A1 for one correct term + A1 for other two

SC B1 for $\frac{1}{4}\left(1-\frac{x}{4}\right)^{-1}$; B1 for $\left(1-\frac{x}{4}\right)^{-1}=1+\frac{x}{4}+\frac{x^2}{16}$; M1 for multiplying $(1+ax)$ by their $(4-x)^{-1}$.

If result is $p+qx+rx^2$, then to find $\left(p+qx+rx^2\right)^{\frac{1}{2}}$ award B1 for $p^{\frac{1}{2}}(......)$,

B1 correct 1st & 2nd terms of expansion, B1 correct 3rd term; A1,A1 as before, for correct answers.

$\boxed{8}$

7 Attempt to sep variables in format $\int py^2\,(\mathrm{d}y) = \int \dfrac{q}{x+2}(\mathrm{d}x)$ M1 where constants p and/or q may be wrong

Either y^3 & $\ln(x+2)$ or $\frac{1}{3}y^3$ & $\frac{1}{3}\ln(x+2)$ A1+A1 Accept $\frac{1}{3}\ln(3x+6)$ for $\frac{1}{3}\ln(x+2)$ & $|\ |$ for ( )

If indefinite integrals are being used (most likely scenario)

Substitute $x=1, y=2$ into an eqn containing '+const' M1

Sub y = 1.5 and their value of 'const' & solve for *x or q* M1

x or q = –1.97 only A2

[SC x or q = –1.970 or –1.971 or –1.9705 or –1.9706 A1] **7**

If definite integrals are used (less likely scenario)

Use $\int_{1.5}^{2}...\mathrm{d}y = \int_{q}^{1}...\mathrm{d}x$ where 2 corresponds with 1..... M2 & 1.5 corresp with q (at top/bottom or v.v.)

Then A2 or SC A1 as above

Use $\int_{1.5}^{2}...\mathrm{d}y = \int_{1}^{q}...\mathrm{d}x$ where 2 corresponds with q….. M1 & 1.5 corresp with 1 (at top/bottom or v.v.)

Then A1 for 1.97 only

$\boxed{7}$

8 **Cartesian equation may be used in parts (i) - (iii) and corresponding marks awarded**

(i)	Sub parametric eqns into $y = 3x$ & produce $t = -2$			
	OR sub $t = -2$ into para eqs, obtain $(-1, -3)$ & state $y = 3x$			
	OR other similar methods producing (or verifying) $t = -2$	B1		
	Value of t at other point is 2	B1	**2**	$t = \pm 2$ is sufficient for B1+B1
(ii)	Use (not just quote) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	M1		
	$= -(t+1)^2$	A1		or $\frac{-1}{x^2}$ or $\frac{-(2+y)}{x}$
	Attempt to use $-\frac{1}{\frac{dy}{dx}}$ for gradient of normal	M1		
	Gradient normal = 1 cao	A1		
	Subst $t = -2$ into the parametric eqns.	M1		to find pt at which normal is drawn
	Produce $y = x - 2$ as equation of the normal WWW	A1	**6**	'A' marks in **(ii)** are dep on prev 'A'
(iii)	Substitute the parametric values into their eqn of normal	M1		
	Produce $t = 0$ as final answer cao	A1	**2**	This is dep on final A1 in (ii)

N.B. If $y = x - 2$ is found fortuitously in (ii) (& $\therefore$ given A0 in (ii)), you must award A0 here in (iii).

(iv)	Attempt to eliminate t from the parametric equations	M1		
	Produce any correct equation	A1		e.g. $x = \frac{1}{y+2}$
	Produce $y = \frac{1}{x} - 2$ or $y = \frac{1-2x}{x}$ ISW	A1	**3**	Must be seen in (iv)

{N.B. Candidate producing only $y = \frac{1}{x} - 2$ is awarded both A1 marks.}

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9 (i) Treat $x \ln x$ as a product	M1		If $\int \ln x$, use parts $u = \ln x$, $dv = 1$
Obtain $x.\frac{1}{x} + \ln x$	A1		$x \ln x - \int 1\,dx = x \ln x - x$
Show $x.\frac{1}{x} + \ln x - 1 = \ln x$ WWW **AG**	A1	**3**	And state given result
(ii)(a) Part (a) is mainly based on the indef integral $\int (\ln x)^2\,dx$			
[A candidate stating e.g. $\int (\ln x)^2\,dx = \int 2\ln x\,dx$ or $= \int (\ln x - x)^2\,dx$ is awarded 0 for (ii)(a)]			
Correct use of $\int \ln x\,dx = x\ln x - x$ anywhere in this part	B1		Quoted from (i) or derived
Use integ by parts on $\int (\ln x)^2\,dx$ with $u = \ln x, dv = \ln x$	M1		or $u = (\ln x)^2, dv = 1$
[For 'integration by parts, candidates must get to a 1st stage with format $f(x) +/- \int g(x)\,dx$]			
1st stage $= \ln x(x\ln x - x) - \int \frac{1}{x}(x\ln x - x)\,dx$ soi	A1		$x(\ln x)^2 - \int x.\frac{2}{x}\ln x\,dx$
2nd stage $= x(\ln x)^2 - 2x\ln x + 2x$ AEF (unsimplified)	A1		
$\therefore$ Value of definite integral between 1 & e $= e - 2$ cao	A1		Use limits on 2nd stage & produce cao
Volume $= \pi(e-2)$ ISW	A1	**6**	Answer as decimal value (only) $\rightarrow$ A0
Alternative method when subst. $u = \ln x$ used			
Attempt to connect dx and du	M1		
Becomes $\int u^2 e^u\,du$	A1		
First stage $u^2e^u - \int 2u\,e^u du$	A1		
Third stage $(u^2 - 2u + 2)e^u$	A1		
Final A1 A1 available as before			
(b) Indication that reqd vol = vol cylinder – vol inner solid	M1		
Clear demonstration of either vol of cylinder being πe^2 (including reason for height $= \ln e$) or rotation of $x = e$ about the y-axis (including upper limit of $y = \ln e$)	A1		Could appear as $\pi\int_0^1 e^2\,dy$
$(\pi)\int x^2\,dy = (\pi)\int e^{2y}dy$	B1		
$\frac{\pi(e^2+1)}{2}$ or 13.2 or 13.18 or better	B1	**4**	May be from graphical calculator
		$\boxed{13}$	

Possible helpful points

1. M is Method; does the candidate know what he/she should be doing? It does not ask how accurate it is..
   e.g. in Qu.4, a candidate saying $\frac{dx}{d\theta} = -\frac{1}{3}\cos\theta$ is awarded M1.
2. When checking if decimal places are acceptable, accept both rounding & truncation.
3. In general we ISW unless otherwise stated.
4. The symbol √ is sometimes used to indicate 'follow-through' in this scheme.

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