

Mathematics

Advanced GCE

Unit **4736**: Decision Mathematics 1

Mark Scheme for June 2011

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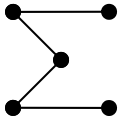
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1	(i)	$y \geq x$ $x + y \leq 8$ $y \leq 2x$	M1 A1	[2]	Any two boundary lines correct (any form) All three inequalities correct (any form)	Allow = or wrong inequality sign for method mark Ignore $x \geq 0, y \geq 0$, if given
	(ii)	$(2\frac{2}{3}, 5\frac{1}{3})$ $P_2 = 13\frac{1}{3}$	B1 B1	[2]	Coordinates of point A seen, cao, fractions or 2.6 to 2.7 and 5.3 to 5.4 cao, fractions or 13.3 to 13.4	Must be written down, allow $x = 2\frac{2}{3}, y = 5\frac{1}{3}$ or as decimals 2.6 to 2.7, 5.3 to 5.4 BOD if not identified as A or as optimal
	(iii) a b	$m \geq 1$ $m \leq 1$	B1 B1	[2]	Condone > 1 Condone < 1	Do not accept 1, 2, 3 ... (integer valued) Do not accept 0, -1, ... (integer valued) $m = 1$ is not enough for either mark

2	(i)	<table><tr><th>R</th><th>S</th></tr><tr><td>8</td><td>5</td></tr><tr><td>5</td><td>4.1</td></tr><tr><td>4.1</td><td>4.0012 (4.00)</td></tr><tr><td>4.0012 (4.00)</td><td>3.9988 (4.00)</td></tr></table> Output 4.00	R	S	8	5	5	4.1	4.1	4.0012 (4.00)	4.0012 (4.00)	3.9988 (4.00)	M1	[2]	(N = 16) 8, 5 and 4.1 seen	Need not be in table form
R	S															
8	5															
5	4.1															
4.1	4.0012 (4.00)															
4.0012 (4.00)	3.9988 (4.00)															
	(ii)	<table><tr><th>R</th><th>S</th></tr><tr><td>1</td><td>1.5</td></tr><tr><td>1.5</td><td>1.4167 (1.42)</td></tr><tr><td>1.4167 (1.42)</td><td>1.4142 (1.41)</td></tr><tr><td>1.4142 (1.41)</td><td>1.4142 (1.41)</td></tr></table> Output 1.41	R	S	1	1.5	1.5	1.4167 (1.42)	1.4167 (1.42)	1.4142 (1.41)	1.4142 (1.41)	1.4142 (1.41)	M1 A1	[2]	(N = 2) 1, 1.5 and 1.4167 seen to at least 2 dp accuracy (ie accept 1, 1.5, 1.42) Final value (of R) 1.41 (or anything that rounds to 1.41) after 4 iterations	Need not be in table form 1.4166... = $\frac{17}{12}$ as a fraction Watch out for truncation on S = 1.4167
R	S															
1	1.5															
1.5	1.4167 (1.42)															
1.4167 (1.42)	1.4142 (1.41)															
1.4142 (1.41)	1.4142 (1.41)															
	(iii)	Output is the square root of input (to 2 dp)	B1	[1]	Square root, $\sqrt{\quad}$, root											
	(iv)	<table><tr><th>R</th><th>S</th></tr><tr><td>-2</td><td>0</td></tr><tr><td>0</td><td>undefined, error or ?</td></tr></table>	R	S	-2	0	0	undefined, error or ?	B1	[1]	(N = -4) Stops when R = 0 (without wrong working) in second iteration (division by 0) Do not allow continuing on to further R values	‘Division by 0’ $\Rightarrow$ B1 (without wrong working) Do not give mark for talking about square root of a negative number, unless evidence of division by 0 is also seen in working				
R	S															
-2	0															
0	undefined, error or ?															
	(v)	<table><tr><th>R</th><th>S</th></tr><tr><td>-1</td><td>0.5</td></tr><tr><td>0.5</td><td>-1.75</td></tr><tr><td>-1.75</td><td>-0.3036</td></tr><tr><td>-0.3036</td><td>3.1423</td></tr></table> The algorithm does not terminate (chaotic) Build in a stopping condition, such as counting how many times step 3 has been carried out	R	S	-1	0.5	0.5	-1.75	-1.75	-0.3036	-0.3036	3.1423	B1 B1	[2]	(N = -2) Values not necessarily shown, but if comments are made then they must be true for the <u>correct</u> values The values never agree (to 2 dp) Values keep changing/do not settle down Needs a stopping condition, so that algorithm works for $N \geq 0$ and stops if $N < 0$	Allow correct listed values (ignoring comments) -1, 0.5, -1.75, -0.30 ($-\frac{17}{56}$) (or better) Allow ‘oscillate’ Not ‘use modulus of N’ Not suggesting changes to ‘S =’ line Not ‘if $N < 0$ then ...’
R	S															
-1	0.5															
0.5	-1.75															
-1.75	-0.3036															
-0.3036	3.1423															

3	(i)	Must have an even number of odd vertices Total order must be even	B1	[1]	(Cannot have) three odd vertices (It would have) three odd nodes $15 \div 2 = 7.5$ and cannot have half an arc	Note: graph need not be simple or connected Do not consider descriptions of specific cases
	(ii)	The vertex of <u>order 5</u> would either connect to at least one of the other vertices twice or connect to itself Graph is not simple	B1 B1	[2]	Identifying that vertex of order 5 causes problem, with a valid reason ('order 5 and repeated arcs' or 'order 5 and loop') Stating 'simple' (simple cannot be achieved)	Not a description of a specific case 'simple graph with 5 vertices means no vertex can have order greater than 4' $\Rightarrow$ B1 B1 But not if also claim graph is not connected
	(iii)	5 arcs A tree with five vertices only has 4 arcs (<u>must be using number of arcs</u>)	B1 B1	[2]	$10 \div 2 = 5$ Tree has 4 arcs, or sum of vertex orders = 8 Explaining why 5 arcs means this is not a tree A tree has fewer arcs than vertices	$5 > n-1$ (but $n-1$ on its own is not enough) Not a description of a specific case
	(iv)	 A tree must have at least two nodes of order 1, an Eulerian graph has no odd nodes so has no nodes of order 1	B1 B1	[2]	Any tree with five vertices that is a 'string' Any of: Eulerian so all even orders Eulerian so no nodes of order 1 A cycle cannot be a tree Two (or more) regions so not a tree	Any of: A tree cannot contain a cycle A tree must have at least two nodes of order 1
	(v)	Would need either an Eulerian or a semi-Eulerian graph $\Rightarrow$ 0 or 2 odd nodes, but the graph has 6 odd nodes Need to repeat 2 arcs	B1 B1	[2]	Accept 'needs 0 odd nodes' or 'needs 2 odd nodes' or both Cannot have more than 2 odd nodes 2 (allow two arcs written, using four nodes, eg <i>AB</i> and <i>CD</i>)	Condone '6 odd nodes' Condone 'all odd' [Note: route need not be a closed cycle]

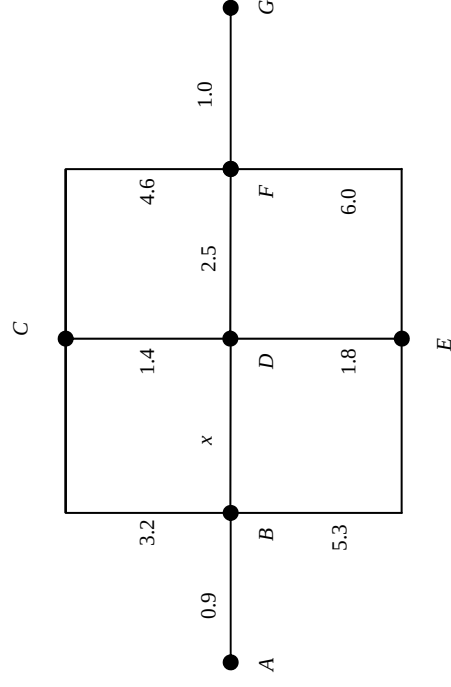
4	(i)	<table><tr><th>P</th><th>w</th><th>x</th><th>y</th><th>z</th><th>s</th><th>t</th><th>u</th><th>RHS</th></tr><tr><td>1</td><td>3</td><td>-5</td><td>7</td><td>-2</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>2</td><td>-2</td><td>-1</td><td>1</td><td>0</td><td>0</td><td>10</td></tr><tr><td>0</td><td>2</td><td>0</td><td>3</td><td>-4</td><td>0</td><td>1</td><td>0</td><td>12</td></tr><tr><td>0</td><td>4</td><td>5</td><td>1</td><td>0</td><td>0</td><td>0</td><td>1</td><td>30</td></tr></table> Pivot must be chosen from a column that has a negative entry in the objective row, so pivot cannot be chosen from the w column or the y column (both) Pivot entry cannot be negative or zero, there are no positive entries in the z column so the pivot cannot be chosen from the z column	P	w	x	y	z	s	t	u	RHS	1	3	-5	7	-2	0	0	0	0	0	1	2	-2	-1	1	0	0	10	0	2	0	3	-4	0	1	0	12	0	4	5	1	0	0	0	1	30	B1 B1 B1 B1	[4]	Objective row correct (3, -5, 7, -2) Constraint rows correct, with three slack variable columns x has a negative value (in the objective row) Identifying the value -5 BOD: x most negative (in objective row) Using signs of entries in columns to show that pivot cannot come from z column Identifying the values -1 and -4 Allow numbers in z column are 'negative' x is the <u>only</u> column with a negative in objective row and positives in column $\Rightarrow$ B1, B1	Tableau may be credited if seen in part (ii), but only if not attempted in part (i) Rows and columns may be in any order Condone P column missing here Only follow through their initial tableau if it leads to x as a valid pivot column But not just: x is the minimum value in the objective row (unless also say negative) Only follow through their initial tableau if it leads to x as the only valid pivot column, but values on objective row led to two (or more) possible pivot columns x column has a negative value in objective row and positives in column $\Rightarrow$ B1, B0
P	w	x	y	z	s	t	u	RHS																																											
1	3	-5	7	-2	0	0	0	0																																											
0	1	2	-2	-1	1	0	0	10																																											
0	2	0	3	-4	0	1	0	12																																											
0	4	5	1	0	0	0	1	30																																											
	(ii)	<table><tr><th>P</th><th>w</th><th>x</th><th>y</th><th>z</th><th>s</th><th>t</th><th>u</th><th>RHS</th></tr><tr><td>1</td><td>5.5</td><td>0</td><td>2</td><td>-4.5</td><td>2.5</td><td>0</td><td>0</td><td>25</td></tr><tr><td>0</td><td>0.5</td><td>1</td><td>-1</td><td>-0.5</td><td>0.5</td><td>0</td><td>0</td><td>5</td></tr><tr><td>0</td><td>2</td><td>0</td><td>3</td><td>-4</td><td>0</td><td>1</td><td>0</td><td>12</td></tr><tr><td>0</td><td>1.5</td><td>0</td><td>6</td><td>2.5</td><td>-2.5</td><td>0</td><td>1</td><td>5</td></tr></table> Decimals or fractions New row 2 = (row 2) $\div$ 2 New row 1 = row 1 + 5(new row 2) New row 3 = row 3 New row 4 = row 4 - 5(new row 2) (or equivalent, eg row 1 + 5/2$\times$row 2) w = 0, x = 5, y = 0, z = 0 and P = 25	P	w	x	y	z	s	t	u	RHS	1	5.5	0	2	-4.5	2.5	0	0	25	0	0.5	1	-1	-0.5	0.5	0	0	5	0	2	0	3	-4	0	1	0	12	0	1.5	0	6	2.5	-2.5	0	1	5	M1 A1 B1 ft B1	[4]	An augmented tableau with four basis columns, non-negative values in final column and value of objective having not decreased Correct tableau after one iteration (cao) Method seen and correct – usually in margin and abbreviated eg ①+5⑥, R1 + 5$\times$pivot, ①+5② (BOD new ②) but not just +5⑥ or + 5$\times$pivot (condone row 3 and/or pivot row not explained, but do not ignore if explained and wrong) Condone 0 values missing Ignore values for slack variables, if given	P column required in part (ii) A column of 0's $\Rightarrow$ M0 Rows and columns may be in any order Ignore any working lines (or interim tableau) Follow through their pivot choice, even if negative (but not if pivot value = 0) This B mark may be credited if seen in part (i), but only if not attempted in part (ii) x = 5, P = 25 (and others not written), not ft
P	w	x	y	z	s	t	u	RHS																																											
1	5.5	0	2	-4.5	2.5	0	0	25																																											
0	0.5	1	-1	-0.5	0.5	0	0	5																																											
0	2	0	3	-4	0	1	0	12																																											
0	1.5	0	6	2.5	-2.5	0	1	5																																											

(iii)	<table><tr><th>P</th><th>w</th><th>x</th><th>y</th><th>z</th><th>s</th><th>t</th><th>u</th><th>RHS</th></tr><tr><td>1</td><td>8.2</td><td>0</td><td>12.8</td><td>0</td><td>-2</td><td>0</td><td>1.8</td><td>34</td></tr><tr><td>0</td><td>0.8</td><td>1</td><td>0.2</td><td>0</td><td>0</td><td>0</td><td>0.2</td><td>6</td></tr><tr><td>0</td><td>4.4</td><td>0</td><td>12.6</td><td>0</td><td>-4</td><td>1</td><td>1.6</td><td>20</td></tr><tr><td>0</td><td>0.6</td><td>0</td><td>2.4</td><td>1</td><td>-1</td><td>0</td><td>0.4</td><td>2</td></tr></table> Decimals or fractions $w = 0, x = 6, y = 0, z = 2$ and $P = 34$ There is still a negative entry in the objective row <u>but</u> there is no further valid pivot choice The <u>coefficients</u> of z are never positive, so z can be increased without limit, and hence P can also be increased without limit	P	w	x	y	z	s	t	u	RHS	1	8.2	0	12.8	0	-2	0	1.8	34	0	0.8	1	0.2	0	0	0	0.2	6	0	4.4	0	12.6	0	-4	1	1.6	20	0	0.6	0	2.4	1	-1	0	0.4	2	M1 A1 B1 B1 B1	[5]	(answer to (ii) below on scan) An augmented tableau with four basis columns, non-negative values in final column and value of objective having not decreased Correct tableau after two iterations Condone 0 values missing Ignore values for slack variables, if given Have not reached optimum <u>and</u> cannot pivot further z can be arbitrarily large	P column required in part (iii) A column of 0's $\Rightarrow$ M0 Rows and columns may be in any order May be credited if seen in part (ii), but only if not attempted in part (iii), (unless crossed out) Ignore any working lines (or interim tableau) $x = 6, z = 2, P = 34$ (and others not written), not ft From the final tableau Explanation needs to be convincing From the original formulation Explanation needs to be convincing
P	w	x	y	z	s	t	u	RHS																																										
1	8.2	0	12.8	0	-2	0	1.8	34																																										
0	0.8	1	0.2	0	0	0	0.2	6																																										
0	4.4	0	12.6	0	-4	1	1.6	20																																										
0	0.6	0	2.4	1	-1	0	0.4	2																																										

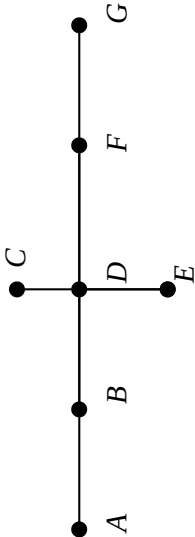
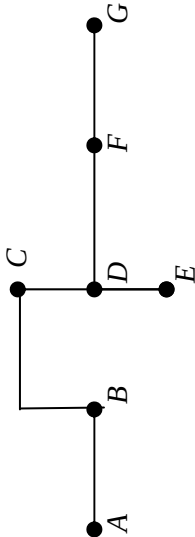
5	(i)	$A \ B \ C \ D \ E \ F \ G$ 9.0 km	M1 A1 M1 A1	[6]	At least four sets of temporary labels (excluding A) correct with no extras All temporary labels correct with no extras At least four permanent labels correct, (excluding A) All permanent labels correct (condone blank at A) and order of labelling correct	Temporary labels correct at (at least four of) B, C, D, E, F, G cao May be seen as permanent labels Permanent labels correct at (at least four of) B, C, D, E, F, G cao
(ii)		$x = 3.2 + 1.4 = 4.6$ $0.9 + x + 2.5 + 1.0 = 4.4 + x$	B1 B1	[2]	Not reversed 9	cao not ft, units not necessary
(iii)(a)		$x + 1.8 < 5.3 \Rightarrow x < 3.5$ Temporary labels are 6.2 and $2.7 + x$	B1 B1	[2]	4.6 seen (ignore what is said about it) $4.4 + x$	cao cao, not a specific case Expression must be simplified
(b)		$x + 1.4 < 3.2 \Rightarrow x < 1.8$ Temporary labels are 4.1 and $2.3 + x$	B1 B1	[2]	Condone 3.5 identified Condone $2.7 + x$ without 6.2 as well	3.5, even with wrong inequality Expressions must be simplified
(iv)		$20 \times \left(\frac{100}{50} \right)^2 = 80 \text{ seconds}$	M1 A1	[2]	Condone 1.8 identified Condone $2.3 + x$ without 4.1 as well	1.8, even with wrong inequality Expressions must be simplified
					Correct calculation 80 (or 1 minute 20 seconds)	Or equivalent correct method eg $100^2 \times 0.008$ Units not necessary

6	(i)				
		Need A and G odd so must pair C with E	B1	Recognising that <u>only</u> C and E need to be paired	May be implied from CE (CDE) seen in subsequent working (without AG) but not implied from 3.2 or from a route written
		$CE = 1.4 + 1.8 = 3.2$	M1	3.2 seen, without having to look for it in a list of additions	Allow $1.4 + 1.8$ seen (may be implied from 29.9) cao
		Shortest route has length $29.9 + x$	A1	29.9+x	
			[3]		

For reference:



Error on question paper. If you see any evidence that a candidate has lost out through this (needs to be a substantial attempt at this part) please check with your team leader. May need to mark crossed out work, even if it has been replaced. Credit work for (a) even when labelled as $x \geq 1.8$, and similarly for (b)

(iii)(a)	 Total weight = $7.6 + x$ km This is the shortest way to connect the vertices, but Shauna needs the shortest cycle so there must be at least one extra arc, and hence her solution must be longer than this	B1		Tree may be difficult to see on printed diagram, give BOD if necessary cao $7.6 + x$ Recognition that Shauna needs a cycle, needs to return to A, or similar Need to travel (some) arcs more than once	If drawn underneath assume labelling is as in question May list arcs instead Not ft Explaining why her solution must be longer than weight of tree Must repeat AB (or FG)
(b)	$A - B - D - C - F - G$ Misses out E	M1		cao (as far as G but not continuing past G) Saying that E is not visited	Must be written here, not just shown on diagram 'Stalls' on its own is not enough 'need to repeat B (or F)' is not enough
(iv)(a)	 Total weight = 10.8 km	B1	[2]	Tree may be difficult to see on printed diagram, give BOD if necessary cao 10.8	If drawn underneath assume labelling is as in question May list arcs instead Not ft
(b)	$A - B - C - D - E - F - G$ Total weight = 14.3 km Use shortcut $E - D - F$ instead of EF to get a total weight of 12.6 km Upper bound = $17.0 + x$	B1 B1		$ABCDEFG$ or AB, BC, CD, DE, EF, FG 14.3 Showing a valid shortcut ('shortcut EF' is not enough) $17 + x$	Must be written here, not just shown on diagram Need not give new weight, but it needs to be a shortcut ie $E-D-F$ Accept 21.6 or 25.2 or 28.6 as alternative upper bounds, but nothing else

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