

**ADVANCED GCE
MATHEMATICS**

Core Mathematics 3

QUESTION PAPER

4723

Candidates answer on the printed answer book.

OCR supplied materials:

- Printed answer book 4723
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Monday 13 June 2011

Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets [] at the end of each question or part question on the question paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

1 Find

(i) $\int 6e^{2x+1} dx,$

(ii) $\int 10(2x + 1)^{-1} dx.$

[5]

2 The curve $y = \ln x$ is transformed by:

a reflection in the x -axis,
followed by a stretch with scale factor 3 parallel to the y -axis,
followed by a translation in the positive y -direction by $\ln 4$.

Find the equation of the resulting curve, giving your answer in the form $y = \ln(f(x))$. [4]

3 (a) Given that $7 \sin 2\alpha = 3 \sin \alpha$, where $0^\circ < \alpha < 90^\circ$, find the exact value of $\cos \alpha$. [3]

(b) Given that $3 \cos 2\beta + 19 \cos \beta + 13 = 0$, where $90^\circ < \beta < 180^\circ$, find the exact value of $\sec \beta$. [5]

4 (i) Show by means of suitable sketch graphs that the equation

$$(x - 2)^4 = x + 16$$

has exactly 2 real roots. [3]

(ii) State the value of the smaller root. [1]

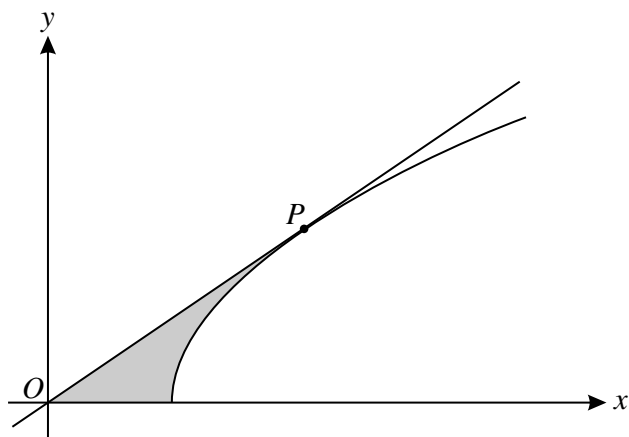
(iii) Use the iterative formula

$$x_{n+1} = 2 + \sqrt[4]{x_n + 16},$$

with a suitable starting value, to find the larger root correct to 3 decimal places. [4]

5 The equation of a curve is $y = x^2 \ln(4x - 3)$. Find the exact value of $\frac{d^2y}{dx^2}$ at the point on the curve for which $x = 2$. [8]

6



The diagram shows the curve with equation $y = \sqrt{3x - 5}$. The tangent to the curve at the point P passes through the origin. The shaded region is bounded by the curve, the x -axis and the line OP . Show that the x -coordinate of P is $\frac{10}{3}$ and hence find the exact area of the shaded region. [9]

7 The functions f , g and h are defined for all real values of x by

$$f(x) = |x|, \quad g(x) = 3x + 5 \quad \text{and} \quad h(x) = gg(x).$$

(i) Solve the equation $g(x + 2) = f(-12)$. [3]

(ii) Find $h^{-1}(x)$. [3]

(iii) Determine the values of x for which

$$x + f(x) = 0. \quad [2]$$

8 An experiment involves two substances, Substance 1 and Substance 2, whose masses are changing. The mass, M_1 grams, of Substance 1 at time t hours is given by

$$M_1 = 400e^{-0.014t}.$$

The mass, M_2 grams, of Substance 2 is increasing exponentially and the mass at certain times is shown in the following table.

t (hours)	0	10	20
M_2 (grams)	75	120	192

A critical stage in the experiment is reached at time T hours when the masses of the two substances are equal.

(i) Find the rate at which the mass of Substance 1 is decreasing when $t = 10$, giving your answer in grams per hour correct to 2 significant figures. [3]

(ii) Show that T is the root of an equation of the form $e^{kt} = c$, where the values of the constants k and c are to be stated. [5]

(iii) Hence find the value of T correct to 3 significant figures. [2]

[Question 9 is printed overleaf.]

9 (i) Prove that $\frac{\sin(\theta - \alpha) + 3 \sin \theta + \sin(\theta + \alpha)}{\cos(\theta - \alpha) + 3 \cos \theta + \cos(\theta + \alpha)} \equiv \tan \theta$ for all values of α . [5]

(ii) Find the exact value of $\frac{4 \sin 149^\circ + 12 \sin 150^\circ + 4 \sin 151^\circ}{3 \cos 149^\circ + 9 \cos 150^\circ + 3 \cos 151^\circ}$. [3]

(iii) It is given that k is a positive constant. Solve, for $0^\circ < \theta < 60^\circ$ and in terms of k , the equation

$$\frac{\sin(6\theta - 15^\circ) + 3 \sin 6\theta + \sin(6\theta + 15^\circ)}{\cos(6\theta - 15^\circ) + 3 \cos 6\theta + \cos(6\theta + 15^\circ)} = k. \quad [4]$$

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