

**ADVANCED GCE
MATHEMATICS**

Core Mathematics 4

QUESTION PAPER

4724

Candidates answer on the printed answer book.

OCR supplied materials:

- Printed answer book 4724
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

**Thursday 16 June 2011
Afternoon**

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets [] at the end of each question or part question on the question paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

1 Simplify $\frac{x^4 - 10x^2 + 9}{(x^2 - 2x - 3)(x^2 + 8x + 15)}$. [4]

2 Find the unit vector in the direction of $\begin{pmatrix} 2 \\ -3 \\ \sqrt{12} \end{pmatrix}$. [3]

3 (i) Find the quotient when $3x^3 - x^2 + 10x - 3$ is divided by $x^2 + 3$, and show that the remainder is x . [4]

(ii) Hence find the exact value of

$$\int_0^1 \frac{3x^3 - x^2 + 10x - 3}{x^2 + 3} dx. \quad [4]$$

4 Use the substitution $x = \frac{1}{3} \sin \theta$ to find the exact value of

$$\int_0^{\frac{1}{6}} \frac{1}{(1 - 9x^2)^{\frac{3}{2}}} dx. \quad [6]$$

5 The lines l_1 and l_2 have equations

$$\mathbf{r} = \begin{pmatrix} 4 \\ 6 \\ 4 \end{pmatrix} + s \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

respectively.

(i) Show that l_1 and l_2 are skew. [3]

(ii) Find the acute angle between l_1 and l_2 . [4]

(iii) The point A lies on l_1 and OA is perpendicular to l_1 , where O is the origin. Find the position vector of A . [3]

6 Find the coefficient of x^2 in the expansion in ascending powers of x of

$$\sqrt{\frac{1+ax}{4-x}},$$

giving your answer in terms of a . [8]

7 The gradient of a curve at the point (x, y) , where $x > -2$, is given by

$$\frac{dy}{dx} = \frac{1}{3y^2(x+2)}.$$

The points $(1, 2)$ and $(q, 1.5)$ lie on the curve. Find the value of q , giving your answer correct to 3 significant figures. [7]

8 A curve has parametric equations

$$x = \frac{1}{t+1}, \quad y = t - 1.$$

The line $y = 3x$ intersects the curve at two points.

(i) Show that the value of t at one of these points is -2 and find the value of t at the other point. [2]

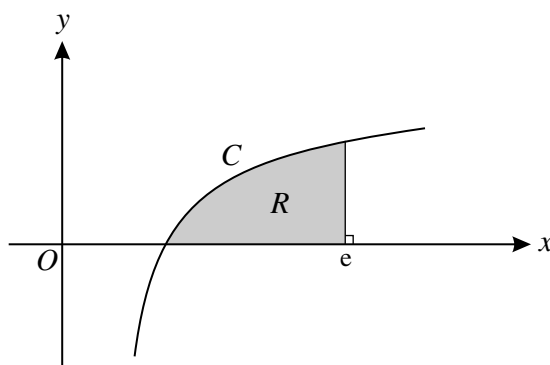
(ii) Find the equation of the normal to the curve at the point for which $t = -2$. [6]

(iii) Find the value of t at the point where this normal meets the curve again. [2]

(iv) Find a cartesian equation of the curve, giving your answer in the form $y = f(x)$. [3]

9 (i) Show that $\frac{d}{dx}(x \ln x - x) = \ln x$. [3]

(ii)



In the diagram, C is the curve $y = \ln x$. The region R is bounded by C , the x -axis and the line $x = e$.

(a) Find the exact volume of the solid of revolution formed by rotating R completely about the x -axis. [6]

(b) The region R is rotated completely about the y -axis. Explain why the volume of the solid of revolution formed is given by

$$\pi e^2 - \pi \int_0^1 e^{2y} dy,$$

and find this volume. [4]

**Copyright Information**

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.